

This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

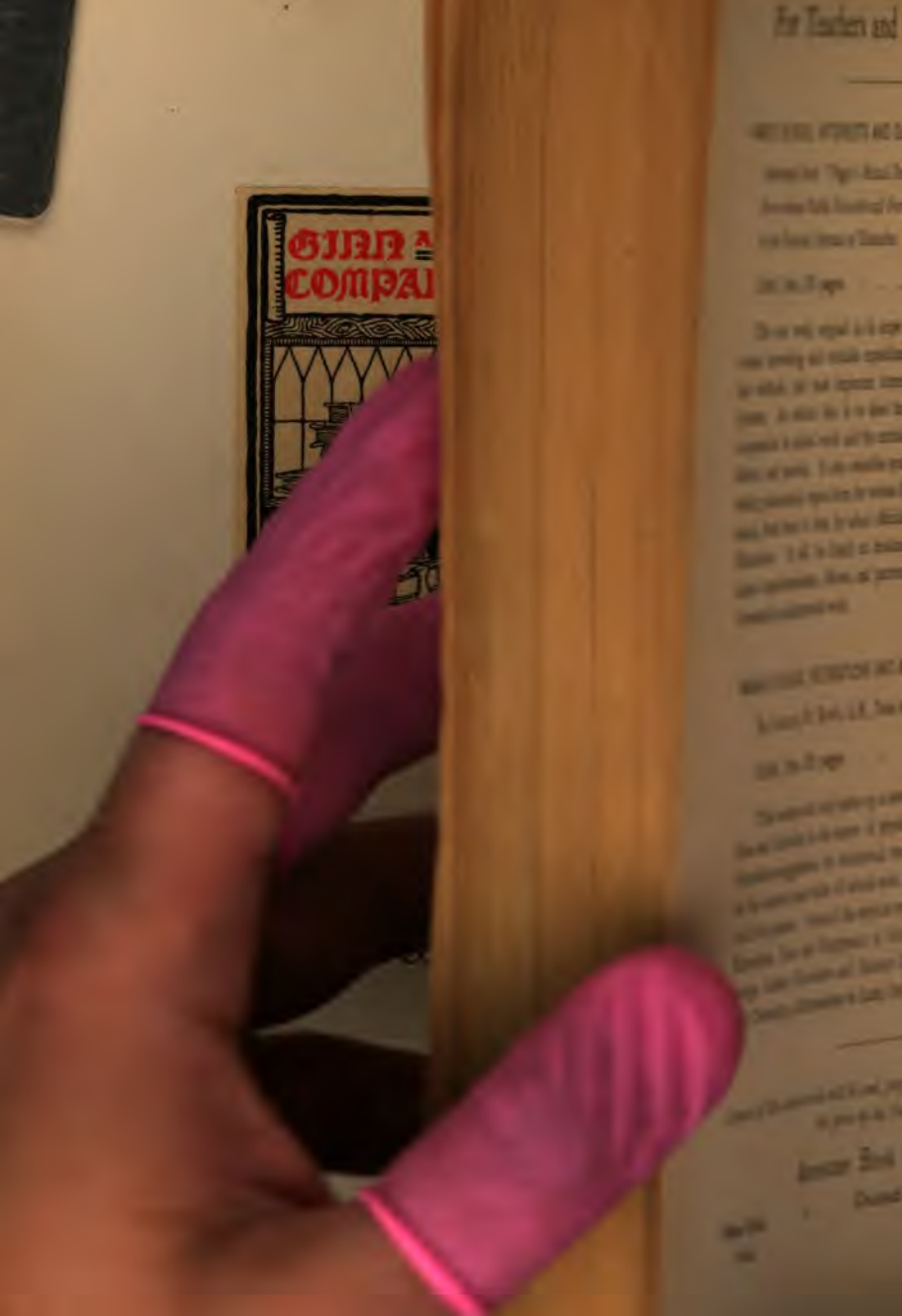
About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NEW PRACTICAL ARITHMETIC

DUBBS

AMERICAN · BOOK · COMPANY
NEW YORK · CINCINNATI · CHICAGO


GINN &
COMPA



CT 119.01.335



Harvard College Library
THE GIFT OF
GINN AND COMPANY


3 2044 097 004 790

NEW

PRACTICAL ARITHMETIC

BY

EUGENE L. DUBBS

NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

Educ T 119.01.335

MARCH

COPYRIGHT, 1901, BY
EUGENE L. DUBBS.

ENTERED AT STATIONERS' HALL, LONDON.

DUBBS'S NEW PRACT. AR.

E-P 1

PREFATORY NOTE

THE aim of this book is threefold: first, to cultivate habits of accuracy and rapidity in arithmetical computation; second, to develop the reasoning powers; and third, to make the pupils familiar with the ordinary commercial applications of arithmetic.

The chief means adopted to secure the end in view are clearness of explanation, conciseness of statement, and thoroughness of drill.

Each subject is introduced by carefully worded definitions which familiarize the students with the terminology. Then follows a statement of the principles involved in the processes to be developed. The next step is a model example with a written solution, accompanied by a clear explanation of the operations. When the process is thoroughly understood, it is generalized and fixed firmly in the mind by a briefly stated rule. After this follow the problems bearing on the subject. These, by their practical character as well as by their great number and variety, furnish the drill necessary to produce a ready skill in dealing with numbers as they present themselves in everyday life.

The matter throughout is carefully graded, and so arranged that each subject leads naturally to the one that follows. The frequency of reviews is a feature that will commend itself to all who realize that reiteration is an indispensable factor in the development of mathematical skill.

CONTENTS

	PAGE
NOTATION AND NUMERATION	7
Definitions	7
Arabic System of Notation	7
Roman System of Notation	11
ADDITION	13
Definitions	13
Addition of Simple Numbers	14
SUBTRACTION	21
Definitions	21
Subtraction of Simple Numbers	22
Addition and Subtraction	27
MULTIPLICATION	28
Definitions	28
Multiplication of Simple Numbers	29
DIVISION	35
Definitions	35
Short Division	36
Long Division	41
Contracted Division	47
GENERAL REVIEW	49
USE OF THE SIGNS	52
UNITED STATES MONEY	54
Definitions	54
Reduction	56
Addition	57
Subtraction	59
Multiplication	61
Division	64
Review of United States Money	67
MERCHANTS' BILLS	73
COMPOUND NUMBERS	75
Definitions	75
Reduction of Compound Numbers	75
Review Problems in Tables	103
Addition of Compound Numbers	108
Subtraction of Compound Numbers	112
Multiplication of Compound Numbers	117
Division of Compound Numbers	119
Longitude and Time	122
FACTORING	126
Definitions	126
Greatest Common Divisor	130
Least Common Multiple	133
Cancellation	136
FRACTIONS	139
Definitions	139
Reduction of Fractions	143
Addition of Fractions	152
Subtraction of Fractions	155
Multiplication of Fractions	158
Division of Fractions	163
Reduction of Fractional Concrete Numbers	173
Review of Fractions	178
DECIMAL FRACTIONS	188
Definitions	188

	PAGE
Notation and Numeration of Decimals	190
Reduction of Decimals	194
Addition of Decimals	197
Subtraction of Decimals	200
Multiplication of Decimals	202
Division of Decimals	205
Reduction of Decimal Concrete Numbers	209
Review of Decimals	212
THE METRIC SYSTEM	219
Definitions	219
Review of Metric System	225
PERCENTAGE	227
Definitions	227
Four Cases of Percentage	228
Review of Percentage	247
PROFIT AND LOSS	251
Definitions and Problems	251
TRADE DISCOUNT	255
Definitions and Problems	255
COMMISSION	259
Definitions and Problems	259
STOCK TRANSACTIONS	263
Definitions	263
Brokerage	264
Assessments and Dividends	265
Stock Values	267
Stock Investments	270
Review of Stocks	272
INSURANCE	275
Definitions and Problems	275
Life Insurance	278
TAXES	280
Definitions and Problems	280
Internal Revenue	286
Duties or Customs	287
INTEREST	289
Definitions	289
Simple Interest	291
Review of Simple Interest	311
Partial Payments	312
DISCOUNT	319
Definitions	319
Bank Discount	319
True Discount	329
EXCHANGE	333
Definitions	333
Domestic Exchange	334
Foreign Exchange	336
RATIO AND PROPORTION	339
Ratio	339
Simple Proportion	343
PARTNERSHIP	351
Simple Partnership	351
Bankruptcy	354
INVOLUTION	356
Definitions	356
EVOLUTION	359
Definitions	359
Square Root	360
Similar Figures	368
Review of Square Root	370
Cube Root	372
Similar Solids	379
Review of Cube Root	380
MENSURATION	383
Surfaces	383
Solids	395
Applications of Mensuration	402
MISCELLANEOUS PROBLEMS	407
Answers	421

NOTATION AND NUMERATION

DEFINITIONS

1. 1. A *unit* is a single thing.

2. A *number* is an expression denoting one or more units.

3. *Numbers* are either abstract or concrete.

4. An *abstract number* is one whose unit is not expressed; as, 7, 12, 40.

5. A *concrete number* is one whose unit is expressed; as, 4 men, 16 pounds, 25 dollars.

6. *Arithmetic* is the art of computing by figures.

7. *Notation* is the method of expressing numbers by *figures* or by *letters*. The former is called the *Arabic system;* the latter is termed the *Roman system.*

THE ARABIC SYSTEM OF NOTATION

2. The Arabic system of notation employs ten *figures;* namely, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first nine of these are called *significant figures*, because they express *value*. The last figure, "0," is called *naught, cipher*, or *zero*. It has no value, and is used merely to supply vacant orders.

3. 1. Figures have two values, simple and local.

2. The *simple* value of a figure is that which it repre-

sents when it stands alone, or when it occupies the units' order of a number.

3. The *local* value of a figure depends upon the position which the figure occupies in a number. Thus, in the number 453, the local value of the figure 4 is *four hundred*, and that of the figure 5 is *fifty*. In the number 608, the cipher has no value, but it is used to show that the tens' order is vacant.

4. The *order* of a figure is its position in a number.

4. 1. The figures of which numbers are composed are arranged in orders. A figure in the *first* order denotes *units;* in the *second* order, *tens;* in the *third* order, *hundreds;* in the *fourth* order, *thousands*, etc.

2. The *names* of the first ten numbers are *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*, *ten;* and the figures which represent them are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

3. Each successive number is greater by *one* than that which precedes it.

5. In the *Arabic system of notation, ten units of any order always make a unit of the next higher order.* That is, ten *units* make one *ten;* ten *tens* make one *hundred;* ten *hundreds* make one *thousand;* and so on to *tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, billions*, etc.

6. 1. A *period* is a group of *three figures*, comprising the hundreds', tens', and units' orders of any denomination.

REMARK. In learning to read and write large numbers, it is customary to separate the periods by commas. This is called *point-*

ing off into periods. It frequently happens that the *left* period contains only *one* or *two* figures, but all other periods must contain *three* figures.

2. The *names* of the first four periods are *units, thousands, millions, billions.*

7. The Arabic system of notation is illustrated by the following

TABLE

BILLIONS			MILLIONS			THOUSANDS			UNITS		
HUNDREDS	TENS	UNITS	HUNDREDS	TENS	UNITS	HUNDREDS	TENS	UNITS	HUNDREDS	TENS	UNITS
5	1	2,	8	3	7,	6	4	9,	2	5	0

The above number should be read: Five hundred and twelve billion eight hundred and thirty-seven million six hundred and forty-nine thousand two hundred and fifty.

RULE. *Begin at the left, and write the hundreds, tens, and units of each period, supplying all vacant periods and orders with ciphers.*

Write the following numbers:

1. One hundred and twenty-three; one hundred and five.
2. One hundred and fifty; two hundred and forty-one.
3. Three hundred and seventeen; four hundred and fifty-six.
4. Five hundred and thirty-eight; six hundred and nine.

5. Seven hundred and forty; eight hundred and sixty-two.

6. Nine hundred and seventy-four; one thousand five hundred.

7. One thousand and ten; six thousand two hundred and fifty.

8. Ten thousand six hundred and twenty-five; ten thousand and one.

9. Two hundred and forty thousand seven hundred and ninety-eight.

10. Eight million three hundred and twelve thousand and sixty.

11. Fifteen million nine hundred and seventy-six thousand one hundred.

12. Nine hundred and eighty-seven million six hundred and fifty-four thousand three hundred and twenty-one.

8. *Numeration* is the art of reading numbers when expressed by figures or by letters.

RULE. *Beginning at the right, point off the number into periods of three figures each; then, beginning at the left, read each period as a single number, adding the name of the period.*

REMARK. Omit the name of the units' period in reading numbers.

Read the following numbers:

1. 125, 207, 840, 1639, 2701, 3512, 4096.
2. 10250, 18324, 36517, 50608, 72005.
3. 187420, 256000, 349943, 403201, 560073.
4. 725096, 800080, 908070, 612735, 987654.
5. 1240625, 2504789, 3456007, 4010105, 5006007.

6. 6070080, 39260450, 70403029, 98765432.
7. 123456789, 300400500, 405607809, 528000364.
8. 10010, 1001, 10101, 100100, 1010001, 100001.
9. 8040, 2490127, 610024000, 803025, 75004.
10. 500016043, 7456912, 39056048, 1234567890.

THE ROMAN SYSTEM OF NOTATION

9. 1. The Roman system of notation uses the capital letters I, V, X, L, C, D, M to express numbers. It is chiefly employed in numbering the chapters of books, and is occasionally used to designate the years of the Christian era.

2. The *values* of the Roman numerals are as follows:

I denotes *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

3. By the use of combinations or repetitions of these letters any number may be expressed.

TABLE

I	one	XIV	fourteen	XC	ninety
II	two	XV	fifteen	C	one hundred
III	three	XVI	sixteen	CC	two hundred
IV	four	XVII	seventeen	CCC	three hundred
V	five	XVIII	eighteen	CCCC	four hundred
VI	six	XIX	nineteen	D	five hundred
VII	seven	XX	twenty	DC	six hundred
VIII	eight	XXX	thirty	DCC	seven hundred
IX	nine	XL	forty	DCCC	eight hundred
X	ten	L	fifty	DCCCC	nine hundred
XI	eleven	LX	sixty	M	one thousand
XII	twelve	LXX	seventy	MD	fifteen hundred
XIII	thirteen	LXXX	eighty	MDC	sixteen hundred

4. The *value* of a letter is repeated as often as the letter itself is repeated; as, XX, *twenty;* CCC, *three hundred.*

5. When a letter is written *before* another of *greater* value, the value of the former must be *subtracted* from that of the latter; as, V, *five;* IV, *four;* C, *one hundred;* XC, *ninety.*

6. When a letter is written *after* another of *greater* value, the value of the former must be added to that of the latter; as, X, *ten;* XV, *fifteen;* L, *fifty;* LX, *sixty.*

7. A *bar* placed over a number *multiplies its value by one thousand;* as, X, *ten;* $\overline{\text{X}}$, *ten thousand;* LIX, *fifty-nine;* $\overline{\text{LIX}}$, *fifty-nine thousand.*

Write the following numbers in the Roman notation:

1. 25, 28, 34, 39, 43, 46, 57.
2. 62, 71, 80, 99, 104, 111, 126.
3. 133, 144, 159, 167, 175, 181, 196.
4. 202, 318, 423, 567, 654, 729, 846.
5. 917, 1024, 2125, 3156, 4762, 5489.

Read the following numbers:

1. XXVII, XLIX, LIV, LXXVI, XCVIII, CXII, CXVI.
2. CLXIV, CLXXIX, CXCVI, CCIII, CCXLIX.
3. CCCXXXIII, DLV, DCXXV, DCCXLIV, DCCCL.
4. MCXI, MDCXV, MDCCLXXVI, MDCCCXC.
5. $\overline{\text{V}}$DCXXV, $\overline{\text{XV}}$CCL, $\overline{\text{XL}}$DXXIV, $\overline{\text{LXI}}$DXLVII.

ADDITION

DEFINITIONS

10. 1. All of the processes of arithmetic are based upon four *fundamental operations*. These are the operations of *addition*, *subtraction*, *multiplication*, and *division*.

2. *Addition* is the process of finding the *sum* of two or more numbers, or of uniting two or more numbers into one number.

3. The *sum* (or *amount*) is the result obtained by addition.

4. The *sign* of addition is +, called *plus*. It indicates that the numbers between which it is placed are to be united into one sum.

Thus, $1 + 3 + 5$ denotes that 1, 3, and 5 are to be added together.

5. The *sign of equality* is =. It is read, *equals*, or *is equal to*, and signifies that the quantities between which it is placed are *equal*.

Thus, $6 + 2 = 8$, and is read, *six plus two equals eight*.

6. A *simple number* is a single number of any kind; as, 7, 12 pounds, $100.

REMARK. The sign $ is read *dollar* or *dollars*.

ADDITION OF SIMPLE NUMBERS

11. Add 2457, 1680, 349, and 71.

OPERATION

$$\begin{array}{r} 2457 \\ 1680 \\ 349 \\ 71 \\ \hline 4557 \end{array}$$

EXPLANATION. Write the numbers, placing units of the same order in the same column. Then, commencing at the right and adding, 1 and 9 are 10, and 7 are 17 *units*, which are equal to 1 *ten* and 7 units. Write 7 under the units' column, and add 1 ten to the tens' column. 1 and 7 are 8, and 4 are 12, and 8 are 20, and 5 are 25 *tens*, which are equal to 2 *hundreds* and 5 tens. Write 5 under the tens' column, and add 2 hundreds to the hundreds' column. 2 and 3 are 5, and 6 are 11, and 4 are 15 *hundreds*, which are equal to 1 *thousand* and 5 hundreds. Write 5 under the hundreds' column, and add 1 thousand to the thousands' column; 1 and 1 are two, and 2 are 4 *thousands*. Write 4 under the thousands' column.

12. RULE. 1. *Write the numbers to be added, placing units of the same order in the same column.*

2. *Beginning at the right, add each column separately, and if the sum is less than ten, write it under the column added.*

3. *If the sum of the numbers in any column is ten or more, write the units' figure alone, and add the tens to the next column.*

4. *Write the entire sum of the numbers in the last column.*

PROOF. Begin at the units' column, and add downward.

REMARK. Should the sum of the numbers in any column end with a cipher, write 0 under that column, and proceed as before.

NOTE. Only numbers of the *same kind* can be added.

Add the following:

1.	2.	3.	4.	5.	6.
201	325	1042	2415	4250	52431
312	230	3524	3201	5637	20576
123	412	2130	4563	6012	83354
231	131	6321	6127	3408	75129

7.	8.	9.	10.	11.	12.
42735	71542	53627	97564	48625	86512
36287	32718	84901	38210	48625	74309
94521	64085	47385	25973	48625	68425
68273	89242	62019	84601	48625	93781
51842	53524	79483	53896	48625	76973

13.	14.	15.	16.	17.	18.
987654	872493	6437	8765	732659	654321
32109	657	560892	43210	84107	987654
8765	48362	365	987654	9362	321098
432	39	75249	32109	746	765432
10	7548	76	8765	64581	109876
9	963215	356981	402	987654	543210

19. 2374 + 9586 + 4107 + 6359 + 5248 + 7693 = what?

20. 538764 + 921058 + 743629 + 854302 + 672895 + 469382 = what?

21. 987 + 5643 + 21098 + 765432 + 10987 + 6543 + 210 = what?

22. 654321 + 78905 + 3456 + 298 + 5647 + 92834 + 769582 = what?

23. 34728 + 56 + 8372 + 541 + 927895 + 5283 + 94637 = what?

24. John weighs 64 pounds, James 73 pounds, George 85 pounds, and Robert 98 pounds. What is their total weight?

25. The contents of four cisterns are as follows: 284 barrels, 356 barrels, 409 barrels, and 538 barrels. What are their entire contents?

26. A man dying left his property to his four sons, giving to the oldest $3250, to the next $2750, to the third $2250, and to the youngest $1750. What was the value of his property?

27. Four vessels were loaded with the following quantities of wheat: 12,375 bushels, 10,625 bushels, 9750 bushels, and 11,125 bushels. What was the total quantity?

28. Five houses contained the following numbers of bricks: 28,564, 31,709, 42,680, 37,091, and 25,846. How many bricks were there altogether?

29. A steamer made six trips from New Orleans to Cincinnati with the following cargoes of sugar: 2385, 3417, 2691, 3056, 2948, and 3572 barrels. What was the total number of barrels?

30. Five companies of men bought public lands: the first, 7436 acres; the second, 8295 acres; the third, 11,748 acres; the fourth, 9654 acres; the fifth, 12,867 acres. How many acres were bought?

31. An oil refinery made six shipments of 4852, 5326, 6097, 7243, 6581, and 5794 gallons. How many gallons were shipped?

32. Seven divisions of an army contained respectively 13,750, 11,963, 12,487, 14,520, 15,839, 16,378, and 15,062 men. What was the entire number?

33. The population of a city in 1897 was 47,523, and during the next five years increased as follows: 1609, 974, 1456, 937, and 1265. What was the population in 1902?

34. The number of pupils enrolled in the six districts of a city is as follows: 847, 1053, 962, 1185, 876, and 1094. What was the total enrollment?

35. Seven barges were loaded with the following quantities of coal: 9483, 11,029, 8567, 10,391, 9876, 12,438, and 7912 bushels. What was the entire number?

36. A railroad company's receipts for five months were: $134,706, $127,485, $142,897, $138,659, and $129,378. What were the total receipts?

37. The area of the New England states in square miles is as follows: Maine, 33,040; New Hampshire, 9305; Vermont, 9565; Massachusetts, 8315; Rhode Island, 1250; Connecticut, 4990. What is their combined area?

38. A merchant built a block of stores on a lot worth $10,500. He paid $2756 for the foundation, $4139 for lumber, $12,487 for brick work, $3691 for plastering, $5235 for labor, and $7942 for other expenses. He then sold the property at a gain of $3250. How much did he receive for it?

39. The supplies of beef for an army in seven days were 31,469, 28,570, 30,625, 29,347, 31,096, 28,753, and 30,140 pounds. How much beef was consumed?

40. In 1892 the population of Ohio was 3,672,316; of Indiana, 2,192,404; of Illinois, 3,826,351; of Michigan, 2,093,889; and of Wisconsin, 1,686,880. What was the total population of the five states?

41. Write 142,857 seven times, and find the sum.

42. The taxable property of six towns was valued at $239,170, $196,540, $217,850, $184,230, $211,690, and $200,520. What was the entire valuation?

43. During the last eight days of a state fair the attendance was 32,753, 29,846, 30,527, 28,465, 31,980, 27,396, 34,672, and 36,109. Find the total attendance.

44. Seven boat loads of lumber measured respectively 73,649, 82,510, 67,438, 59,782, 85,247, 74,826, and 68,973 feet. Find the entire quantity.

45. A manufactory produced the following quantities of paper in six months: 251,916, 247,385, 239,847, 260,438, 228,659, and 253,764 pounds. What was the total product?

46. Eight tracts of land contained respectively 346,829, 275,164, 92,758, 68,572, 59,437, 87,945, 24,096, and 432,853 square feet. How many square feet were in all?

47. The water supply of a town for ten days was: 37,416, 29,850, 46,793, 58,967, 65,384, 70,548, 82,675, 94,231, 73,029, and 78,402 gallons. How many gallons were consumed?

48. A owns 120 acres of land, B has 24 acres more than A, C owns 31 acres more than B, and D has 26 acres more than C. How many acres have all?

49. A grain dealer sold 2764 bushels of wheat for 259,816 cents, 3128 bushels of corn for 153,272 cents, 4059 bushels of barley for 150,183 cents, and 5267 bushels of oats for 131,665 cents. How many bushels of grain did he sell, and what amount did he receive?

50. Five lots cost \$500, \$625, \$750, \$875, and \$1000, and the houses upon them cost \$1250, \$1750, \$2250, \$2750, and \$3250. A profit of \$375 was made on the first, \$500 on the second, \$625 on the third, \$750 on the fourth, and \$875 on the fifth. What amount was paid to the owners?

When long columns are to be added, the work should be performed *twice* to insure accuracy. In such cases it is best not to write the result before the proof is made, lest an error should require the alteration of one or more figures. The columns should be added in the usual way and the several results written in another column. The amount of the last column, with the right-hand figures of the column of totals annexed, will be the answer.

5462	
4793	
7621	
3584	
6945	
2376	
8467	
1259	
9782	
5473	
7645	
3287	
8596	
9879	
4351	
9788	
7962	95
4648	129
8977	118
120895	120

EXPLANATION. The sum of the numbers in the first column is 95; write 95, and add 9 to the second column. The sum is 129; write 129 under 95, and add 12 to the third column. The sum is 118; write 118 under 129, and add 11 to the fourth column. The sum is 120; write 120 under 118, and then add the columns downward. As the results are the same, *write the sum of the last column under that column, and annex the right-hand figures of the column of totals* (8, 9, 5) *in their proper order.*

Copy the following, and add:

51.	52.	53.	54.	55.	56.
396	5678	7249	98765	649327	893627
472	8901	865	4320	856439	435
851	2345	3724	87654	532974	76294
639	6789	638	3201	763256	8473
247	1234	72	76543	427843	652948
764	5678	583	2987	385692	756
125	9012	9461	65432	978561	47
986	3456	89	748	294785	95862
492	7890	6375	4926	536429	589
648	2345	856	59754	748256	749625
753	6789	8947	8597	487624	68
874	7654	64	659	853962	97487
529	3210	793	72436	675483	5376
967	9876	5629	45869	329748	468942
435	5432	4852	81347	592857	859
691	4567	98	57625	768534	34
764	8910	7586	93781	546379	34765
843	2345	437	32576	987216	9876
562	6781	9889	68259	453962	58
987	9876	678	79864	875645	587293

SUBTRACTION

DEFINITIONS

13. 1. *Subtraction* is the process of finding the *difference* between two numbers.

2. The greater number is called the *minuend;* the smaller number is called the *subtrahend;* and the number left after subtraction is the *difference* or *remainder.*

3. The *sign* of subtraction is $-$, called *minus*, which means *less.* When placed between two numbers, it indicates that the number on the right of it is to be taken from the other number. Thus, $7 - 4 = 3$, and is read *seven minus four equals three.*

4. When two or more numbers are included in a *parenthesis*, ( ), or in a *vinculum*, $\overline{\quad}$, they are to be considered as a single quantity. Thus, $17 - (6 + 4) = 7$. This shows that the *sum* of 6 and 4 is to be subtracted from 17. Also, $16 - \overline{10 - 3} = 9$. This shows that the *difference* between 10 and 3 is to be subtracted from 16.

14. PRINCIPLES. 1. *The minuend is equal to the sum of the subtrahend and the remainder.*

2. *The subtrahend is equal to the difference between the minuend and the remainder.*

SUBTRACTION OF SIMPLE NUMBERS

15. In subtraction of simple numbers, when the value of each figure in the subtrahend is *less* than that of the corresponding figure in the minuend, the difference is written beneath them as follows:

What is the difference between 746 and 325?

OPERATION.
746 minuend
325 subtrahend
421 remainder

EXPLANATION. The subtrahend is written under the minuend, with units of the same order in the same column.

Commencing at the right, 5 from 6 leaves 1. Write 1 under the column of units. 2 from 4 leaves 2. Write 2 under the column of tens. 3 from 7 leaves 4. Write 4 under the column of hundreds. The remainder is 4 hundreds, 2 tens, and 1 unit, or 421.

16. When the value of any figure in the subtrahend is *greater* than that of the corresponding figure in the minuend, the operation is performed as follows:

Subtract 5847 from 8351.

OPERATION.
8351 minuend
5847 subtrahend
2504 remainder

EXPLANATION. As 7 units cannot be taken from 1 unit, borrow 1 ten from the 5 tens. Adding the 10 (units) to 1 unit, the sum is 11 units. Subtracting 7 units from 11 units, the remainder is 4 units, which is written under the column of units. As 1 ten has been taken from the 5 tens, there remain 4 tens. 4 tens from 4 tens leave naught, which is written under the column of tens. As 8 hundreds cannot be taken from 3 hundreds, borrow 1 thousand (or 10 hundreds) from the 8 thousands. Adding 10 hundreds to 3 hun-

dreds, the sum is 13 hundreds. Subtracting 8 hundreds from 13 hundreds, the remainder is 5 hundreds, which is written under the column of hundreds. As 1 thousand has been taken from the 8 thousands, there remain 7 thousands. 5 thousands from 7 thousands leave 2 thousands, which is written under the column of thousands. The remainder is 2 thousands, 5 hundreds, and 4 units, or 2504.

17. There is another method of subtraction, which is sometimes used. It depends on the principle that the same number added to both minuend and subtrahend does not alter the remainder.

Take 7689 from 10,453.

OPERATION.

10453 minuend
7689 subtrahend
2764 remainder

EXPLANATION. As 9 units cannot be taken from 3 units, add 1 ten or 10 units to 3 units making 13 units. Subtracting, 9 from 13 leaves 4. Having added 1 ten to the minuend, 1 ten must be added to the subtrahend, making 9 tens. Subtracting, 9 from 15 leaves 6. Add 1 (hundred) to the 6 hundreds. Subtracting, 7 from 14 leaves 7. Finally, add 1 (thousand) to the 7 thousands. Subtracting, 8 from 10 leaves 2, and the entire remainder is 2764.

18. RULE. 1. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

2. *Begin at the right, and if possible subtract the value of each figure of the subtrahend from that of the corresponding figure of the minuend, and write the remainder below.*

3. *When any figure of the subtrahend has a greater value than the corresponding figure of the minuend, add ten to the value of the latter, and subtract. Then, either subtract one from the next order of the minuend, or add*

one to the next order of the subtrahend, and proceed as before.

PROOF. Add the remainder to the subtrahend.

NOTE. Only numbers of the *same kind* can be subtracted.

1. From 875 take 460.
2. From 7962 take 3641.
3. From 9537 take 6475.
4. From 6041 take 2708.
5. From 8372 take 5916.
6. From 10,625 take 4978.
7. From 18,053 take 7954.
8. From 27,625 take 16,750.
9. From 34,192 take 26,847.
10. From 43,826 take 30,918.
11. From 50,000 take 39,899.
12. From 98,765 take 56,789.
13. From 101,010 take 95,385.
14. From 201,234 take 182,484.
15. Take 296,079 from 404,706.
16. Take 498,756 from 523,610.
17. Take 609,857 from 615,243.
18. Take 456,789 from 987,654.
19. Take 304,052 from 1,020,301.
20. Take 98,765 from 4,615,203.

21. What is the difference between one hundred thousand two hundred and fifty, and seventy-six thousand three hundred and seventy-five?

22. How much greater is six hundred twenty-five thousand four hundred and twelve, than 62,542?

23. How much less is nine hundred thirteen thousand eight hundred and sixty-nine, than 1,234,567?

24. How much greater than 9,699,796 is 10 million 256 thousand 473?

25. How much less than four million 810 thousand 572 is 793,648?

26. What number must be added to 1,907,589 to make 2,031,045?

27. What number must be subtracted from 5,683,720 to make 3,926,740?

28. To what number must 5,094,876 be added to equal 8,162,735?

29. 28,436,051 minus 26,528,746 = how many?

30. 1,234,567 less 586,392 less 608,695 = what?

31. A merchant having $2745, spent $1495 for groceries, and the remainder he devoted to a charitable institution. Find how much the institution received.

32. Two ships sail from the same point for the same port, 5280 miles distant. When the first arrives, the other is 687 miles behind it. How far had the second sailed?

33. A dealer having 31,642 bushels of coal, sold a part, and had 10,792 bushels remaining. How much did he sell?

34. A merchant imported goods which he sold at a profit of $10,124. If he received $50,000 for them, how much did they cost?

35. The enrollment in the public schools of a city in 1901 was 59,824; in 1902, 62,371. What was the increase?

36. A starch factory consumed 43,967 bushels of corn in January, and 35,482 bushels in February. What was the difference?

37. A's fortune is $75,000, which is $14,625 more than B's. How much has B?

38. One reservoir contains 225,317 barrels of water, which is 87,293 barrels more than another contains. What are the contents of the second?

39. A sugar refinery shipped 180,460 pounds at one time, and 27,589 pounds less at another. What was the weight of the second shipment?

40. The area of the state of Texas is 265,780 square miles, and that of California is 158,360 square miles. What is the difference in their areas?

41. The Boston post office mailed 361,527 letters on Monday, which was 73,964 more than were mailed on Tuesday. How many were mailed on Tuesday?

42. 590,843 ties were used in building one railroad, and 452,769 in constructing another. How many more ties did the first require than the second?

43. One tract of government land contains 634,592 acres, and another contains 98,765 acres less. How many acres are in the second tract?

44. The population of the state of New York in 1890 was 5,997,853; that of Pennsylvania, 5,258,014. What was the difference in population?

45. The cotton crop of the United States in 1879 was 5,757,397 bales, which was 2,602,451 bales more than the crop of 1869. How many bales were produced in 1869?

ADDITION AND SUBTRACTION

19. 1. $425 + 396 - 539 + 718 =$ what?

2. $2604 - 1528 - 587 + 1086 =$ what?

3. $3750 - 1875 + 3125 - 1250 =$ what?

4. $53742 + 31860 - 46987 - 9876 =$ what?

5. $102010 - 56987 - 39416 + 6738 =$ what?

6. From 43,574 take the sum of 26,910 and 8732.

7. From 51,026 take the difference between 34,712 and 18,965.

8. From the sum of 76,839 and 51,025 take their difference.

9. A owns 125 acres of land, B owns 26 acres less than A, and C owns 64 acres less than A and B together. How many acres does C own?

10. Charles I. was born in the year 1600. Shakespeare was born 36 years earlier, and died on the anniversary of his birth in 1616. How old was he?

11. A merchant began business with $18,750. His profits for the first year were $3750, and for the second year $1250 less than the first. How much money was he then worth?

12. A bookseller's stock on December 31 was 10,594 volumes. He bought 2376 volumes in January, 1948 in February, and 3095 in March. His sales for the same months were 3427, 2851, and 3690 volumes. How many volumes remained?

13. A has 3250 bushels of wheat, B has 625 bushels less than A, and C has 1750 bushels less than A and B together. How many bushels have all?

MULTIPLICATION

DEFINITIONS

20. 1. *Multiplication* is the process of taking a number as many times as there are units in another number.

2. The *multiplicand* is the number that is to be multiplied.

3. The *multiplier* is the number by which to multiply. It shows *how many times* the multiplicand is to be taken.

4. The *product* is the result of multiplication.

5. The multiplicand and multiplier are called the *factors* of the product.

6. The *sign* of multiplication is $\times$. When placed between two numbers it indicates that they are to be multiplied together. Thus, $5 \times 2 = 10$, and is read, *five multiplied by two equals ten.*

21. PRINCIPLES. 1. *If either factor is taken for the multiplier, the result will not be changed.*

2. *The multiplier is always an abstract number.*

3. *When the multiplicand is a concrete number, the product is always of the same denomination as that of the multiplicand.*

MULTIPLICATION OF SIMPLE NUMBERS

22. When the multiplier consists of one figure.

Multiply 432 by 5.

OPERATION.
432 multiplicand.
5 multiplier.
2160 product.

EXPLANATION. Write the multiplier under the units' figure of the multiplicand. Beginning at the right and multiplying, 5 times 2 (units) are 10 (units), or 1 *ten* and no units. Write a cipher in the units' order in the product, and reserve 1 ten to add to the next product. 5 times 3 (tens) are 15 (tens), and 1 ten added are 16 (tens), or 1 *hundred* plus 6 tens. Write 6 in the tens' order, and reserve 1 (hundred) to add to the next product. 5 times 4 (hundreds) are 20 (hundreds), and 1 (hundred) added are 21 (hundreds), or 2 *thousands* plus 1 hundred. Write 1 in the hundreds' order and 2 in the thousands' order. The product is 2160.

RULE. 1. *Write the multiplier under the units' figure of the multiplicand, and draw a line beneath them.*

2. *Beginning at the right, multiply the value of each figure in the multiplicand by the multiplier, and carry as in addition.*

Multiply:

1. 523 by 2.
2. 462 by 3.
3. 617 by 4.
4. 356 by 5.
5. 784 by 6.
6. 875 by 7.
7. 984 by 8.
8. 2361 by 9.
9. 6471 by 3.
10. 7825 by 4.
11. 24673 by 5.
12. 49156 by 6.
13. 62738 by 7.
14. 74598 by 8.
15. 98765 by 9.

16. 769 boxes of starch each contained 8 five-pound packages. What was the total weight?

23. When the multiplier consists of two or more figures.

Multiply 356 by 412.

OPERATION.

356	multiplicand.
412	multiplier.
712	1st partial product.
356	2d partial product.
1424	3d partial product.
146672	entire product.

EXPLANATION. For convenience, write the orders of the multiplier under the corresponding orders of the multiplicand. Beginning at the right, multiply the multiplicand by each figure of the multiplier, taken in order. The first partial product is 712 *units*; the second, 356 *tens*; and the third, 1424 *hundreds*. Write the right-hand figure of the first partial product in *units'* order; the right-hand figure of the second partial product in *tens'* order; and the right-hand figure of the third partial product in *hundreds'* order. That is, *the right-hand figure of each partial product must be written directly beneath the order which produced it.* Add the several partial products. Their sum will be the entire product, which is 146,672.

GENERAL RULE. 1. *Write the multiplier under the corresponding orders of the multiplicand, and draw a line beneath them.*

2. *Beginning with units, multiply the multiplicand by each figure of the multiplier taken in order, placing the right-hand figure of each partial product directly beneath that figure of the multiplier which produced it.*

3. *Find the sum of the partial products. The result will be the required product.*

PROOF. Multiply the multiplier by the multiplicand.

REMARK. When the multiplier has ciphers between its significant figures, their products will be zeros, and should not be written.

1. $142 \times 15 = ?$
2. $203 \times 24 = ?$
3. $346 \times 37 = ?$
4. $458 \times 46 = ?$
5. $587 \times 53 = ?$
6. $629 \times 68 = ?$
7. $762 \times 79 = ?$
8. $875 \times 84 = ?$
9. $964 \times 92 = ?$
10. $999 \times 99 = ?$
11. $1250 \times 128 = ?$
12. $2473 \times 245 = ?$
13. $3684 \times 317 = ?$
14. $4129 \times 456 = ?$
15. $5625 \times 572 = ?$
16. $6483 \times 625 = ?$
17. $7294 \times 729 = ?$
18. $8376 \times 863 = ?$

19. $9547 \times 908 = ?$
20. $6250 \times 512 = ?$
21. $10296 \times 245 = ?$
22. $17468 \times 392 = ?$
23. $25389 \times 476 = ?$
24. $42637 \times 519 = ?$
25. $56250 \times 648 = ?$
26. $2075 \times 1463 = ?$
27. $3407 \times 2056 = ?$
28. $4260 \times 3175 = ?$
29. $3125 \times 4096 = ?$
30. $9876 \times 6789 = ?$

31. How many packages of baking powder are there in 25 boxes, each containing 24 packages?

32. Find the cost of 48 acres of land at $\$45$ an acre.

33. A company of soldiers consumed 160 pounds of beef daily for 63 days. How many pounds were required?

34. How many bushels of potatoes could be raised on 78 acres at the rate of 104 bushels per acre?

35. What is the value of 96 barrels of flour at 475 cents a barrel?

36. A city lot has a frontage of 89 feet, and is valued at $\$325$ a front foot. How much is the lot worth?

37. If a pipe discharges 257 gallons in one minute, how many gallons will it discharge in 108 minutes?

38. There are 480 sheets of paper in a ream. How many sheets are there in 357 reams?

39. An express train made daily trips between two cities 468 miles apart. How many miles did it run in 312 days?

40. A tract of land was divided into 248 building lots, which were sold at $425 each. What sum was realized?

41. A crop of tobacco filled 239 hogsheads of 547 pounds each. What was the entire weight?

42. There are 1440 minutes in one day. How many minutes are there in 365 days?

43. An edition of a dictionary comprised 8765 volumes of 973 pages each. What was the entire number of pages?

44. If the water supply for a village is 16,384 gallons daily, how many gallons would be used in 279 days?

45. How many ems can a printer set in 26 days of 9 hours each, at the rate of 876 ems per hour?

46. 125 acres of corn yielded 63 bushels per acre. If the entire crop was sold at 49 cents a bushel, how much was received for it?

47. If a man can plant 1250 hills of corn in a day, how many hills could 16 men plant in 25 days?

48. A wholesale house imported 192 bolts of broadcloth, each containing 36 yards, all of which was sold at 275 cents a yard. What amount was received for it?

49. 28 cars were each loaded with 480 boxes of starch, weighing 42 pounds to the box. What was the total weight?

50. Find the value of 864 bales of cotton, each weighing 459 pounds, at 27 cents a pound.

51. If a laborer can dig 19 cubic feet of ditch in an hour, how many cubic feet can 87 laborers dig in 75 days of 9 hours each?

52. If there are 640 peas in a pint, how many peas would fill 625 barrels each containing 2 bushels, allowing 64 pints to each bushel?

53. There are 9 square feet in one square yard, and 4840 square yards in one acre. How many square feet are there in twelve farms each containing 36 acres?

54. A party of 48 gold miners worked 52 weeks of 6 days each, at the rate of 12 hours per day. If each one gathered 32 grains per hour, what was the value of all their gold at 5 cents a grain?

55. An encyclopedia comprises 25 volumes of 1250 pages, with 2 columns to a page, 64 lines to a column, and 25 letters to a line. How many pages, columns, lines, and letters does it include?

24. To multiply by 1 with one or more ciphers annexed.

Multiply 46 by 100.

OPERATION. $46 \times 100 = 4600$.

PRINCIPLE. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

25. To multiply when there are ciphers upon the right of either or both of the factors.

(*a*) Multiply 428 by 400.

OPERATION.
$$\begin{array}{r} 428 \\ 400 \\ \hline 171200 \end{array}$$

(*b*) Multiply 3600 by 480.

OPERATION.

$$\begin{array}{r} 3600 \\ 480 \\ \hline 288 \\ 144 \\ \hline 1728000 \end{array}$$

RULE. *Multiply the significant figures of the multiplicand by those of the multiplier, and to the product annex as many ciphers as are on the right of both factors.*

1. $729 \times 60 = ?$
2. $864 \times 300 = ?$
3. $1500 \times 450 = ?$
4. $4800 \times 720 = ?$
5. $5640 \times 6800 = ?$
6. $17500 \times 5700 = ?$
7. $31250 \times 3200 = ?$
8. $45000 \times 2500 = ?$
9. $68750 \times 4090 = ?$
10. $20350 \times 5460 = ?$

11. Find the value of 6250 bushels of wheat at 80 cents a bushel.

12. There are 5760 grains in a Troy pound. How many grains are there in 240 Troy pounds?

13. A laborer can dig 250 cubic feet of ditch in a day. How many cubic feet could 250 such laborers dig in 160 days?

14. If a machine can make 120 square feet of paper in a minute, how many square feet could it make in 150 days of 1440 minutes each?

15. What would be the value of 1560 bales of cotton each weighing 480 pounds, at 20 cents a pound?

DIVISION

DEFINITIONS

26. 1. *Division* is the process of finding how many times one number is contained in another number, or of separating a number into a given number of equal parts.

2. The *dividend* is the number to be divided.

3. The *divisor* is the number by which the dividend is divided.

4. The *quotient* is the result obtained by division. It shows the *number of times* the divisor is contained in the dividend.

5. A *remainder* is that part of the dividend remaining when the division is not exact.

6. The *sign* of division is $\div$. When placed between two numbers it indicates that the number on the left of it is to be divided by the other number. Thus, $72 \div 8 = 9$, and is read, 72 *divided by* 8 *equals* 9.

27. Division is usually indicated by writing the divisor at the left of the dividend, and drawing a curved line between them. Thus, 72 divided by 8 is written 8)72. It may also be expressed in the form of a fraction by writing the divisor under the dividend, and drawing a short line between them. Thus, $\frac{72}{8}$ indicates that 72 is to be divided by 8.

28. As two factors multiplied together form a product, it is evident that *if the product be divided by either of those factors the result will be the other factor.* Hence division is the reverse of multiplication. In the latter, two factors are given to find a product; in the former, a product and one of its factors are given to find the other factor. The *dividend* corresponds to the *product* and the *divisor* and *quotient* to the *multiplicand* and *multiplier.*

29. PRINCIPLES. 1. *When the divisor and the dividend are of the same denomination the quotient is an abstract number.*

2. *If the divisor is an abstract number, the dividend and the quotient are of the same denomination.*

3. *The dividend is equal to the product of the divisor and the quotient, plus the remainder, if there is any.*

SHORT DIVISION

30. *Short division* is the method of performing the operation mentally, and writing only the result. This method is used when the divisor is not greater than 12. Each successive figure of the quotient is written directly under that part of the dividend from which it is derived, without writing the several multiplications and subtractions.

Divide 875 by 5.

Divisor Dividend

OPERATION. $5\overline{)875}$

$\underline{175}$ quotient

EXPLANATION. 875 may be regarded as the *sum of three numbers*, 800, 70, 5. 5 is contained in 800, 160 times; in 70, 14 times; in 5, 1 time. Therefore it is contained in 875, $(160 + 14 + 1) = 175$ times.

31. Short division may also be performed by fractional analysis.

Divide 984 by 4.

Divisor Dividend
OPERATION. 4)984
246 quotient

EXPLANATION. One fourth of 9 (hundreds) is 2 (hundreds) with a remainder of 1 (hundred). Write 2 in hundreds' order in the quotient, and add the 1 hundred (or 10 tens) to the 8 tens, making 18 tens. One fourth of 18 (tens) is 4 (tens), with a remainder of 2 (tens). Write 4 in tens' order in the quotient, and add the 2 tens (or 20 units) to the 4 units, making 24 units. One fourth of 24 units is 6 units. Write 6 in units' order. The quotient is 246.

32. When the divisor is not contained in the dividend an exact number of times, there is a remainder which is written over the figure of the divisor, directly after the last figure of the quotient.

Divide 9978 by 7.

OPERATION. 7)9978
1425$\frac{3}{7}$

RULE. 1. *Write the divisor at the left of the dividend, with a curved line between them, and draw a line under the dividend.*

2. *Beginning at the left, divide the value of each figure of the dividend in succession, and write the result in the same order in the quotient.*

3. *Should a remainder occur after any division, prefix it to the next figure of the dividend, and divide as before.*

4. *When the number in any order of the dividend is less than the divisor, write a cipher in the quotient, prefix*

the number to the next figure of the dividend, and continue the division.

5. *If there is a remainder after dividing the last figure, write it over the figures of the divisor, directly after the last figure of the quotient.*

PROOF. Multiply the quotient by the divisor, and when there is a remainder, add it to the product. (Art. 29, 3.)

Divide:

1.	846 by 2.	**7.**	5231 by 8.	**13.**	315924 by 7.
2.	954 by 3.	**8.**	6709 by 9.	**14.**	434568 by 8.
3.	1260 by 4.	**9.**	74352 by 3.	**15.**	562500 by 9.
4.	2375 by 5.	**10.**	81496 by 4.	**16.**	617283 by 10.
5.	3084 by 6.	**11.**	93708 by 5.	**17.**	793485 by 11.
6.	4578 by 7.	**12.**	256427 by 6.	**18.**	984720 by 12.

19. What is one fourth of 3,627,500?

20. What is one fifth of 1,728,493?

21. What is one sixth of 2,491,578?

22. What is one seventh of 4,379,375?

23. What is one eighth of 5,664,723?

24. What is one ninth of 7,888,887?

25. What is one tenth of 1,010,101?

26. There are 2 pints in one quart. How many quarts are in 256 pints?

27. Three feet make a yard. How many yards are there in 2961 feet?

28. Four men bought a farm for $5940. How much did each pay?

29. Six equal loads of hay weighed 12,960 pounds, what was the weight of each load?

30. How many eight-pound balls would be equal in weight to a mass of iron weighing 42,608 pounds?

31. How many $5 bills are equal in value to $31,250?

32. Seven city lots cost $21,875. What was their average price?

33. 78,876 bricks were placed in nine equal piles. How many were there in each pile?

34. A certain number multiplied by six gave 93,750 for the product. What was the number?

35. If a pipe discharges eight gallons in a second, in how many seconds will it discharge 100,000 gallons?

36. A fortune of $410,751 was equally divided among nine heirs. What was the share of each?

37. 271,082 passengers were carried over a railroad in seven months. What was the monthly average?

38. If 8 is the multiplier and 612,344 the product, what is the multiplicand?

39. If a wagon-wheel passes over eleven feet in making one revolution, in how many revolutions will it traverse 690,635 feet?

40. I bought 302,976 feet of lumber. If each board was twelve feet long, what was the number of boards?

41. Five men laid 48,750 bricks in 6 days. How many bricks did each lay daily?

42. Eight cars were loaded with sacks of salt, each weighing 4 pounds. If the entire weight was 171,648 pounds, how many sacks were there in each car?

43. Seven laborers were employed for 6 months, and their wages amounted to 191,100 cents. How much did each earn monthly?

44. Nine pecks of beans were put into a barrel. There are 8 quarts in a peck. If the total number of beans was 100,008, how many were there in one quart?

45. Eleven dray-loads of flour, of 7 barrels each, were placed in each of six cars. If the entire quantity weighed 99,792 pounds, what was the weight of every barrel?

46. A certain number of government horses consumed 201,216 bushels of corn in 12 months. If the monthly allowance of each was 8 bushels, how many horses were there?

47. A tract of land was divided into 9 blocks, and each block into 12 lots. If the lots were sold for 337,500 cents, what was the average selling price of the lots?

48. A vessel made 7 trips a month for 12 months between two cities. If the entire distance traveled was 113,400 miles, how far apart were the cities?

49. 360,448 nails were used in shoeing the horses of 11 cavalry regiments. If each shoe required 8 nails, how many horses were in each regiment?

50. A quantity of starch was packed in boxes, and sold for 952,080 cents. There were 8 packages of 6 pounds each in every box, and the entire quantity was sold at 5 cents a pound. How many boxes were there?

LONG DIVISION

33. *Long division* is the process of writing the entire work of division, and is usually employed when the divisor is greater than 12.

Divide 8192 by 32.

Divisor Dividend Quotient

OPERATION. 32)8192(256

64

179

160

192

192

EXPLANATION. 32 is not contained in 8 (thousands), therefore the next figure of the dividend, 1 (hundred), is united to 8 (thousands), making 81 (hundreds). 32 is contained in 81 (hundreds) 2 (hundreds) times. Write 2 (hundreds) in the quotient, and multiply the divisor by it. The product is 64 (hundreds), which is written under 81 (hundreds) and subtracted. The remainder is 17 (hundreds), to which the next figure of the dividend, 9 (tens), is united, making 179 (tens). 32 is contained in 179 (tens) 5 (tens) times. Write 5 (tens) in the quotient, and multiply the divisor by it. The product is 160 (tens), which is written under 179 (tens) and subtracted. The remainder is 19 (tens), to which the next figure of the dividend, 2 (units), is united, making 192 (units). 32 is contained in 192 (units) 6 (units) times. Write 6 (units) in the quotient, and multiply the divisor by it. The product is 192 (units), which equals the last partial dividend. Therefore, the quotient is 256.

RULE. 1. *Write the divisor at the left of the dividend, and draw a curved line between them.*

2. *Find how many times the divisor is contained in the fewest figures in the left side of the dividend that will contain it, and write the result in the quotient at the right of the dividend.*

3. *Multiply the divisor by this quotient figure, and write the product under the figures of the dividend already divided. Subtract this product from the partial dividend above it, and to the remainder annex the next figure of the dividend.*

4. *Divide as before, until all the figures of the dividend have been used.*

5. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

6. *When there is a remainder after dividing the last partial dividend, write it over the figures of the divisor directly after the last figure of the quotient.*

PROOF. The same as in short division.

Divide:

1. 1722 by 14.
2. 3690 by 15.
3. 6025 by 17.
4. 7416 by 18.
5. 10317 by 19.
6. 14375 by 23.
7. 18356 by 25.
8. 23968 by 28.
9. 31552 by 34.
10. 37888 by 37.
11. 90720 by 42.
12. 159691 by 46.
13. 240674 by 55.
14. 331875 by 59.
15. 375821 by 61.
16. 512818 by 67.
17. 586189 by 73.
18. 755586 by 78.
19. 698968 by 82.
20. 687822 by 87.
21. 604221 by 89.
22. 774720 by 96.
23. 977724 by 99.
24. 1020301 by 101.
25. 184500 by 246.
26. 261564 by 307.

27. 452814 by 463.
28. 559104 by 546.
29. 800000 by 625.
30. 1950972 by 734.
31. 3246915 by 867.
32. 4537088 by 958.
33. 5760000 by 1024.
34. 13707360 by 2160.
35. 24609375 by 3125.
36. 35794944 by 4096.
37. 27878400 by 5280.
38. 39778750 by 6875.
39. 51021564 by 7654.
40. 80000000 by 5120.

41. 26 bales of cotton weighed 12,350 pounds. What was their average weight?

42. 20,416 rails were used in laying 58 miles of track. How many rails were required for each mile?

43. 31,442 bushels of wheat were shipped in 79 cars. How many bushels were there in each car?

44. In grading a street, 63,406 cubic feet of earth were removed by 98 men. How much did each remove?

45. A railroad company paid $188,375 for 137 freight cars. What was the average value of the cars?

46. An author wrote a book containing 73,216 lines, at the rate of 286 lines per day. How long was he thus employed?

47. The product of two numbers is 257,821. One of them is 347. What is the other?

48. A vat holds 458 gallons. How many times can it be filled from a reservoir that contains 744,250 gallons?

49. $800,000 in gold was placed in sacks each containing $625. How many sacks were required?

50. A town was laid out in 748 lots, which were valued at $561,000. What was the average value of the lots?

51. 864 is contained in 598,751 a certain number of times with a remainder. What is the remainder?

52. A flouring mill's product for one week was shipped in 987 barrels, whose total weight was 213,192 pounds. How much did each barrel weigh?

53. The population of the state of Connecticut in 1890 was 627,000, which was an average of 132 people to each square mile. What is the area of that state?

54. A regiment of soldiers consumed 710,655 pounds of beef in 365 days. What was its daily allowance?

55. An express train ran a total distance of 206,784 miles at the rate of 576 miles daily. How many days were required?

56. There are 640 acres in one square mile. How many square miles are equal in area to 160,000 acres?

57. In a city school of 748 pupils, the expense for a certain year was 1,626,900 cents. What was the average expense per pupil?

58. If iron rails weigh 468 pounds apiece, how many rails would be required to lay a track whose total weight is 3,492,216 pounds?

59. The dividend is 4,054,016, and the quotient 856. What is the divisor?

60. $110,160 was paid for 1296 acres of land. What was the cost per acre?

61. There are 1760 yards in a mile. How many miles are equal to 1,138,720 yards?

62. 1,600,000 pounds of cotton were pressed into 3125 bales of equal size. What was the weight of each bale?

63. If the flouring mills of a city consume 7058 bushels of wheat daily, in how many days will they consume 2,209,154 bushels?

64. If $9,568,325 were equally divided among 6487 people, how much would each receive?

65. There are 5760 grains in a Troy pound. How many Troy pounds are there in 3,317,760 grains?

66. Seventy-eight laborers earned $84,240 in twenty-four months. What was the average monthly wages of each?

67. 1,310,400 bricks were laid by 32 workmen in 26 days. What was an average day's work for each?

68. A planter sold 496 hogsheads of molasses, each containing 63 gallons, for 1,406,160 cents. What was its value per gallon?

69. During a certain month a factory shipped 75 car loads of soap, whose net weight was 1,785,000 pounds. Allowing 56 pounds to a box, how many boxes were in each car?

70. A wholesale house imported 248 bolts of cloth, each measuring 34 yards, at a total cost of 1,652,672 cents. What was the cost per yard?

71. There are 160 square rods in an acre. If one square rod produces 25 pounds of hay, how many acres should produce one million pounds?

72. There are 144 pens in a gross. How many packages, each containing 144 gross, would be required for 2,985,984 pens?

73. An encyclopedia averages 672 pages to a volume and 168 lines to a page. If the entire number of lines is 2,709,504, how many volumes are there?

74. If a laborer can dig 24 cubic feet of ditch in an hour, how many laborers can dig 3,172,608 cubic feet in 216 days of 9 hours each?

75. 1728 cubic inches make a cubic foot, and 128 cubic feet make a cord. How many cords would be equal in volume to 21,897,216 cubic inches?

34. 1. Multiply 96 by 63, and divide the product by 84.

2. Divide 26,568 by the product of 18×12.

3. Multiply the sum of 312 and 168 by their difference, and divide the product by 135.

4. Divide the product of 216 times 392 by the product of 24×21.

5. If 96 acres of land produce 4320 bushels of barley, how many bushels should 85 acres produce?

6. A vessel sailed 17,472 miles in 56 days. At the same rate, how far could it sail in 45 days?

7. 69 city lots valued at $425 apiece were exchanged for land at $75 an acre. How many acres were required?

8. A railroad was constructed by 657 men in 285 days. In what time could 513 men have done the same work?

9. A dividend is 124,416, and the quotient 72. If the quotient were multiplied by 8, what would the divisor be?

10. An acre contains 4840 square yards. What is the value of 1,161,600 square yards of land at $50 an acre?

CONTRACTED DIVISION

35. When the divisor has ciphers on the right.

(*a*) Divide 2345 by 100.

OPERATION. $1|00\overline{)}23|45$

$23\frac{45}{100}$

EXPLANATION. Since division is the reverse of multiplication (Art. 28) a number is divided by 10, 100, 1000, etc., *by cutting off as many figures from the right of the dividend as there are ciphers in the divisor, and writing the figures cut off over the figures of the divisor, directly after the last figure of the quotient.*

(*b*) Divide 48,257 by 1500.

OPERATION. $15|00\overline{)}482|57(32\frac{257}{1500}$

$\begin{array}{r} 45 \\ \hline 32 \\ 30 \\ \hline 2 \end{array}$

EXPLANATION. Cutting off two figures from the right of the dividend divides it by 100. Dividing the remaining figures of the dividend by 15 the quotient is 32, with a remainder of 2 (hundreds). Annexing the figures cut off from the dividend (57) to 2 (hundreds) gives the true remainder, 257, which is written in the quotient over the figures of the divisor.

(*c*) Divide 94,300 by 2500.

OPERATION. $25|00\overline{)}943|00(37\frac{1800}{2500}$

$\begin{array}{r} 75 \\ \hline 193 \\ 175 \\ \hline 18 \end{array}$

EXPLANATION. The same as in the preceding example.

RULE. 1. *Cut off the ciphers at the right of the divisor, and the same number of figures from the right of the dividend.*

2. *Divide the figures remaining in the dividend by those remaining in the divisor.*

3. *Annex to the remainder the figures cut off from the dividend, and write the result in the quotient, over the figures of the divisor.*

Divide:

1. 1256 by 10.
2. 24349 by 100.
3. 367485 by 1000.
4. 4062317 by 10000.
5. 538624 by 100.
6. 6940718 by 1000.
7. 71523 by 40.
8. 846902 by 250.
9. 9573816 by 1800.
10. 3456789 by 5000.
11. 467200 by 80.
12. 1598400 by 900.
13. 6825000 by 1250.
14. 9435000 by 15000.

GENERAL REVIEW

36. 1. How much less will 148 mules cost at $38 each than 225 cows at $25 apiece?

2. If 1025 bushels of wheat cost 107,625 cents, how many bushels could be bought for 104,895 cents?

3. The sum of two numbers is 189, and the less is 64. What is the product of the two numbers?

4. A has 112 acres and B has 144 acres. If A buys one ninth of B's land, how many acres will each have?

5. Find the number which when divided by 225, will give a quotient of 224 and a remainder of 223.

6. I sold 248 cords of wood for 105,400 cents, thereby losing 6200 cents. For how much per cord should I have sold it to gain 7440 cents?

7. A has 287 sheep; B twice as many, less 209; C 303 less than A and B together. How many have all?

8. A cistern will hold 12,096 gallons of water. If it receives 1000 gallons per hour by one pipe, and discharges 748 gallons per hour by another, in what time will it be filled?

9. If a certain number is multiplied by 72, and 432 added to the product, the sum divided by 528 will give 45 for the quotient. What is the number?

10. 168 men have provisions for 135 days. In how many days could 216 men consume them?

11. Two vessels are 12,540 miles apart and sail toward each other, one at the rate of 336 miles per day, the other at the rate of 324 miles per day. In how many days will they meet?

12. A trader bought an equal number of horses and mules for $9375. He paid $75 for a horse and $50 for a mule. How many of each did he buy?

13. The product of three numbers is 941,094. Two of them are 97 and 98. What is the third number?

14. A speculator sold a quantity of pork for 343,750 cents, thereby gaining 62,500 cents. If he received 1375 cents a barrel for it, what was the cost per barrel?

15. A contractor agreed to construct a building which would employ 108 men 120 days. 12 of the men failed to come. How long were the others employed?

16. How many sacks of coffee, each weighing 180 lb., at 18 cents a lb., would be equal in value to 48 chests of tea, each containing 36 lb., at 45 cents a lb.?

17. A vessel sailed 22,440 miles in 85 days. At that rate, how far could it sail in 96 days?

18. Express $\overline{\text{XC}}\text{I}\overline{\text{C}}\text{XXV}$ and MMXXV in the Arabic notation, divide the former by the latter, and express the quotient in the Roman notation.

19. 325 barrels of flour, each containing 196 pounds, were emptied into 35-pound sacks and sold at $1 per sack. How much was realized?

20. A drover paid $11,340 for cattle at $15 per head. 49 strayed away and 77 were stolen. At what price per

head did he sell the remainder if he gained $1260 on the cost of all?

21. What was the cost of 225 acres of land, if $625 was lost by selling 125 acres for $5625?

22. Divide the product of 729 times 864 by the difference between 20,025 and 18,729.

23. A man sold 385 hogsheads of tobacco at $99 each. He then borrowed enough money to buy 715 bales of cotton at $56 a bale. How much did he borrow?

24. If 257 barrels of wine cost $10,023, how many barrels would cost $14,235?

25. A man paid $12,150 for horses at $50 each. 18 died, and the others were sold at $54 apiece. What was the gain or loss on the whole transaction?

26. A and B engaged in business, each with $9375. A lost $625 a year for five years, while B gained $1875 a year for the same time. How much was each then worth?

27. If 120 be added to a certain number, the sum multiplied by 248, 1728 subtracted from the product, and the remainder divided by 320, the result will be 441. What is the number?

28. 312 rows of cabbages were planted with 288 to each row. Had 256 been planted in a row, how many rows would there have been?

29. A street was paved with granite blocks which cost 4,752,000 cents at 10 cents apiece. If each square yard required 36 blocks, how many square yards were paved?

30. If 48 men can husk 48,384 bushels of corn in 28 days, how many bushels could 39 men husk in 50 days?

USE OF THE SIGNS

37. A *term* is a quantity included between two *principal signs.*

+ and − are called *principal signs.* They indicate that the quantities between which they are placed are to be considered as *single terms.* Thus, $24+15\times5=99$, *not* 195. The operation 15×5 should be performed *before* the addition, since 15×5 is regarded as a single term. $108 - 63 \div 9 = 101$, *not* 5. Similarly, $120 + 90 \div 3 \times 6 = 125$; and $144 - 6 \times 16 \div 4 \times 12 = 142$. That is, *the part of the term on the left of the sign of division is the dividend, and the part on the right of it is the divisor.* By insisting upon the observance of these very simple principles, all uncertainty is overcome, and the student will be prepared to surmount many of the obstacles to be encountered in the higher branches of mathematics.

REMARKS. 1. A quantity included in a parenthesis or a vinculum should be reduced to its simplest form before performing the other operations indicated.

2. Each term should be reduced separately, and the results united by the proper signs. *This expression reduced to a single term will be the answer.*

$480 + 15 \times 16 + 12 \times 5 - 45 \times (36 - 7 \times 4) =$ what?

Here are *three terms*. The first is 480; the second, $240 \div 60$, or 4; the third, 45×8, or 360. Then, $480 + 4 - 360 = 124$.

1. $450 + 25 \times 20 - 75 \times 9 =$ what?

2. $369 - 18 \times 12 + 288 \div 8 \times 6 =$ what?

3. $1293 + 648 \div 9 \times 24 - 1920 \div 15 \times 32 - 28 \times 41 =$ what?

4. $256 \times 36 - 95 \times \overline{51 + 45} + 16 \times \overline{84 - 75} - 4 \times (8750 \div 7 \times 25) =$ what?

5. $4225 \div 25 \times \overline{97 - 7 \times 12} + 216 \times 720 \div 12 \times 15 - 12 \times \overline{44 + 29} =$ what?

6. $960 - 4320 \div \overline{18 + 17 \times 6} - 48 \times \overline{327 - 24 \times 13} - 8 \times 2002 \div 14 \times 11 =$ what?

7. $8640 \div 12 \times (24 + 12 \times 13) + 2250 \div (875 \div \overline{99 + 76}) + 11 \times (480 - 26 \times 15) =$ what?

8. $3125 - (5280 \div 33 \times 16 + 615) - (67 \times 37 + 10) =$ what?

9. $12800 - 96 \times (720 - 17 \times 35) + 1680 \div (11 + 600 \div 25) - 8 \times (12 \times 7 + 22) =$ what?

10. $2250 \div \{25 \times (17 \times 26 + 8) \div (22500 \div \overline{54 + 96})\} + 5 \times (1872 \div 18 \times 13 + 1) =$ what?

11. $1180 + 240 \times 48 \div 16 \times 36 - 75 \times 336 \times 105 \div 45 \times 49 =$ what?

12. $324 \times 150 \div (750 \times 360 \div \overline{105 \times 84 + 180}) - (53 \times 33 - 130) + 48 \times (4410 - \overline{99 \times 44 + 54}) =$ what?

13. $24 \times (1728 - 96 \times 324 \div \overline{45 + 108 \times 105 \div 45 \times 84}) \div \{21600 - 20736 \div [512 - \overline{49 + 21 \times 22}] + 27 \times (2485 - 63 \times 39)\} =$ what?

UNITED STATES MONEY

DEFINITIONS

38. *United States* or *Federal money* is the currency of the United States.

TABLE

10 mills (m.)	= 1 cent . . .	¢ or ct.
10 cents	= 1 dime . . .	d.
10 dimes	= 1 dollar . . .	$
10 dollars	= 1 eagle . . .	E.

REMARKS 1. *Currency* consists of *coin* and *paper money*. Coin is stamped pieces of metal whose value is fixed by the government. Paper money consists of notes issued by the government and by national banks. The *gold* coins are the double eagle, eagle, half-eagle, and quarter-eagle. The *silver* coins are the dollar, half-dollar, quarter-dollar, and dime. The five-cent piece is *nickel*, and the one-cent piece *bronze*.

2. The denominations *eagles* and *dimes* are not used in computations. Eagles are expressed as tens of dollars, and dimes as tens of cents. The *mill* is an imaginary coin, and is used only for greater accuracy in calculations.

39. 1. All sums of United States money *expressed by figures* must be preceded by the *dollar sign* ($).

2. When cents, or cents and mills, are written in any sum, the cents must be separated from the dollars by a period (.), which is called the *decimal point;* as, $3.375.

3. As cents occupy *two places*, a cipher must be written in the tens' place when the number of cents is less than 10; as, $16.05.

4. In business computations 5 mills or more in the result are considered a cent; less than 5 mills are not considered.

Write in figures:

1. Sixteen dollars twenty-seven cents.
2. Forty-nine dollars thirty cents.
3. Eighty-four dollars five cents.
4. Ninety-six dollars ten cents three mills.
5. One hundred and seventy-five dollars sixty cents eight mills.
6. Six hundred dollars six cents six mills.
7. Five hundred and sixty-seven dollars eighty-nine cents.
8. Nine hundred dollars ninety cents nine mills.
9. One thousand dollars ten cents one mill.
10. Seven thousand two hundred and fifty dollars seventy-five cents.
11. Ten thousand dollars one cent one mill.
12. Three thousand four hundred and fifty-six dollars seventy-eight cents.
13. Seventeen thousand and seventeen dollars one cent seven mills.
14. Sixty-eight thousand seven hundred and eighty-nine dollars ten cents.
15. One hundred thousand one hundred dollars ten cents one mill.

Read the following:

16.	17.	18.	19.
$10.47	$101.01	$1234.567	$10101.01
$18.324	$247.512	$2049.20	$26049.175
$26.125	$360.84	$3750.375	$32518.49
$39.08	$419.326	$4102.85	$47256.381
$45.101	$572.10	$5625.625	$53724.50
$57.89	$634.745	$6381.49	$69380.254
$64.936	$728.63	$7890.012	$75492.14
$72.008	$856.971	$8573.94	$81603.729
$83.24	$985.48	$9467.123	$98175.63
$91.012	$1010.101	$9009.09	$101010.101

REDUCTION

40. 1. As 10 mills make 1 cent, and 100 cents make 1 dollar, there are 10×100, or 1000 mills, in 1 dollar. Therefore, to reduce dollars to cents, *annex two ciphers;* to reduce dollars to mills, *annex three ciphers;* and to reduce cents to mills, *annex one cipher.* This is the same as multiplying by 100, 1000, and 10.

2. To reduce cents to dollars, *point off two figures from the right;* to reduce mills to dollars, *point off three figures from the right;* and to reduce mills to cents, *cut off one figure from the right.* This is the same as dividing by 100, 1000, and 10.

3. To reduce dollars and cents to *cents*, or dollars, cents, and mills to *mills*, *remove the dollar sign and the decimal point.*

4. To reduce *mills to dollars*, place the decimal point between the third and fourth figures from the right; to reduce *cents to dollars*, place the decimal point between the second and third figures from the right.

Reduce:

1. $12 to cents.
2. 1500 cents to dollars.
3. 68 cents to mills.
4. 947 mills to cents.
5. 15625 cents to dollars.
6. $48 to mills.
7. 34567 mills to dollars.
8. $43.75 to cents.
9. $516 to mills.
10. $600 to cents.
11. 62500 mills to dollars.
12. 100 dollars 10 cents 1 mill to mills.
13. 48 dollars 4 cents 8 mills to mills.
14. 550500 mills to dollars.
15. $78125 to cents.
16. 390625 cents to dollars.
17. $606, 60 cents, 6 mills to mills.
18. 2020202 mills to dollars.
19. 125 dollars 5 cents to mills.
20. One million mills to dollars.

ADDITION

41. Find the sum of $8.412, $12.50, $7.964, $53.048, $6.75, and $109.876.

OPERATION

$	¢	m.
8.	4	1 2
12.	5	0
7.	9	6 4
53.	0	4 8
6.	7	5
109.	8	7 6
$198.	5	5 0

RULE. *Write the numbers as in simple addition; find their sum, and place the decimal point under the column of decimal points.*

1. Find the sum of 13 dollars 26 cents, 21 dollars 50 cents, 30 dollars 7 cents, 5 dollars 83 cents, and 29 dollars 34 cents.

2. Add $10.01, $21.12, $32.23, $43.34, $54.45, $65.56, and $76.67.

3. $4.04 + $44.40 + $40.004 + $4.44 + $40.404 + $44.44 + $4.40 + $40.444 = ?

4. A merchant's sales for six days were as follows: $182.07, $216.45, $196.53, $200.10, $183.09, and $241.76. What were his entire receipts?

5. Eight car loads of lumber were sold for the following sums: $379.48, $421.56, $390.84, $500.73, $462.08, $386.27, $515.09, and $458.65. What was the entire value?

6. Nine mechanics earned the following sums in one year: $748.15, $681.39, $830.42, $756.93, $924.78, $675, $812.57, $769.84, and $820.92. What were their total earnings?

7. Mr. Roberts built a house, paying $561.97 for lumber, $243.80 for stonework, $94.62 for brickwork, $308.35 for plastering, $175 for painting, $627.50 for labor, $182.73 for slating, $56.48 for grading, and $249.55 for other expenses. What was the cost of the house?

8. The expenses of a school district for ten months were as follows: $256.74, $301.92, $237.68, $324.56, $285.43, $310.09, $278.31, $342.87, $293.15, and $300. What amount was expended?

9. The receipts of a church fair for six days were $193.57, $218.49, $174.83, $256.74, $329.65, and $407.92. What were the total receipts?

10. Seven invoices of goods were sold respectively for $1364.29, $1500.46, $971.85, $2049.07, $856.54, $1627.13, and $2135.71. How much was realized?

11. Seven bars of gold were assayed as follows: $537.628, $546.374, $529.816, $551.463, $538.042, $542.75, and $503.927. What was the total value?

12. The gatherings of a company of miners for eight weeks were $2841.56, $3107.19, $2564.87, $1953.48, $3426.93, $2682.74, $3795.65, and $4279.31. What was the entire sum?

13. Eight boat loads of wheat were sold respectively for $1050.45, $963.17, $1209.54, $897.63, $1126.39, $974.26, $1048.82, and $875.99. What sum was realized?

14. The receipts of nine excursion trains were returned as follows: $729.45, $864.70, $695.05, $918.25, $752.80, $847.35, $683.50, $920.75, and $836.90. What were the total receipts?

15. The taxes of a certain village for ten consecutive years were $3694.617, $3937.865, $4281.27, $4573.542, $4865.258, $5150.18, $5429.723, $5718.491, $6042.30, and $6306.754. What amount of tax was assessed?

SUBTRACTION

42. What is the difference between 412 dollars 10 cents 5 mills, and 239 dollars 78 cents 6 mills?

OPERATION

$	ct.	m.
412.	10	5
239.	78	6
$172.	31	9

RULE. *Write the less number under the greater, as in simple subtraction; find the difference, and place the decimal point under those above.*

1. From $249.60 take $85.73.

2. From $306.24 take $205.34.

3. Take $456.78 from $876.54.

4. Take $93.75 from $750.

5. Subtract $574.96 from $932.81.

6. From $646.125 subtract $570.375.

7. What is the difference between 450 dollars and 450 cents?

8. $1000 − $512.875 = what?

9. $1246.06 less $1059.63 = what?

10. $2100.36 minus $1532.47 = what?

11. I sold a lot of grain for $493.75, which was $97.50 more than I paid for it. What was the cost?

12. A speculator began trading with $2250, and quit with $1754.60. How much did he lose?

13. A trader's profits for a certain year were $1897.50, and he then had $5250.35. How much had he at the beginning of the year?

14. A drover sold a herd of cattle for $2701.25, which cost him $2160.75. Find his gain.

15. A merchant purchased $346.48 worth of goods at one time, and $437.32 at another, giving a $1000 bill in payment. How much change did he receive?

16. A had $3418.92, B had $2687.41; A spent $1624.49, and B spent $885.48. How much more did B then have than A?

17. $5000 − $1047.32 − $1134.79 − $1256.58 − $1300.56 = what?

18. By selling a lot of lumber for $3261.84, a dealer gained $543.64. How much would he have gained by selling it for $3397.75?

19. A sold B a cargo of wheat, and gained $260.10; B sold it to C for $2124.15, and gained $303.45. How much did A pay for it?

20. A man dying left to his son $5184.75, which was $1157.25 more than he left to his daughter. The widow's share was $739.75 less than the daughter's. What was the value of the estate?

MULTIPLICATION

43. Multiply $39.256 by 24.

OPERATION

$39.256
24
157 024
785 12
$942.144

RULE. *Multiply as in simple multiplication, and place the decimal point between the dollars and cents in the product.*

Multiply:

1. $9.375 by 17.
2. $16.256 by 45.
3. $31.25 by 48.
4. $54.875 by 63.
5. $78.125 by 72.
6. $87.648 by 105.
7. $15.625 by 128.
8. $8.192 by 225.
9. $1.286 by 96.
10. $46.875 by 320.

11. I bought 256 barrels of flour at $5.625 per barrel. How much did I pay for it?

12. A dealer sold 84 cords of wood at $4.375 a cord. How much did he receive for it?

13. A contractor furnished 192 horses at $68.75 apiece. What was their total value?

14. I paid $0.875 a bushel for 450 bushels of wheat. What was the entire cost?

15. A dealer bought 480 gallons of wine at $1.125 a gallon. How much did he pay for it?

16. 17,280 feet of lumber, worth $0.028 a foot, was used in building a house. How much was paid for it?

17. 84,480 square feet of cement sidewalk, costing $0.225 a square foot, was laid in a certain village. What was its value?

18. Find the cost of 384 gallons of petroleum at $0.125 a gallon.

19. I sold 160 acres of pasture land at $31.25 an acre. How much did it bring?

20. A dealer ordered 18,625 bushels of coal at $0.108 a bushel. What was the bill?

21. Find the value of 12 piles of wood, each measuring 27 cords, at $4.80 a cord.

22. A merchant imported 36 bolts of broadcloth, each containing 48 yards, at $1.875 a yard. How much did it cost?

23. A mechanic worked 72 days of 12 hours each, at $0.25 an hour. How much did he earn?

24. Find the cost of 45 bundles of paper, each containing 960 sheets, at 4 mills per sheet.

25. A manufacturer sold 144 boxes of soap, each containing 48 bars, at $0.055 a bar. How much did he realize?

26. A fruit dealer bought 240 boxes of oranges, each containing 15 dozen, at 17 ct. 5 m. per dozen. How much did he pay for them?

27. A starch company received an order for 750 boxes of starch of 40 pounds each, at $0.045 a pound. How much was the amount of the bill?

28. A wholesale grocer imported 216 sacks of Mocha coffee, each holding 128 pounds, at a cost of 32 ct. 5 m. per pound. What was the value of the coffee?

29. A planter sold 324 hogsheads of molasses, each containing 63 gallons, at $0.375 a gallon. How much did he receive for it?

30. Twelve carpenters built a house in 16 weeks, working 6 days each week. If their wages were $1.625 apiece per day, what were their total earnings?

31. A jeweler bought 49 gold chains, each weighing 25 pennyweights, at $1.25 a pennyweight. What was their value?

32. A wholesale druggist compounded 1250 boxes of pills. Each box contained 20 pills weighing .8 grains apiece, which were sold at the rate of 3 mills per grain. How much did he receive for them?

33. A ship with a crew of 144 men sailed on a twelve months' cruise. If each man was allowed 32 pounds of beef per month, how much did it all cost at $0.05 a pound?

34. Sixteen car loads of sugar, each containing 80 barrels of 250 pounds per barrel, were sold at $0.045 a pound. How much did it bring?

35. Eight equal tracts of land were each subdivided into 64 lots. The lots were 50 feet wide, and were sold for $6.25 a front foot. What amount was realized?

36. A merchant imported 36 cases of brandy, each containing 12 2-quart packages. If he paid $1.225 a quart for it, what was the value of the consignment?

37. 1296 government horses were each allowed a peck of corn daily for 108 days. If the corn cost $0.135 a peck, what was the total expense?

38. In constructing a road, 49 teams were employed for 28 days, each hauling 18 loads of earth daily, at $0.175 a load. What were their total earnings?

39. Thirty-eight cars were each loaded with 72 barrels of salt weighing 240 pounds per barrel, all of which was sold at 5 mills per pound. What sum was paid for it?

40. A company of 24 gold miners worked 9 hours a day for 248 days. If each one's gatherings averaged 65 grains per hour, what was it all worth at 4 cents per grain?

DIVISION

44. PRINCIPLES. 1. *When the divisor is an abstract number, separate dollars from cents in the quotient, and prefix the dollar sign.*

2. *When the divisor is a sum of money, reduce the divisor and dividend to the same denomination before dividing. The quotient will be an abstract number.*

REMARK. If the dividend is a number of dollars, and a remainder should occur, reduce the dollars to *cents*, or, if necessary, to *mills*, by annexing *two* or *three ciphers*. Then, if the division is not exact, the remainder should be indicated by writing + after the last figure of the quotient.

(*a*) 7 loads of coal cost $26.25, what was the cost of one load?

OPERATION. 7)$26.25
$3.75

EXPLANATION. $26.25 may be regarded as 2625 *cents*. The cost of 1 barrel was one seventh of 2625 cents = 375 cents = $3.75.

(*b*) At $1.375 a yard, how many yards of carpet will cost $66?

OPERATION. $1.375)$66.000(48 yards
55 00
11 000
11 000

EXPLANATION. As the divisor is *a sum of money*, both divisor and dividend are reduced to the same denomination (mills), which is done by annexing *three ciphers* to the dividend. 66000 mills ÷ 1375 mills = 48. Therefore, it will require as many yards *as the value of one yard is contained times in* $66 = 48 *yards*.

Divide:

1. $389.60 by 16.
2. $843.75 by 45.
3. $1174.68 by 78.
4. $227.465 by 97.
5. $337.50 by 108.
6. $1485 by 264.
7. $4559.544 by 456.
8. $1469.664 by 729.
9. $8023.25 by 958.
10. $100000 by 256.
11. 4032 cents by 28 cents.
12. 18816 mills by 48 mills.
13. $306.36 by 69 cents.
14. $2625 by 84 cents.
15. $1080 by $1.44.

16. $2850 by $4.56.

17. $1600 by 51 cents 2 mills.

18. $1200 by $1, 87 cents, 5 mills.

19. $1260 by $1, 8 mills.

20. $7006.652 by $5.678.

21. How many bushels of corn at 56 cents a bushel can be bought for $126?

22. How many gallons of coal oil at 10¢ 5 m. a gallon will cost $26.25?

23. A man paid $3600 for a farm of 64 acres. What was the price per acre?

24. 324 barrels of flour were sold for $1579.50. What was its value per barrel?

25. If turkeys sell for $1.375 each, how many could be bought for $99?

26. If a car load of freight costs $0.275 a mile, how far could it be carried for $59.40?

27. $900 was paid for 480 perches of broken stone. What was the cost per perch?

28. I paid $18.90 for 84 dozen eggs. How much was one dozen worth?

29. How many yards of rope at 4¢ 5 m. a yard would cost $79.20?

30. How many feet of lumber could be bought for $144 at 2¢ 5 m. a foot?

31. 720 cans of apricots were sold for $234. What was the value per can?

32. 1024 gross of steel pens cost $640. What was the cost per gross?

33. A wholesale dealer paid $486 for imported cheese at $0.375 a pound. How many pounds did he buy?

34. A speculator paid $10,000 for cattle at $15.625 apiece. How many did he buy?

35. 96 boxes of sapolio, each containing 48 packages, were bought for $345.60. What was the cost per package?

36. A merchant imported 16 casks of wine for which he paid $1092, at $1.625 a gallon. How many gallons were there in each cask?

37. A wholesale dealer ordered 64 rolls of carpet, each measuring 85 yards, for which he paid $6120. What was the cost per yard?

38. A number of bales of cotton, each weighing 320 pounds, was sold for $10,000 at $0.125 a pound. How many bales were sold?

39. Sixty-nine laborers were employed for 108 days, and their wages amounted to $12,295.80. How much did each earn per day?

40. A crop of wheat was sold for $600 at 96¢ a bushel. Allowing 160 square rods to an acre, and 32 quarts to a bushel, how many acres were in the tract if each square rod produced 5 quarts of wheat, and how many bushels did each acre produce?

REVIEW OF UNITED STATES MONEY

45. **1.** I paid $18.75 for a chain, and 8 times as much for a watch. What was the total cost?

2. An agent's collections for six days were: $318.67, $269.15, $375.89, $247.56, $426.03, and $362.70. What amount did he collect?

3. Five men bought a cargo of grain for $5250. The first paid $942.16; the second, $1167.58; the third, $859.45; the fourth, $1216.72. How much did the fifth pay?

4. A dealer sold 437 bushels of wheat at $1.08 a bushel; 874 bushels of corn at $0.625; 758 bushels of rye at $0.875; and 623 bushels of oats at $0.48 a bushel. How much did he receive for all?

5. If flour is $4.50 a barrel, and apples are $1.75 a barrel, how many barrels of each could be bought for $150?

6. I sold a lot of lumber for $1643.85, thereby losing $182.65. How much should I have gained by selling it for $2009.15?

7. I sold 75 barrels of apples for $135, thereby gaining $22.50. What was the cost per barrel?

8. A manufacturer sold 24 cases of muslin, each containing 16 bolts of 64 yards per bolt. What was the entire value at $0.075 a yard?

9. I bought a bill of goods as follows: furniture, $32.45; stove, $17.50; carpet, $24.75; chinaware, $10.60; pictures, $12.55. I gave the clerk a $100 note, and received $3.25 change. How much was this incorrect?

10. A man lost $97.60; he then borrowed $156.85, spent $279.20, and had $280.05 left. How much had he at first?

11. I bought 48 barrels of molasses for $777.60 at 36 cents a gallon. How many gallons were there in each barrel?

12. A man exchanged 36 bales of cotton, each weighing 240 pounds, at \$0.225 a pound, for beef at \$13.50 a barrel. How many barrels were required?

13. A dealer sold an equal number of bushels of wheat at 99¢, corn at 64¢, barley at 45¢, and oats at 37¢. If he received \$600.25, how many bushels of each were sold?

14. A merchant bought flour at \$3.625 a barrel, and sold it at \$4.25 a barrel, thereby gaining \$50. How many barrels were sold?

15. A trader having \$3374.50 bought 26 horses at \$55 per head, 39 cows at \$24.50 per head, 97 hogs at \$5.75 each, and spent the remainder for sheep at \$3.125 apiece. How many sheep did he buy?

16. I bought 144 acres of land at \$68.75 an acre, and sold a part of it for \$3600, at \$56.25 an acre, and the remainder at \$80 an acre. How much did I gain or lose on the transaction?

17. A merchant having \$1750, spent \$849.65 for groceries, \$654.50 for dry goods, \$170.85 for boots and shoes, and the remainder for 25 barrels of petroleum at \$0.075 a gallon. How many gallons were there in each barrel?

18. A man sold 36 horses for \$2070, thereby losing \$11.25 on each. What was the cost per head?

19. If 48 mules cost \$1800, for how much must each be sold to gain \$12.50 per head?

20. If 84 acres of land were bought for \$2835, and 56 of them were sold for \$2310, how much was gained on the quantity sold?

21. Four men bought a vessel for $13,500. A paid $3150, which was $225 less than B paid. If B paid $350 less than C, how much did D pay?

22. The gravel for a public road cost $793.80, at $0.024 a cubic foot. If there were 27 cubic feet in each load, how many loads were required?

23. A farmer sold 49 barrels of apples at $2.25 a barrel, and a lot of potatoes at $0.75 a bushel. If he received $150 for all, how many bu. of potatoes had he?

24. I paid $1521 for cattle, and lost $1.75 a head by selling them for $1384.50. What was the cost per head?

25. How many hours a day must a carpenter work to earn $864 in 256 days at $0.375 an hour?

26. A man bought 28 barrels of cider, and sold it all at a profit of $0.045 per gallon, thereby gaining $52.92. How many gallons were in each barrel?

27. 39 barrels of molasses were sold for $659.88, by which $73.32 was lost. If there were 47 gallons in each barrel, what was the cost per gallon?

28. A speculator sold 24 village lots at $187.50 apiece, and, after depositing $1125 in bank, he invested the remainder in an equal number of horses at $49.50, cows at $19.75, and hogs at $5.75. How many of each were bought?

29. I bought a tract of land for $4000, and sold it at $43.75 an acre, thereby losing $500. Had I sold it at $56.25 an acre, how much should I have received?

30. I bought wine at $1.50 a gallon and sold it at a profit of $0.125 a gallon, receiving $520 for the entire quantity. How much did I pay for it?

31. If 37 men can earn $6055.42 in 98 days, how much could 69 men earn in 87 days at the same rate?

32. A merchant paid $2539.68 for 52 bales of cotton, at the rate of 16 cents 5 mills per pound. What was the average weight of the bales?

33. How many boxes of lemons, each containing 16 dozen, worth $0.135 a dozen, should be given in exchange for 40 boxes of canned fruit, each containing 18 cans worth $0.105 a can?

34. What was the cost of 126 acres of land if $525 was gained by selling 84 acres for $4095?

35. How many barrels of flour, each containing 196 pounds worth $0.025 a pound, should be given in exchange for 24 tons of hay worth $14.40 a ton, and $66 in money?

36. A dealer bought 16 car loads of coal of 312 bushels each, at 9¢ 5 m. per bushel. The expense for freight was $3.34 per car, and one cent a bushel for hauling. If he sold it all at $0.135 a bushel, what was his profit?

37. A farmer sold 485 bushels of wheat at $1.25 a bushel, and received in exchange 9375 feet of lumber at $0.018 a foot, and the balance in cows at $17.50 apiece. What was the number of cows?

38. 64 acres of meadow land yielded two tons of hay per acre. The crop was sold at $13.50 a ton, and the money invested in land at 45¢ a square rod. Allowing 160 square rods to an acre, how many acres were bought?

39. C and D built a fence for $47.25; C worked 15 days and built 180 rods. D worked as many days as C built rods per day. How much did each earn?

40. A merchant bought 12 barrels of vinegar, each containing 36 gallons, at $0.175 a gallon, but 20 gallons leaked out. At what price per gallon must he sell the remainder to gain $27.40 on the cost of all?

41. If $365 is added to my money, the sum multiplied by 49, and $5930 subtracted from the product, the remainder divided by 87 will equal $450. How much have I?

42. A man can earn $0.325 an hour, and his son $0.15. If they receive $49.40 for doing a certain work, what is the man's share?

43. I sold a lot of wheat for $1275 at 85¢ a bushel, thereby losing $37.50. Had I sold it at a gain of $112.50, how much per bu. should I have received?

44. A huckster bought an equal number of turkeys at $1, geese at $0.85, ducks at $0.30, and chickens at $0.25 apiece. He sold the entire lot for $207, thereby gaining $41.40. How many of each were there?

45. What was the cost of 2250 bushels of corn, if $27.93 was lost by selling 798 bushels at 52¢ 5 m. per bushel?

46. 84 men were employed upon a railroad for a certain time, and their wages, at $1.45 per day each, amounted to $11,692.80. How many days did they work?

47. A man paid $8250 for land at $68.75 an acre. He sold a part of it for $4050 at $56.25 an acre, and the remainder at $87.50 an acre. How much was gained or lost on the whole transaction?

MERCHANTS' BILLS

46. 1. A *bill* is a written statement of merchandise bought or sold, naming the articles, their prices, and the entire cost, or a statement of the price of services rendered.

2. The *debtor* is the person who owes. The *creditor* is the person to whom the money is due.

Find the amounts of the following bills:

1.

MR. J. M. STRATTON, CLEVELAND, O., JUNE 12, 1901.

To GILBERT & MEYERS, *Dr.*

To 16 yd. silk	at $1.20	. . . .	$
" 5 yd. lining	" .13	. . . .	
" 7 yd. muslin	" .10	. . . .	
" 4 doz. buttons	" .20	. . . .	
			$

2.

JOHN ADAMS & CO. LOUISVILLE, KY., NOV. 16, 1900.

Bought of ROBERTS & MAYHEW.

32 bbl. flour	at $4.80	. . . .	$
12 bbl. sugar	" $8.45	. . . .	
7 boxes soap	" $2.25	. . . .	
24 boxes starch	" $1.35	. . . .	
144 lb. tobacco	" .40	. . . .	
			$

Received Payment,

ROBERTS & MAYHEW.

3.

BUFFALO, N.Y., JAN. 25, 1899.

MR. GEO. W. HALLECK,

To ROCKWELL & SHEPHERD, *Dr.*

To 16 lb. coffee	at	$0.24	$
" 3 lb. tea	"	.75	
" 28 lb. sugar	"	.06	
" 6 gal. oil	"	.09	
" 11 lb. bacon	"	.12	
" 4 lb. cheese	"	.19	
			$

Received Payment,

4.

CHICAGO, ILL., JULY 13, 1900.

DAY & KNIGHT,

Bought of RANDALL, PARKER & CO.

240 arithmetics	at	$0.54	$
162 grammars	"	.45	
125 readers	"	.80	
104 histories	"	.96	
108 geographies	"	1.10	
196 copy books	"	.06	
			$

Received Payment,

RANDALL, PARKER & CO.

5.

Henry Graham bought of Raymond & Hunt, of Boston, Aug. 7, 1901, 168 yards Brussels carpet, at $1.125; 96 yards ingrain carpet, at $0.625; 8 rugs, at $2.50; 36 yards oilcloth, at $0.45; 10 pair lace curtains, at $1.48. What is the amount of the bill? Make it out and receipt it.

COMPOUND NUMBERS

DEFINITIONS

47. A *compound number* consists of two or more concrete numbers of the *same class*, but of *different denominations;* as, 4 gallons 3 quarts 1 pint, or 2 yards 1 foot 5 inches.

REMARKS. 1. 5 pounds 6 inches is *not* a compound number, as pounds and inches do not belong to the same class.

2. With the exceptions of *United States money* and the *metric system of weights and measures*, the denominations of compound numbers do *not* increase or decrease by the *scale of ten*.

48. A *standard unit* is one which is taken as a standard of comparison for other units of the same class in assigning numerical values to them. It may be regarded as the fundamental unit adopted in any system of weights, measures, or money by which the denominations of that system are regulated.

REDUCTION

49. 1. *Reduction of compound numbers* is the process of changing them from one denomination to another without altering their value.

2. *Reduction descending* is changing the denomination of a number to another that is lower.

3. *Reduction ascending* is changing the denomination of a number to another that is higher.

Dry Measure

50. *Dry Measure* is used for measuring grain, fruits, etc.

TABLE

2 pints (pt.)	= 1 quart . . .	qt.
8 quarts	= 1 peck . . .	pk.
4 pecks	= 1 bushel . . .	bu.

REMARK. The *standard unit* of dry measure is the *bushel*, which contains 2150$\frac{4}{5}$ cubic inches.

(*a*) Reduce 4 bu. 3 pk. 2 qt. 1 pt. to pints.

OPERATION

bu.	4 pk.	8 qt.	2 pt.
4	3	2	1
4			
19 pk.			
8			
154 qt.			
2			
309 pt.			

EXPLANATION. As there are 4 pecks in 1 bushel, any number of bushels is equal to 4 times that number of pecks. 4 bushels = 16 pecks, and 3 pecks added make 19 pecks. As there are 8 quarts in 1 peck, any number of pecks is equal to 8 times that number of quarts. 19 pecks = 152 quarts, and 2 quarts added make 154 quarts. As there are 2 pints in 1 quart, any number of quarts is equal to 2 times that number of pints. 154 quarts = 308 pints, and 1 pint added make 309 pints. Hence, 4 bushels 3 pecks 2 quarts 1 pt. = 309 pints. This operation illustrates *reduction descending*.

(*b*) Reduce 309 pt. to bushels.

EXPLANATION. As there are 2 pints in 1 quart, 309 pints are equal to one half that number of quarts, = 154 quarts, with a remainder of 1 pint. 154 quarts are equal to one eighth of that number of pecks, = 19 pecks, with a remainder of 2 quarts. 19 pecks are equal to one fourth of that number of bushels, = 4 bushels, with a remainder of 3 pecks. Hence, 309 pints are equal to 4 bushels 3 pecks 2 quarts 1 pint. This operation illustrates *reduction ascending.*

OPERATION

2|309 pt.
8|154 qt. + 1 pt.
4| 19 pk. + 2 qt.
4 bu. + 3 pk.

I. Rule for Reduction Descending

51. 1. *Write the given quantity in the order of its denominations, beginning with the highest, supplying vacant denominations with ciphers.*

2. *Multiply the highest denomination by the number that will reduce it to the next lower denomination, and add to the product the units of the lower denomination, if there are any.*

3. *Proceed in the same manner until the entire quantity is reduced to the required denomination.*

II. Rule for Reduction Ascending

1. *Divide the given denomination by the number that will reduce it to the next higher denomination. Divide the quotient in the same manner, and continue the operation until the entire quantity is reduced.*

2. *To the last quotient annex the several remainders in their proper order. The result will be the answer.*

REMARK. In reduction descending it will be found convenient to write over each denomination of the quantity to be reduced the

number of units of that denomination that makes a unit of the next higher denomination. These numbers are used as the multipliers. (See Example 1.)

Reduce:

1. 2 bu. 3 pk. 4 qt. 1 pt. to pints.
2. 11 bu. 1 pk. 1 pt. to pints.
3. 40 bu. 6 qt. to quarts.
4. 75 bu. 1 pk. 4 qt. 1 pt. to pints.
5. 248 bu. 3 pk. to quarts.
6. 494 bu. 4 qt. 1 pt. to pints.
7. 9984 qt. to bushels.
8. 50000 pt. to bushels.
9. 29876 pt. to bushels.
10. 63999 pints to bushels.
11. A dealer bought 12 bushels of blackberries at 3 cents a pint. How much did he pay for them?
12. I sold 5 bu. 1 pk. 7 qt. of cherries at 5 cents a quart. How much did I receive?
13. Find the value of 31 bu. 1 pk. of wheat at $0.02 a pint.
14. $11.25 was paid for hickory nuts at $0.045 a quart. What quantity was bought?
15. 48 bushels of pears were packed in boxes, each holding 1 pk. 4 qt. How many boxes were required?
16. A lot of peaches was shipped in 144 baskets, each containing 1 pk. 7 qt. How many bushels were there?

17. A crop of pecans yielded 346 bu. 3 pk. 4 qt. and was shipped in barrels, each holding 2 bu. 1 pk. 2 qt. How many barrels were shipped?

18. 54 bushels of plums were preserved in 3-pint jars, which were sold at $3.60 a dozen. What was their entire value?

19. 48 cases of canned fruits were sold for $151.20, at $0.175 a quart. How many 1-quart cans were in each case?

20. A crop of pears was packed in baskets, each containing 1 pk. 2 qt. and sold for $250 at $0.625 per basket. What was the number of baskets and of bushels?

Avoirdupois Weight

52. *Avoirdupois weight* is used for weighing heavy articles, as grain, groceries, coarse metals, etc.

TABLE

16 ounces (oz.)	= 1 pound . . . lb.
100 pounds	= 1 hundredweight . . . cwt.
20 cwt.	= 1 ton . . . T.

Remarks. 1. The *standard avoirdupois pound* is equal in weight to 7000 grains Troy.

2. In computing duties on imports, 2240 pounds are considered a ton.

Reduce:

1. 12 cwt. 37 lb. 8 oz. to ounces.
2. 6 T. 3 cwt. 45 lb. to pounds.
3. 9 T. 10 cwt. 11 lb. 12 oz. to ounces.
4. 48 T. 8 cwt. 75 lb. to pounds.

5. 15 T. 12 cwt. 50 lb. to ounces.

6. 30000 oz. to hundredweight.

7. 98125 lb. to tons.

8. 27471 oz. to hundredweight.

9. 102486 oz. to tons.

10. 1000000 oz. to tons.

11. A load of hay weighing 1 ton 2 cwt. 50 lb. was sold at the rate of $0.006 per pound. What was its value?

12. If a cubic foot of marble weighs 192 lb. and is worth $4.50, what would be the value of a block of marble weighing 2 T. 6 cwt. 8 lb.?

13. A bushel of barley weighs 48 lb. How many wagonloads of 50 bu. each would be required to haul 36 tons of barley?

14. 750 boxes, each containing 10 four-pound packages, were sent from Chicago to St. Louis. What was the total net weight?

15. 5 tons 4 cwt. of starch was packed in boxes, each containing 16 two-pound packages, and sold at $1.50 a box. What was its value?

16. Thirteen car loads of coal were sold at the rate of $3.75 a ton. Reckoning 25 bu. to a ton, and 275 bu. to a car, what was the value of all?

17. There are 5280 feet in a mile. If steel rails weigh 17 lb. per foot, what would be the weight of the rails in 75 miles of railroad?

18. A manufactory received an order for a number of boxes of soap, each containing 25 two-pound bars,

which were sold for $1080, at $0.045 a pound. How many boxes were ordered?

19. A barrel of pork weighs 200 lb. Find the value of the pork required by a company of 120 men for 80 days, pork being worth $12.50 a barrel, and allowing each man 1 lb. 2 oz. daily.

20. A quantity of baking powder was shipped in boxes each containing 24 eight-ounce packages, and sold for $450 at 15¢ a package. How many boxes were required, and what was the total net weight?

Troy Weight

53. *Troy weight* is used for weighing gold, silver, and precious stones.

TABLE.

24 grains (gr.)	= 1 pennyweight . .	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

REMARK. The *standard unit* of weight in the United States is the *Troy pound*, which contains 5760 grains.

Reduce:

1. 9 oz. 19 pwt. 20 gr. to grains.
2. 24 lb. 8 oz. 12 pwt. to pennyweights.
3. 50 lb. 10 oz. 15 pwt. 15 gr. to grains.
4. 216 lb. 6 oz. 16 pwt. to pennyweights.
5. 214 lb. 4 oz. 7 gr. to grains.
6. 18324 pwt. to pounds.
7. 24000 gr. to ounces.
8. 74649 gr. to pounds.

9. 287999 gr. to pounds.

10. 639360 gr. to pounds.

11. A jeweler converted 18 lb. of silver into teaspoons weighing 9 pwt. each. What was their value at $0.75 apiece?

12. 50 silver cups were offered as prizes at a state fair. If their value was $180, and the silver cost 6 cents a pwt., what was the weight of each cup?

13. The gatherings of a company of gold miners averaged 20 grains per minute for 312 days of 9 hours each. What was the value of all the gold at $240 a pound?

14. The communion plate of a cathedral cost $2850 at $0.95 a pwt. How much did it weigh?

15. The American bronze cent weighs 48 grains Troy. What would be the value of the number of bronze cents that would be equal in weight to 625 pounds Troy?

16. $10,125 was paid for a number of gold medals awarded at an Exposition. If the gold was worth $225 a pound, and the medals weighed 12 pwt. each, how many were awarded?

17. A goldsmith manufactured a quantity of gold into watch cases weighing 1 oz. 5 pwt. each, which he sold at the rate of $0.04 a gr. If he received $2016 for the lot, how many cases were there?

18. A jeweler bought two pounds of gold at $18.75 an ounce, and made it into rings weighing 5 pwt. each, which he sold at $6.25 apiece. Find his entire gain.

19. The estimated value of a silver brick was $105 at $0.042 a pwt. What was the weight of the brick?

20. A silversmith manufactured 22 lb. 6 oz. of silver into tablespoons weighing 18 pwt. each. If he sold them all at $7.50 per set of 6 spoons, how much did he receive for them?

Apothecaries' Weight

54. *Apothecaries' weight* is used by druggists in compounding medicines.

TABLE

20 grains (gr.)	= 1 scruple . . .	℈.
3 scruples	= 1 dram	ʒ.
8 drams	= 1 ounce	℥.
12 ounces	= 1 pound	℔.

Remark. The grain, ounce, and pound are alike in Troy and Apothecaries' weights, but the *ounce* is differently divided.

Reduce:

1. 11 ℔ 10 ℥ 7 ʒ 1 ℈ to scruples.
2. 10 ℥ 5 ʒ 2 ℈ 11 gr. to grains.
3. 8 ℔ 8 ℥ 1 ʒ 1 ℈ to grains.
4. 25 ℔ 5 ʒ 10 gr. to grains.
5. 113 ℔ 7 ℥ 1 ʒ 1 ℈ 1 gr. to grains.
6. 5371 gr. to ounces.
7. 12960 ℈ to pounds.
8. 30941 gr. to pounds.
9. 287999 gr. to pounds.
10. 190080 gr. to pounds.

11. A druggist sold 5 ℥ 3 ʒ 1 ℈ of morphine at $0.045 a scruple. How much did he receive for it?

12. An apothecary bought calomel at $7.20 a pound and sold it at $0.05 a scruple. What was his gain per pound?

13. A quantity of medicine was compounded into three-grain pills, which were sold for $48.75, at $0.015 each. What was the weight of the medicine?

14. 6 ℔ 3 ℥ of quinine was made into four-grain pills, which were put into boxes each containing 30 pills, and sold at $0.25 per box. How much was received for the entire quantity?

15. A druggist received an order for a number of five-grain powders, for which he charged $34.50, at the rate of $0.025 each. What quantity of medicine was required?

16. An infected district was supplied with 324,000 eight-grain capsules. What was their value at $14.40 a pound?

17. An institute manufactured 43 ℔ 4 ℥ 6 ʒ 2 ℈ of drugs into pellets, each containing one eighth of a grain. How many pellets were made?

18. A quantity of calomel was made into ten-grain powders and sold for $19.20 at the rate of $0.04 a powder. How much calomel was used?

19. A lot of medicine weighing 2 ℔ 6 ℥ was compounded into capsules and sold for $37.50 at $0.50 per box. If every capsule weighed 8 gr., how many were in each box?

Apothecaries' Fluid Measure

TABLE

60 minims (♏)	= 1 fluid dram	fl. ʒ.
8 fluid drams	= 1 fluid ounce	fl. ℥.
16 fluid ounces	= 1 pint (octarius)	O.
8 pints	= 1 gallon (congius) . . .	cong.

Liquid Measure

55. *Liquid measure* is used for measuring wine, milk, and other liquids.

TABLE

4 gills (gi.)	= 1 pint	pt.
2 pints	= 1 quart	qt.
4 quarts	= 1 gallon	gal.

Remarks. 1. The *standard unit* of liquid measure is the *gallon*, which contains 231 cubic inches.

2. In measuring *wine* it is customary to consider 31½ gallons as a barrel and 63 gallons as a hogshead.

Reduce:

1. 31 gal. 1 qt. to pints.
2. 70 gal. 1qt. 2 gi. to gills.
3. 349 gal. 1 pt. to pints.
4. 488 gal. 1 qt. to gills.
5. 1080 gal. 1 pt. 3 gi. to gills.
6. 725 pt. to gallons.
7. 1440 gi. to gallons.
8. 6048 pt. to hogsheads.
9. 31625 gi. to hogsheads.
10. 43 hhd. 31 gal. 2 qt. to gills.

11. A grocer bought 16 barrels of petroleum, each containing 45 gallons, at the rate of $0.015 a pint. How much did he pay for it?

12. A wine merchant sold 62 gal. 2 qt. of California wine at the rate of $0.025 a gill. What was its value?

13. A man bought 250 gal. of molasses at 4¢ a pint, and sold it at $0.10 a quart. Find the entire gain.

14. Find the cost of 150 gal. of cider at 6 mills per gill.

15. A dealer paid $225 for French brandy at $4.50 a gal. and sold it at 75¢ a pint. Find the total gain.

16. 630 gallons of wine were drawn into one-pint bottles, which were sold at $2.40 per dozen. How much was received?

17. Find the cost of 4 casks of vinegar, each containing 32 gal., at the rate of 5 mills per gill.

18. A man sold an invoice of patent medicine in half-pint bottles at 40 cents each, thereby realizing $320. How many gallons were sold?

19. $97.20 was paid for 18 cases of jelly, each containing 12 three-pint jars. What was the price per gallon?

Long Measure

56. *Long measure* is used for measuring length or distance.

TABLE

12 inches (in.)	= 1 foot . . .	ft.
3 feet	= 1 yard . . .	yd.
5½ yards, or 16½ feet,	= 1 rod . . .	rd.
320 rods, or 1760 yards,	= 1 mile . . .	mi.

REMARK. The *standard unit* of length is the *yard.*

Reduce:

1. 32 mi. 10 rd. to rods.

2. 270 yd. 2 ft. 6 in. to inches.

3. 1759 yd. 2 ft. to inches.

4. 132000 ft. to miles.

5. 2090880 in. to miles.

6. 56250 in. to yards.

7. 31779 rd. to miles.

8. 62211 in. to yards.

9. 999 yd. to inches.

10. 156 mi. 80 rd. to rods.

11. If a horse can trot 72 rods per minute, in what time can he trot 9 miles?

12. What would be the value of 125 miles of telephone wire at $0.008 per foot?

13. In Ex. 12, if the telephone poles were 250 ft. apart, what would be their value at $6.25 each?

14. A fence extends along both sides of a railroad 80 miles in length. Find its cost at $0.60 a rod.

15. The cement sidewalks of a certain village have a total length of 8 miles. How much did they cost at $0.625 a linear foot?

16. If the driving wheels of a locomotive are 14 ft. in circumference, how many revolutions would each make in a trip between two stations 35 miles apart?

17. $211.20 was paid for three-inch nails at $0.002 apiece. What length of wire was required for them?

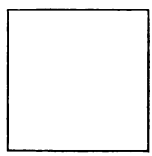
18. Sixteen silk flags exactly alike cost $180 at $0.125 per linear inch. How long was each flag?

Square Measure

57. 1. *Square measure* is used for measuring surfaces.

2. A *surface* has two dimensions, *length* and *breadth.*

3. A *square* is a figure that has four *equal sides* and four *right angles.*



4. The *unit of measure* for surfaces is a square, each of whose sides is a linear unit. Thus, a *square inch* is a square, each of whose sides is one inch long; a *square foot* is a square, each of whose sides is one foot long, etc.

5. The *area* of a square is the product of two of its sides. Thus, the area of a surface 3 feet square is $3 \times 3 = 9$ square feet.

TABLE

144 square inches (sq. in)	= 1 square foot . . . sq. ft.
9 square feet	= 1 square yard . . . sq. yd.
30¼ square yards	= 1 square rod . . . sq. rd.
160 square rods	= 1 acre . . . . . A.
640 acres	= 1 square mile . . . sq. mi.

Reduce:

1. 49 A. 50 sq. rd. to square rods.
2. 29 sq. yd. 6 sq. ft. 72 sq. in. to square inches.
3. 3 sq. mi. 427 A. 105 sq. rd. to square rods.
4. 99 sq. yd. 8 sq. ft. 143 sq. in. to square inches.
5. 25 A. to square feet. (1 acre = 43560 sq. ft.)
6. 51840 sq. rd. to acres.
7. 37325 sq. in. to square yards.
8. 409600 sq. rd. to square miles.
9. 566280 sq. ft. to acres.
10. 69696000 sq. ft. to square miles.

11. A drawing-room required 68 sq. yd. 6 sq. ft. 108 sq. in. of carpet. What was the value of the carpet at $0.002 a square inch?

12. A man bought a farm containing 43 A. 120 sq. rd. at $0.375 a square rod. How much did it cost?

13. A school building has 8 rooms, each requiring 26 sq. yd. 6 sq. ft. of slate blackboard. What was its cost at 25¢ a sq. ft.?

14. A tract of land containing 80 acres was divided into lots of 40 sq. rd. each, after allowing 17 A. 80 sq. rd. for streets and alleys. If the lots were all sold for $330 apiece, how much was realized?

15. A cellar required 166 sq. yd. 6 sq. ft. of cement floor. How much did it cost at $0.125 a square foot?

16. The stone sidewalks in a village cost $28,512 at $0.225 a sq. ft. How many square yards were laid?

17. I paid $3840 for a farm at the rate of $0.30 a sq. rd. How many acres were in the farm?

18. The slate roof for a public building cost $810 at $0.08 a sq. ft. How many sq. yd. were covered?

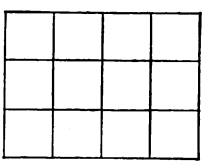
19. The floor of a dining hall measures 105 sq. yd., and is laid with marble tiles 9 inches square. Find their value at $0.375 apiece.

20. A crop of potatoes was sold for $2000 at $0.625 a bushel. If each square rod of the land produced two pecks, how many acres were planted?

58. 1. A *rectangle* is a figure that has four straight sides and four right angles.

2. The *area* of a rectangle is the extent of its surface, or the number of square units which it contains.

ILLUSTRATION. A rectangle is 4 feet long and 3 feet wide. What is its area?



EXPLANATION. The rectangle may be divided into 3 rows, and each row into 4 squares. Then the number of squares is $4 \times 3 = 12$; and as each square is 1 foot long and 1 foot wide, its *area* is 1 *square foot.* (Art. 57, 4.) Therefore the area of the rectangle is $1 \times 12 = 12$ square feet. Hence,

To find the area of a rectangle.

RULE. *Multiply the length by the breadth expressed in units of the same denomination.*

1. A reception room is 30 feet long and 24 feet wide. What is the area of its floor?

2. A garden is 24 rods wide and 40 rods long. How many acres does it contain?

3. A ceiling is 33 feet long and 21 feet wide. Find the cost of painting it at $0.18 a square yard.

4. A park is 32 rd. wide by 35 rd. long. Find its value at $250 an acre.

5. A platform is 34 yd. long and 15 yd. wide. How much did it cost at $0.075 a square foot?

6. What would be the value of a marble tablet 3 ft. 9 in. long by 2 ft. 8 in. wide at 85 cents a square foot?

7. A dining hall 38 ft. long and 20 ft. wide was floored with tiles 8 inches square at 15¢ apiece. What was the cost of the floor?

8. A field 48 rd. long by 40 rd. wide yielded 80 bu. of potatoes per acre. What was the value of the crop at $0.65 a bushel?

9. The stone sidewalks in a village are 5 ft. in width and have a total length of 6 miles. How much did they cost at $1.75 a square yard?

10. A ball park 650 feet square is surrounded by a close board fence 9 ft. high. What was the cost of the fence at $3.25 per 100 square feet?

59. As the area of a rectangle is found by taking the product of the numbers representing its length and breadth, it is evident that if the area be divided by either of those numbers, the quotient will be the other number. Hence, to find either side of a rectangle when its area and the other side are given:

RULE. *Divide the area by the given side. The quotient will be the required side.*

1. A parlor is 21 ft. long, and its area is 35 sq. yd. What is its width?

2. The area of a town lot is 800 sq. yd., and its width is 48 ft. What is its length?

3. The cement floor of a cellar cost $151.20 at $1.20 a sq. yd. If the floor was 42 ft. long, what was its width?

4. $450 was paid for a pasture 32 rd. wide, at $50 per acre. What was the length of the pasture?

5. The ceiling of a concert hall cost $107.80, at $0.35 a sq. yard. If the hall was 66 ft. long, how wide was it?

6. A meadow is 45 rd. wide, and cost $1125 at $62.50 per acre. Had it been 5 rods wider, how much would it have cost?

7. A village lot 135 ft. long has an area of 675 sq. yd., and is surrounded by a close board fence 5 ft. high. Find the value of the fence at $0.035 a sq. ft.

8. A pasture cost $600 at $60 an acre. If its width is 32 rods, how much did the fence around it cost at $0.875 a rod?

9. The area of a field is 22 A. 55 sq. rd., and its length is 65 rods. It is inclosed by a wire fence of 4 strands. What length of wire was used?

10. The floor of a barn 50 ft. long is made of boards 10 ft. long by 15 inches wide, costing $0.25 apiece. If the total cost was $30, how wide was the floor?

11. A drawing-room is 51 ft. long by 33 ft. wide, and 15 ft. high. How much would it cost to paper its walls at $0.125 a sq. yard?

12. Find the value of a farm 80 rods square at $80 an acre.

13. How many bricks would be required to pave a sidewalk 480 ft. long, and 6 ft. wide, allowing 36 bricks to a square yard?

14. The fence around a field whose area is 15 A. 36 sq. rd. cost $150. If the field was 58 rd. long, what was the cost per rod?

15. How many sods 20 in. long, and 12 in. wide, would be required for a lawn 50 ft. square?

16. A church is 69 ft. long by 42 ft. wide, and 24 ft. high. Find the cost of painting the outside walls at $0.30 a sq. yd.

17. How many tiles 9 inches square would cover the floor of an office 15 ft. long by 12 ft. wide?

18. A parlor is 15 ft. wide by 20 ft. long. Find the cost of covering the floor with rugs 4 ft. long by 2 ft. 6 in. wide, at $2.50 apiece.

19. A roll of paper measures 240 yards in length, and 4 ft. in width. How many posters, each 2 ft. long and 15 in. wide, could be cut from it?

20. A lecture room is 60 ft. long, 36 ft. wide, and 15 ft. high. Find the cost of plastering it at $0.375 a sq. yd.

21. How many boards 10 ft. long and 15 in. wide would be necessary to inclose the sides of a barn 64 ft. long, 36 ft. wide, and 20 ft. high?

22. An improvement company divided a 40-acre farm into lots 3 rods wide by 8 rods long, after allowing one fourth of the entire tract for streets. If they sold all the lots at $250 apiece, what sum was realized?

23. How many panes of glass, each 9 in. by 12 in., would be used in glazing 32 window sashes, each 3 feet square?

24. The fence around a pasture cost $141.10 at $0.85 a rd. If the pasture was 35 rd. wide, what was its area?

25. The floor of an art gallery 75 ft. long by 40 ft. wide is to be made of marble tiles 8 in. by 12 in. at $0.375 each. Find the cost of the floor.

26. A stable is 60 ft. long, and the rafters on each side are 18 ft. in length. How much would it cost to cover the roof with slates 9 in. by 15 in. at 4¢ apiece, allowing one half for overlapping?

27. A crop of corn from a field 48 rd. long and 40 rd. wide was sold for $422.40, at 55¢ a bushel. How many bushels did each acre produce?

28. 120 boards, each 12 ft. long and 10 in. wide, were sawn into palings 3 ft. long by 2 in. wide, which were sold for $2.50 per hundred. What was their value?

29. A lodge room is 75 ft. long, 45 ft. wide, and 18 ft. high. It is to be plastered at 35¢ a sq. yd., and the floor covered with carpet costing $0.65 a sq. yd. What will be the amount of the bills?

30. A lot is 135 ft. long and its area is 675 sq. yards. It is surrounded by a fence 4 boards high, each board being 6 in. wide. Find the value of the lumber at $0.02 a square foot.

60. Surveyors' Square Measure

TABLE

625 square links (sq. l.)	= 1 square rod . . .	sq. rd.
16 square rods	= 1 square chain . .	sq. ch.
10 square chains	= 1 acre	A.
640 acres	= 1 square mile . .	sq. mi.

Remark. A *surveyor's chain* is 66 feet long, and is divided into 100 links of $7\frac{92}{100}$ inches each.

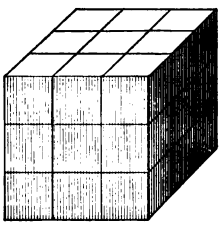
Cubic Measure

61. 1. *Cubic measure* is used for measuring solids.

2. A *solid* has three dimensions, *length*, *breadth*, and *thickness* or *height*.

3. A *cube* is a solid that has *six equal square faces*.

4. A cube is *named* from the length of its edge. Thus, if its edge is 1 inch long, it is a *cubic inch;* if 1 foot long, a *cubic foot*, etc.

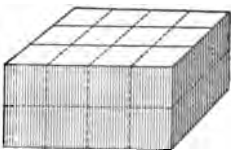


The accompanying figure represents a cubic yard. As each of its faces is 3 feet long and 3 feet wide, the area of its base is $3 \times 3 = 9$ square feet. Then, a section 1 foot high contains 9 *cubic feet;* and, as there are *three* such sections, the solid contents of the cube are $9 \times 3 = 27$ *cubic feet*. Therefore, *to find the solid contents of a cube:*

Rule. *Multiply together the length, breadth, and thickness or height.*

5. A rectangular solid is a solid having *six rectangular faces.*

The figure represents a rectangular solid 4 feet long, 3 feet wide, and 2 feet high. Reasoning as before, the area of its base is $4 \times 3 = 12$ square feet. Then a section 1 foot high contains 12 *cubic feet*, and as there are *two* such sections, the contents of the solid are $12 \times 2 = 24$ *cubic feet.* Hence, *to find the solid contents of a rectangular solid:*



RULE. *Multiply together the numbers representing its dimensions, expressed in units of the same denomination.*

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot . . .	cu. ft.
27 cubic feet	= 1 cubic yard . . .	cu. yd.
128 cubic feet	= 1 cord	C.

REMARKS. 1. The *unit of measure* for solids is a *cube* whose edge is a linear inch, foot, or yard.

2. A cord is 8 feet long, 4 feet wide, and 4 feet high.

3. A *perch of stone* or *gravel* is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide, and 1 foot high. It contains $24\frac{3}{4}$ cubic feet.

Reduce:

1. 24 cords 24 cu. ft. to cubic feet.
2. 2 cords 16 cu. ft. 444 cu. in. to cubic inches.
3. 5 cu. yd. 9 cu. ft. 1168 cu. in. to cubic inches.
4. 96 cords 49 cu. ft. to cubic yards.
5. 4 cords 66 cu. ft. 1216 cu. in. to cu. in.

6. 2985984 cu. in. to cubic yards.

7. 12799 cubic feet to cords.

8. 276980 cu. in. to cubic yards.

9. 156 cu. yd. 12 cu. ft. to cords.

10. 1990656 cu. in. to cords.

11. How many cubic yards of earth were removed from a cellar 39 ft. long, 24 ft. wide, and 6 ft. deep?

12. How many cords of wood are in a pile 80 ft. long, 8 ft. wide, and 5 ft. high?

13. How many headstones, each 30 in. long, 18 in. wide, and 4 in. thick would be equal in volume to a block of marble 5 ft. long, 3 ft. wide, and 2 ft. thick?

14. A street 2250 ft. long and 48 ft. wide is to be graded to a depth of 2 feet. How much will the excavation cost at $0.20 a cubic yard?

15. A squared log is 24 ft. long, and 18 in. square at the ends. How many boards 12 ft. long, 6 in. wide, and 1 in. thick can be sawn from it?

16. How many cubical blocks two inches high could be placed in a cubical box 20 inches deep?

17. A tank is 5 ft. 6 in. long, 3 ft. wide, and 2 ft. 4 in. deep. How many gallons will it hold, allowing 231 cubic inches to a gallon?

18. How many cakes of toilet soap 4 inches long, 3 inches wide, and 1 in. thick can be packed in a box 18 in. long, 12 in. wide, and 6 in. deep?

19. A bin is 7 ft. 2 in. long, 3 ft. 9 in. wide, and 3 ft. 4 in. deep. How many bushels of wheat will fill it, allowing 2150 cu. in. to a bushel?

20. How many posts 4 in. square and 5 ft. long could be sawed from a log 20 in. square, and 15 ft. long?

21. How many cans of oysters, each 6 in. long, 5 in. wide, and 2 in. deep can be packed in a box measuring 30 in. in length, 15 in. in width, and 12 in. in depth inside?

22. How many starch boxes, each 2 ft. long, 16 in. wide, and 7 in. high, could be piled in a car 28 ft. long, 8 ft. wide, and 7 ft. high inside?

23. $25 was paid for digging a cellar 30 ft. long by 18 ft. wide, at $0.25 a cubic yard. What was its depth?

24. A reservoir is 105 ft. long, 66 ft. wide, and 20 ft. deep. How long would it require 3 pipes, each discharging 24 gal. per minute, to empty it, allowing 60 minutes to an hour, and 24 hours to a day?

25. A kiln of brick was 64 ft. long, 25 ft. wide, and 10 ft. high. If each brick was 8 in. long, 4 in. wide, and 2 in. thick, what was the value of the kiln at $12.50 per 1000 bricks, allowing one ninth for waste?

REMARK. In reviews, the last 15 of these problems may be quickly solved by *cancellation*.

TIME MEASURE

62. *Time* is the measure of duration.

TABLE

60 seconds (sec.)	= 1 minute . . .	min.
60 minutes	= 1 hour	hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
365 days	= 1 year	yr.
366 days	= 1 leap year.	
12 calendar months	= 1 year.	
100 years	= 1 century . . .	C.

REMARKS. 1. A *year* is the length of time required for one revolution of the earth around the sun. The exact time is 365 days 5 hours 48 minutes 46 seconds, or 11 minutes 14 seconds less than 365¼ days. In four years this excess amounts to almost an extra day, which is given to the month of February. There is an error of 44 minutes 56 seconds by this addition, and to correct it none of the centennial years are regarded as leap years excepting those divisible by 400. Thus, the year 1900 was *not* a leap year, while the year 2000 will be one. Hence, those centennial years whose numbers are divisible by 400, and all other years whose numbers are divisible by 4, are *leap years*.

2. In most civilized countries the year is divided into 12 *calendar months*. The names of these months in their order, and the number of days belonging to each, are as follows:

1st.	January, 31 days.	7th.	July, 31 days.
2d.	February, 28 or 29 days.	8th.	August, 31 days.
3d.	March, 31 days.	9th.	September, 30 days.
4th.	April, 30 days.	10th.	October, 31 days.
5th.	May, 31 days.	11th.	November, 30 days.
6th.	June, 30 days.	12th.	December, 31 days.

REMARK. In business transactions and in interest it is customary to consider 30 days as a month.

1. Reduce 6 da. 22 hr. 40 min. to minutes.

2. Reduce 4 da. 19 hr. 41 min. 40 sec. to seconds.

3. Reduce 1 wk. 2 da. 3 hr. 4 min. 5 sec. to seconds.

4. Reduce 1 leap year to minutes.

5. How many seconds in January, February, and March, 1904?

6. Reduce 604,799 sec. to days.

7. Reduce 2,635,200 min. to leap years.

8. Reduce 27,043,200 sec. to days.

9. Reduce 525,600 min. to weeks.

10. What is the exact number of seconds in a year?

11. There were 52 Sundays and 6 legal holidays in the year 1900. If a mechanic worked 10 hours a day for every working day in that year, and his wages were 25 cents an hour, how much did he earn?

12. Two cities are situated upon opposite sides of the Atlantic, 3000 miles apart. In what time can a vessel sail from one to the other at the rate of 12 miles per hour?

13. A courier carried a message between two stations in 9 hr. 10 min., traveling at the rate of 8 ft. per second. What was the distance?

14. An engine running from Cincinnati to New Orleans consumed 4 tons of coal at the rate of 5 lb. per minute. In what time was the trip made?

15. A reservoir with a capacity of 2,332,800 gallons was filled by three pipes, each discharging 9 gallons per second. In what time was it filled?

16. A farmer mowed a meadow in 12 days of 10 hours each, at the rate of 20 sq. rods per hour. How many acres were in the meadow?

17. A party of laborers dug a channel 3240 yd. long, 48 ft. wide, and 5 ft. deep, at the rate of two cubic yards per minute. If they worked 9 hours a day, how many days were they employed?

18. If an express train runs 3 rods per second, in what time will it run 135 miles?

19. If a man can weave one inch of rag carpet in three minutes, in how many days of 12 hr. each could he weave the carpet for a room 15 ft. long, and 12 ft. wide?

20. A flouring mill ground a quantity of wheat in 15 days, running day and night, at the rate of one pint in two seconds. How many bushels were ground?

Circular Measure

63. 1. *Circular measure* is used for measuring angles, circles, latitude, and longitude.

2. A *circle* is a plane figure bounded by a curved line, every part of which is equally distant from a point within called the *center.*

3. The *circumference* of a circle is the curved line that bounds it. For purposes of measurement it is divided into 360 equal parts called *degrees;* each degree is divided into 60 equal parts called *minutes;* and each minute into 60 equal parts called *seconds.*

TABLE

60 seconds (″) = 1 minute . . . ′.
60 minutes = 1 degree . . . °.
360 degrees = 1 circumference . . . cir.

Remark. The circumference is also divided into four *quadrants* of 90 degrees each, or into twelve *signs* of 30 degrees each.

64. Miscellaneous Tables

1. *Counting.*
12 units = 1 dozen . . . doz.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

2. *Paper.*
24 sheets of paper = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.

3.

100 pounds of grain or seed	= 1 cental.
100 pounds of nails	= 1 keg.
196 pounds of flour	= 1 barrel.
200 pounds of beef or pork	= 1 barrel.
240 pounds of lime	= 1 cask.

1. A bushel of shelled corn weighs 56 lb. What is the value of 84 centals of shelled corn at 67 cents a bushel?

2. A field is 15 chains wide by 22 chains long. If it yields 45 bu. of barley per acre, what would be the value of the crop at $0.45 per bushel?

3. 16 barrels of flour costing $3.85 per bbl., were emptied into 14-pound sacks which were sold at $0.35 per sack. What was the entire profit?

4. A wholesale dealer sold a quantity of beef for $1275 at $0.075 a pound. How many barrels were sold?

5. A merchant sold 25 bundles of paper at $0.006 per sheet, thereby gaining $24. What was the cost per bundle?

6. 80 gross of buttons were sold for $0.08 a dozen, by which $19.20 was lost. What was the cost per dozen?

7. A quantity of silver was manufactured into society badges weighing 5 pwt. each, which were sold for $288 at $57.60 per gross. What was the weight of the silver?

8. I bought 4500 marbles at 3 cents per dozen, and sold them at 5 cents per score. What was my gain or loss?

9. 11 tons 5 cwt. of iron was made into nails which were sold for $4.50 per keg. What was their value?

10. An army of 19,200 men consumed 1 lb. 2 oz. of pork apiece daily on an eight days' march. What was the value of all the pork at $12.50 a barrel?

11. 1 ton 1 cwt. 60 lb. of baking powder was shipped in boxes, each containing two dozen eight-ounce packages. What was its entire value at $4.50 a box?

12. An advertising firm ordered 216,000 circulars. If they cost 50 cents a ream, how much was paid for them?

13. The rations for an army during one week included 16,800 loaves of bread daily. Allowing 12 oz. of flour to each loaf, what was the cost of all the flour at $5 per bbl.?

14. 17 lb. 8 oz. of steel was made into pens. Allowing 14 oz. to a great gross, what was their entire value at 5 ct. a dozen?

REVIEW PROBLEMS IN TABLES

65. *Find the cost of:*

1. 15 bu. of berries at 6 cents a quart.
2. 24 gal. of wine at 5 cents a gill.
3. 5 tons of hay at 6 mills a pound.
4. 20 yards of lace at 3 cents an inch.
5. A lot 50 yards square at 4 ct. a square foot.
6. A cubic yard of marble at 3 mills a cubic inch.
7. 50 days' house rent at 2 mills a minute.
8. 12 pounds silver bullion at 4 ct. a pennyweight.
9. 9 pounds of opium at 4 ct. a scruple.

10. How many three-inch cubes could be placed in a box 15 inches square and 12 inches deep inside?

11. A jeweler made 3 lb. 4 oz. of gold into rings weighing 5 pwt. each, which he sold at $6.25 apiece. What was their entire value?

12. How many dozen quart bottles could be filled from 15 hhd. of wine?

13. How many visiting cards 2 in. wide by 3 in. long could be cut from 25 sq. yd. of cardboard?

14. If a man's income were $0.015 per minute, what would it be for the month of February, 1904?

15. 9 T. 18 cwt. of iron was made into telephone wire weighing 4 oz. to a yard. What was the total length?

16. A druggist compounded 1 ℔ 3 ℥ 5 ʒ of medicines into five-grain tablets, which he sold at 20 ct. a dozen. How much did he receive for them?

17. How much would it cost to paper the walls of a room 19 ft. long, 14 ft. wide, and 9 ft. high, at $0.175 a square yard?

18. 5 barrels of flour were made into loaves of bread. Allowing 14 oz. to each loaf, what was their entire value at 6 cents apiece?

19. How much silver would be required to make 15 dozen spoons, each weighing 16 pwt.?

20. How many tons of coal can be stored in a bin 12 ft. long, 5 ft. wide, and 3 ft. deep, allowing 30 cubic feet to a ton?

21. A tract of land one mile square was divided into farms 80 rods square. What was the value of the farms at $2500 each?

22. 75 bu. of peaches were packed in baskets, each containing 1 pk. 7 qt., and sold at $0.625 a basket. What was their entire value?

23. A block of granite is 6 ft. long, 4 ft. wide, and 3 ft. thick. How many tons does it weigh, allowing 250 lb. to a cubic foot?

24. If an express train runs 3 rods per second, in what time will it travel 810 miles?

25. A field 64 rd. long and 50 rd. wide produced 88 bu. of potatoes per acre. What was the value of the crop at $0.875 per bushel?

26. How many thousand bricks 8 in. long, 4 in. wide, and 2 in. thick, can be piled in 5 cars 30 ft. long, 8 ft. wide, and 2 ft. 6 in. deep inside?

27. A log 25 ft. long and 20 in. square at the ends was sawn into posts 5 in. square and 5 ft. long. What was their value at 25 ct. apiece?

28. How much would it cost to plaster a schoolroom 45 ft. long, 36 ft. wide, and 15 ft. high at $0.325 a square yard?

29. Thirteen bushels of chestnuts cost $62.40. How much was that per pint?

30. If a man's income is 8 mills a minute, what would it be for the year 1908?

31. A rectangular block of marble 5 ft. long, 3 ft. wide, and 2 ft. thick was sawn into tiles one foot square and one inch thick, and all were used for the floor of an office. If the floor was 24 ft. long, what was its width?

32. A rick of straw was sold at 2 mills a pound for $25. How much did it weigh?

33. $30 was paid for four-grain pills at 25 cents a dozen. What was their total weight?

34. If steel rails weigh 50 lb. per yard, how many tons would be required for a railroad track 101 miles in length?

35. If the driving wheels of a locomotive are 15 ft. in circumference, how many revolutions would each make in a trip over the above road?

36. 318 sheets of cardboard, each 20 in. wide by 30 in. long, were cut into packs of 53 cards, each 3 in. wide and 4 in. long, which were sold at $1.25 per dozen packs. How much was realized?

37. 5 hogsheads of wine were drawn into bottles holding 1 pt. 3 gi. each, and packed in baskets, each holding one dozen bottles. What was their total value at $5 a basket?

38. A company of woodcutters earned $607.50 in 5 weeks, at the rate of $1.125 a cord. How many cords did they chop daily?

39. A mechanic agreed to work 8 hr. a day, at the rate of 5 mills per minute. If his wages amounted to $237.60, how many days did he work?

40. What quantity of quinine will be required for the number of four-grain pills that will sell for $36, at 40 ct. per box of 25 pills?

41. One week's rations of bread for a regiment cost $588 at 5 ct. a loaf. If 12 oz. of flour were required for each loaf, how many barrels were used?

42. In the preceding question, if two loaves per day were allowed each man, how many men were there?

43. 45 bu. of pecans cost $1.75 a bushel, and were retailed at 8 cents a quart. How much was gained?

44. 5 pounds of silver was manufactured into badges weighing 6 pwt. apiece, all of which were sold for $100. What was the value of each?

45. If a cubic inch of gold weighs 9 oz. 12 pwt., what would be the value of a bar of gold 6 in. long, 3 in. wide, and 2 in. thick, at 4 ct. a grain?

46. I bought berries at 48 ct. a peck and sold them at four cents a pint, thereby gaining $9.60. How many bushels were sold?

47. A reservoir was emptied in 15 hr. by a pipe that discharged 1 gal. 3 gi. per second. How many barrels of 31 gal. 2 qt. each did it hold?

48. 20 bu. of pears were packed in baskets, each containing 1 pk. 2 qt., and all were sold for $28.80. What was the price per basket?

49. A field is 45 rd. wide, and its area is 21 A. 15 sq. rd. Find the cost of fencing it at 75 ct. a rod.

50. A concert hall is 45 ft. long, 36 ft. wide, and 15 ft. high. How much would it cost to plaster it at 33 ct. a square yard?

51. A pile of wood is 56 ft. long, 8 ft. wide, and 6 ft. high. Find its value at $4.80 per cord.

52. An express train ran a certain distance in 4 hr. 30 min. at the rate of 44 ft. per second. What was the distance?

53. A crop of corn from a field 50 rd. long by 48 rd. wide was sold for $565.50, at 65 ct. a bushel. How many bushels did each acre yield?

54. The slate blackboards for a school building of 12 rooms cost $907.20, at 35 ct. a square foot. How many square yards were required for each room?

55. A hound running 72 rods per minute is chasing a fox whose speed is 60 rd. per minute. If the fox has 300 rods the start, in what time will the hound catch him?

56. A poulterer has 450 turkeys. If he feeds each a pint of corn daily for 80 days, what would be the cost of the corn at $0.48 per bushel?

57. The distance from Cincinnati to Chicago is 315 miles. In what time could a carrier pigeon make the flight at the rate of 4 rd. per second?

58. 75 planks, each 16 ft. long and 15 in. wide, were sawn into palings 4 ft. long by 3 in. wide, which were sold at $3.33 per hundred. What was their value?

59. The fore wheel of a wagon is 9 ft. in circumference, and the hind wheel 11 ft. How many more revolutions will the former make than the latter in traveling 6 miles?

60. A quantity of gold was made into sleeve buttons weighing 3 pwt. 8 gr. each, which were sold for $270, at $7.50 a pair. How much gold was used?

61. A barge was loaded with freestone sills, each 4 ft. long, 8 in. wide, and 4 in. thick. Their total cost was $2400, at $1.25 a cubic foot. What was the number of sills?

62. A twelve-acre field is 48 rd. long, and is surrounded by a wire fence of four strands. What length of wire was used?

63. A cask of wine was bought for $1.20 a gallon and retailed at the rate of 22 ct. 5 m. a pint, thereby yielding a profit of $25.20. How many gallons were in the cask?

64. A reception room is 33 ft. long, 21 ft. wide, and 13 ft. high. Find the total cost of painting the ceiling at 25 ct. a square yard, papering the walls at 15 ct. a square yard, and carpeting the floor at 75 ct. a square yard.

65. 125 gross of colored crayons at 4 ct. a score were exchanged for 4500 steel pens at 6 ct. a dozen and 45 dozen lead pencils. What was the value of each pencil?

66. A reservoir 99 ft. long, 50 ft. wide, and 21 ft. deep was filled by three pipes, each discharging 12 gal. per second. In what time was it filled?

ADDITION OF COMPOUND NUMBERS

66. Addition of compound numbers is the process of finding the sum of two or more numbers of the *same kind*, but of *different denominations*.

Find the sum of 12 bu. 2 pk. 4 qt. 1 pt., 13 bu. 1 pk. 5 qt. 1 pt., and 14 bu. 3 pk. 1 pt.

OPERATION

bu.	pk.	qt.	pt.
12	2	4	1
13	1	5	1
14	3	0	1
40	3	2	1

EXPLANATION. The numbers are written so that units of the same denomination stand in the same column. Beginning at the right and adding, the sum is 3 (pints) equal to 1 qt. 1 pt. Write 1 (pint) under its column, and add 1 (quart) to the next column; the sum is 10 (quarts) equal to 1 pk. with a remainder of 2 quarts. Write 2 (quarts) under its column, and add 1 (peck) to the next column. The sum is 7 (pecks) equal to 1 bu. 3 pk. Write 3 (pecks) under its column, and add 1 (bushel) to the last column. The sum is 40 (bushels), which is written under its column. The result is 40 bu. 3 pk. 2 qt. 1 pt.

RULE. 1. *Write the compound numbers in order, placing units of the same denomination in the same column.*

2. *Begin at the right, add each column separately, and if the sum is less than a unit of the next higher denomination, write it under the column added.*

3. *If the sum in any column is greater than a unit of the next higher denomination, divide it by the number which forms that unit. Write the remainder under the column added, and add the quotient to the next column.*

4. *Proceed in the same manner with all the columns, and write the entire sum under the last.*

REMARKS. 1. In writing the numbers, the places of vacant denominations should be supplied with ciphers.

2. It will be found convenient to write the numbers representing the units of the tables over the quantities in addition, subtraction, multiplication, and division of compound numbers as in the preceding example.

Find the sum of:

1.

bu.	pk.	qt.	pt.
22	3	4	1
24	2	5	0
25	0	7	1
27	2	0	1

2.

gal.	qt.	pt.	gi.
14	1	1	1
17	0	1	2
20	2	0	1
22	3	1	0

3.

T.	cwt.	lb.	oz.
5	15	55	5
6	16	66	6
7	17	77	7
8	18	88	8
9	19	99	9

4.

cu. yd.	cu. ft.	cu. in.
23	14	648
34	17	501
45	12	826
56	10	564
67	26	917

5.

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
13	47	3	7	103
17	56	4	2	99
19	72	5	8	68
23	80	6	5	87
25	65	7	6	75

6.

C.	cu. ft.	cu. in.
10	49	723
12	106	485
14	92	831
17	127	679
19	34	510

7.

wk.	da.	hr.	min.	sec.
8	1	6	46	38
9	2	13	38	56
10	3	9	57	49
11	4	12	29	28
12	3	10	55	59

8.

mi.	rd.	yd.	ft.	in.
17	143	1	1	6
18	87	0	1	4
20	128	1	1	9
21	205	0	1	3
22	76	1	1	8

9.

lb.	oz.	pwt.	gr.
4	5	6	7
7	8	9	10
10	9	8	7
6	5	4	3
3	2	1	5

10.

℔	℥	ʒ	℈	gr.
6	8	6	2	10
8	0	7	1	16
9	6	0	2	15
12	4	3	0	19
13	3	5	1	0

11. Six loads of barley measured as follows: 37 bu. 5 qt. 1 pt.; 41 bu. 2 pk. 1 qt.; 38 bu. 3 pk. 1 pt.; 40 bu. 7 qt. 1 pt.; 39 bu. 1 pk. 6 qt.; 42 bu. 2 pk. 1 pt. What was the total quantity?

12. Seven barrels of cider contained respectively: 42 gal. 2 qt. 3 gi.; 44 gal. 1 pt. 2 gi.; 42 gal. 3 qt. 1 pt.; 44 gal. 1 qt. 1 gi.; 41 gal. 1 pt. 3 gi.; 43 gal. 3 qt. 1 pt.; 40 gal. 2 qt. 2 gi. What were the entire contents?

13. Six loads of castings weighed: 2 T. 12 cwt. 39 lb. 7 oz.; 3 T. 8 cwt. 16 lb. 15 oz.; 1 T. 19 cwt. 85 lb. 14 oz.; 2 T. 15 cwt. 93 lb. 10 oz.; 3 T. 7 cwt. 48 lb. 12 oz.; 1 T. 16 cwt. 15 lb. 6 oz. How much did all weigh?

14. The excavation for a ditch was made in seven days as follows: 54 cu. yd. 26 cu. ft. 912 cu. in.; 49 cu. yd. 15 cu. ft. 1407 cu. in.; 52 cu. yd. 12 cu. ft. 764 cu. in.; 50 cu. yd. 19 cu. ft. 1396 cu. in.; 48 cu. yd. 17 cu. ft.

953 cu. in.; 51 cu. yd. 24 cu. ft. 1681 cu. in.; 52 cu. yd. 10 cu. ft. 1527 cu. in. What was the total excavation?

15. Five piles of wood contained the following quantities; 27 C. 108 cu. ft. 1369 cu. in.; 28 C. 99 cu. ft. 1296 cu. in.; 29 C. 120 cu. ft. 1417 cu. in.; 30 C. 84 cu. ft. 1528 cu. in.; 32 C. 97 cu. ft. 1302 cu. in. What was the amount?

16. Six farms measured respectively: 80 A. 96 sq. rd. 2 sq. yd. 7 sq. ft.; 95 A. 130 sq. rd. 3 sq. yd. 5 sq. ft.; 108 A. 84 sq. rd. 4 sq. yd. 2 sq. ft.; 76 A. 101 sq. rd. 5 sq. yd. 4 sq. ft.; 123 A. 79 sq. rd. 6 sq. yd. 8 sq. ft.; 89 A. 150 sq. rd. 7 sq. yd. 1 sq. ft. What was the total area?

17. The time of an ocean steamer in seven trips was as follows: 10 da. 8 hr. 36 min. 39 sec.; 9 da. 21 hr. 15 min.; 11 da. 6 hr. 50 sec.; 12 da. 24 min. 16 sec.; 10 da. 14 hr. 19 min.; 9 da. 23 hr. 45 sec.; 11 da. 13 min. 30 sec. How much time was consumed?

18. Eight bars of silver weighed respectively: 4 lb. 8 oz. 6 pwt. 7 gr.; 4 lb. 6 oz. 18 pwt. 16 gr.; 4 lb. 10 oz. 12 pwt. 13 gr.; 4 lb. 11 oz. 15 pwt. 21 gr.; 4 lb. 9 oz. 13 pwt. 15 gr.; 4 lb. 4 oz. 9 pwt. 12 gr.; 4 lb. 6 oz. 17 pwt. 19 gr.; 4 lb. 7 oz. 5 pwt. 17 gr. How much did all weigh?

19. Seven shipments of drugs weighed: 10 ℔ 5 ʒ 1 ℈ 12 gr.; 11 ℔ 9 ℥ 2 ℈ 15 gr.; 12 ℔ 8 ℥ 7 ʒ 10 gr.; 13 ℔ 7 ℥ 3 ʒ 1 ℈; 14 ℔ 6 ʒ 1 ℈ 18 gr.; 15 ℔ 10 ℥ 1 ℈ 13 gr.; 16 ℔ 6 ℥ 4 ʒ 7 gr. What was the total weight?

20. A letter carrier walked the following distances in 6 days: 8 mi. 167 rd. 1 ft. 6 in.; 7 mi. 85 rd. 1 yd. 8 in.;

8 mi. 203 rd. 1 ft.; 8 mi. 1 yd. 1 ft. 7 in.; 8 mi. 74 rd. 1 ft. 11 in.; 9 mi. 110 rd. 1 yd. 10 in. How far did he walk altogether?

SUBTRACTION OF COMPOUND NUMBERS

67. Subtraction of compound numbers is the process of finding the difference between two numbers of the *same kind*, but of *different denominations*.

From 14 gal. 1 pt. 1 gi. subtract 8 gal. 1 qt. 2 gi.

OPERATION

	4	2	4
gal.	qt.	pt.	gi.
14	0	1	1
8	1	0	2
5	3	0	3

EXPLANATION. The numbers are written, the less under the greater, so that units of the same denomination stand in the same column. Beginning at the right, 2 gills cannot be subtracted from 1 gill. Borrowing 1 pint, reducing it to gills and adding 1 gill makes 5 gills. Subtracting 2 gills from 5 gills, the remainder is 3 gills, which is written under its column. As the 1 pint in the minuend has been reduced to gills 0 pints remain. Subtracting 0 pints from 0 pints leaves 0 pints, which is written under its column. 1 quart cannot be subtracted from naught. Borrowing 1 gallon and reducing it to quarts makes 4 quarts. Subtracting 1 quart from 4 quarts, the remainder is 3 quarts, which is written under its column. As 1 gallon has been reduced to pints only 13 gallons remain in the minuend. 8 gallons from 13 gallons leaves 5 gallons, which is written under its column. The result is 5 gal. 3 qt. 3 gi.

RULE. 1. *Write the subtrahend under the minuend, placing units of the same denomination in the same column.*

2. *Begin at the right, and if the lower number is less than the one above it, write the difference beneath them.*

3. *If any number in the subtrahend is greater than that above it, borrow a unit from the next higher denomi-*

nation in the minuend, reduce it to the lower denomination, add the given number of the latter denomination, and then subtract.

4. *After decreasing by* 1, *the next number in the minuend, proceed as before with the remaining denominations.*

REMARK. The general principles of addition and subtraction of compound numbers are the same as those of simple numbers, *allowing for the difference of their scales.*

Find the difference between:

1.

bu.	pk.	qt.	pt.
50	3	7	1
17	0	4	1

2.

gal.	qt.	pt.	gi.
46	2	1	1
24	1	1	2

3.

T.	cwt.	lb.	oz.
35	7	49	12
17	13	74	14

4.

cu. yd.	cu. ft.	cu. in.
96	24	1296
47	24	1725

5.

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
159	159	25	4	120
80	99	13	7	121

6.

mi.	rd.	yd.	ft.	in.
79	180	5	1	11
28	250	4	2	6

7.

C.	cu. ft.	cu. in.
100	0	0
99	99	99

8.

wk.	da.	hr.	min.	sec.
50	2	10	37	15
26	3	11	40	30

9.

lb.	oz.	pwt.	gr.
49	10	11	12
38	10	11	13

10.

℔	℥	ʒ	℈	gr.
12	0	5	0	9
9	6	0	2	19

11. A farmer having raised 512 bu. 1 pk. 4 qt. of barley, sold 184 bu. 3 pk. 5 qt. 1 pt. at one time, and 177 bu. 1 pk. 6 qt. 1 pt. at another. How many bushels remained?

12. A dealer having 750 gal. of wine, disposed of it to three customers. The first took 249 gal. 3 qt. 1 gi.; the second, 251 gal. 3 qt. 3 gi. How much did the third take?

13. Three shipments of marble weighed 5 T. 2 cwt. 49 lb. 15 oz. The first weighed 1 T. 12 cwt. 50 lb. 10 oz.; the second, 1 T. 15 cwt. 48 lb. 9 oz. What was the weight of the third?

14. A cyclist traveled 156 mi. 98 rd. 4 yd. in two days. If he went 75 mi. 216 rd. 2 ft. the first day, how far did he travel on the second?

15. A owns 88 A. 72 sq. rd. 17 sq. yd. 5 sq. ft. of land. B owns 17 A. 144 sq. rd. 4 sq. yd. 8 sq. ft. less. How much land have they together?

16. A, B, and C dug a cellar 24 ft. long, 15 ft. wide, and 6 ft. deep. A dug 25 cu. yd. 20 cu. ft. 1024 cu. in.; B dug 26 cu. yd. 15 cu. ft. 1260 cu. in. How much did C dig?

17. C cut 15 cords 99 cu. ft. of wood; D cut 1 cord 108 cu. ft. less than C, and E cut 3 cords 81 cu. ft. less than D. How many cords were cut altogether?

18. An ocean steamer from New York to Queenstown made the trip in 6 da. 12 hr. 49 min. 30 sec., while a sailing vessel occupied 3 wk. 2 da. 5 hr. 30 min. 20 sec. on the same voyage. How much time was gained by the steamer?

19. A druggist compounded 7 ℔ 4 ℥ 7 ʒ 1 ℈ 8 gr. of drugs into medicines, of which he sold 2 ℔ 7 ℥ 6 ʒ 2 ℈ 12 gr. to one physician, and 2 ℔ 10 ℥ 5 ʒ 16 gr. to another? How much remained?

20. A jeweler bought 12 lb. of silver, of which he made 4 lb. 8 oz. 15 pwt. 6 gr. into knives, which was

1 lb. 2 oz. 18 pwt. 12 gr. more than he made into forks. He manufactured spoons of the remainder. What was their weight?

Difference of Time between Two Dates

68. In finding the difference of time between two dates, it is customary to consider 30 days as a month, and twelve months as a year.

What length of time elapsed from July 26, 1839, to May 11, 1900?

OPERATION

yr.	12 mo.	30 da.
1900	5	11
1839	7	26
60	9	15

Explanation. In writing the dates, the numbers representing the years, the months, and the days of the months are written in order. May is the fifth month of the year; July is the seventh. Then the later date is 1900 yr. 5 mo. 11 da., and the earlier one 1839 yr. 7 mo. 26 da. Subtracting, the difference is 60 yr. 9 mo. 15 da.

Rule. *Write the earlier date under the later one, and find the difference of the compound numbers.*

1. President Lincoln was born February 12, 1809, and he died April 15, 1865. What was his age?

2. A note was dated Dec. 12, 1898, and paid Mar. 22, 1903. How long did it run?

3. John Adams signed the Declaration of Independence July 4, 1776, and died July 4, 1826. How long did he live after signing it?

4. What time elapsed from the birth of Shakespeare, April 23, 1564, to that of Robert Burns, Jan. 25, 1759?

5. How long was it from the discovery of America, Oct. 12, 1492, to the beginning of the Civil War, April 12, 1861?

6. Queen Elizabeth was crowned Nov. 17, 1558, and died Mar. 24, 1603. How long did she reign?

7. Washington was inaugurated president April 30, 1789. McKinley, March 4, 1897. What is the difference of the dates?

8. A note was paid June 9, 1904, that had been dated 4 yr. 9 mo. 24 da. before. When was it dated?

9. A man was born May 7, 1874. His sister is 5 yr. 9 mo. 19 days older, and his brother is 4 yr. 7 mo. 27 da. older than the sister. What are the birth-dates of the sister and the brother?

10. The battle of New Orleans was fought Jan. 8, 1815. General Andrew Jackson was born 47 yr. 9 mo. 23 da. before that date, and died June 8, 1845. What was his age?

Difference of Time in Days between Two Dates

69. How many days elapse from April 14 to October 11?

Solution. There are 16 days remaining in April. May has 31 days; June, 30; July, 31; August, 31; September, 30; and there are 11 in October. The sum of these is 180 days.

1. Find the number of days from March 4 to June 23.

2. How many days elapsed from August 7 to January 1?

3. What was the number of days from Jan. 9, 1908, to the 6th of July following?

4. A note dated Oct. 19, 1905, was paid Mar. 18 following. How many days did it run?

5. The battle of Murfreesboro began Dec. 31, 1862; that of Gettysburg, July 1, 1863. How many days after the former was the latter fought?

6. A traveler started on a journey around the world Sept. 4, 1907, and arrived at home June 30 following. How many days was he absent?

7. A salesman was employed Feb. 19, 1909, and discharged on Dec. 31 of the same year. If his salary was $15 per week, how much did he earn?

8. A note was dated May 17, and paid 100 days afterward. What was the date of payment?

9. A Government building was completed on Dec. 31. If 280 days were occupied in constructing it, when was it begun?

10. A clerk was employed Jan. 26, 1900, at $12.50 per week, and discharged when he had earned $550. What was the date of his discharge?

MULTIPLICATION OF COMPOUND NUMBERS

70. Multiplication of compound numbers is the process of taking a compound number a given number of times.

Multiply 6 bu. 2 pk. 3 qt. 1 pt. by 7.

OPERATION

	4	8	2
bu.	pk.	qt.	pt.
6	2	3	1
			7
46	1	0	1

EXPLANATION. Beginning at the right, 7 times 1 pint are 7 pints, equal to 3 quarts 1 pint. Write 1 (pint) under its column, and carry 3 quarts to the product of the quarts. 7 times 3 quarts are 21 quarts, and 3 quarts added are 24 quarts, equal to 3 pecks, with no remainder. Write a cipher under quarts' column, and carry 3 pecks to the product of the pecks. 7 times 2 pecks are 14 pecks, and 3 pecks added are 17 pecks, equal to 4

bushels 1 peck. Write 1 (peck) under its column, and carry 4 bushels to the product of the bushels. 7 times 6 bushels are 42 bushels, and 4 bushels added are 46 bushels, which is written under its column. The result is 46 bu. 1 pk. 1 pt.

RULE. 1. *Write the multiplier under the lowest denominator of the multiplicand.*

2. *Beginning with the lowest denomination, multiply as in simple numbers, and reduce as in addition of compound numbers.*

REMARK. When the multiplier is a large *composite number* it may be separated into factors before multiplying.

Multiply:

1. 7 bu. 3 pk. 4 qt. 1 pt. by 7.
2. 10 gal. 2 qt. 1 pt. 2 gi. by 9.
3. 5 T. 6 cwt. 7 lb. 8 oz. by 12.
4. 21 rd. 1 ft. 2 in. by 15.
5. 32 A. 4 sq. rd. 1 sq. yd. 4 sq. ft. 15 sq. in. by 20.
6. 12 C. 64 cu. ft. 216 cu. in. by 32.
7. 1 wk. 3 da. 3 hr. 30 min. 30 sec. by 36.
8. 4 lb. 8 oz. 10 pwt. 16 gr. by 45.
9. 1 ℔ 6 ℥ 7 ʒ 1 ℈ 10 gr. by 64.
10. 12 cu. yd. 4 cu. ft. 216 cu. in. by 72.
11. 16 acres of barley averaged 48 bu. 1 pk. 4 qt. 1 pt. What was the entire crop?
12. 25 barrels of petroleum averaged 45 gal. 3 qt. 1 pt. 2 gi. What was the total quantity?

13. How many yards of cloth are there in 36 bolts, each containing 27 yd. 2 ft. 4 in.?

14. A crop of hay was hauled to market in 40 loads whose average weight was 1 T. 3 cwt. 43 lb. 12 oz. What was the entire weight?

15. A tract of land was divided into 48 lots, each measuring 50 sq. rd. 5 sq. ft. 90 sq. in. What was the area of the tract?

16. 50 men were required to complete a certain work in 6 da. 6 hr. 14 min. 24 sec. In what time could one man have done it?

17. 1 cu. yd. 15 cu. ft. 324 cu. in. of earth was removed from a cellar in each of 64 loads. If the cellar was 30 ft. long and 15 ft. wide, what was its depth?

18. If a laborer can cut 1 C. 94 cu. ft. 384 cu. in. of wood in a day, how much could he cut in 72 days?

19. A company of 80 gold miners averaged 2 lb. 4 oz. 2 pwt. 12 gr. apiece during a certain month. What was the value of all the gold at $20 an ounce?

20. 96 packages of drugs had an average weight of 1 ℔ 3 ʒ 5 gr. What was the total weight?

DIVISION OF COMPOUND NUMBERS

71. Division of compound numbers is the process of:

1. Separating a compound number into a given number of equal parts.

2. Dividing one compound number by another compound number.

(*a*) Divide 43 bu. 1 pk. 6 qt. 1 pt. by 9.

OPERATION

	4	8	2
bu.	pk.	qt.	pt.
9)43	1	6	1
4	3	2	1

EXPLANATION. Dividing the highest denomination first, $\frac{1}{9}$ of 43 bushels is 4 bushels, with a remainder of 7 bushels. Write 4 under bushels, reduce the remainder to pecks and add 1 peck, making 29 pecks. $\frac{1}{9}$ of 29 pecks is 3 pecks, with a remainder of 2 pecks. Write 3 under pecks, reduce the remainder to quarts and add 6 quarts, making 22 quarts. $\frac{1}{9}$ of 22 quarts is 2 quarts, with a remainder of 4 quarts. Write 2 under quarts, reduce the remainder to pints and add 1 pint, making 9 pints. $\frac{1}{9}$ of 9 pints is 1 pint. Write 1 under pints. The result is 4 bu. 3 pk. 2 qt. 1 pt.

RULE. 1. *Write the divisor at the left of the dividend, as in division of simple numbers.*

2. *Beginning with the highest denomination, divide the numbers in their order, and, if there is no remainder, write the quotients beneath them.*

3. *When a remainder occurs after dividing any denomination, reduce it to the next lower denomination, add to it the number of that denomination, and proceed as before.*

REMARKS. 1. When the divisor is a large composite number, it may be separated into two or more factors before dividing.

2. It should be remembered that in *multiplication* of compound numbers *the remainder is written*, and *the quotient* carried, while in *division the quotient is written*, and *the remainder reduced to the next lower denomination*.

(*b*) How many times can a vessel holding 2 gal. 2 qt. 1 pt. be filled from a tank containing 126 gal.?

SOLUTION. 2 gal. 2 qt. 1 pt. = 21 pt.; 126 gal. = 1008 pt. $1008 \div 21 = 48$ times.

Hence, to divide one compound number by another compound number:

RULE. *Reduce both quantities to the lowest denomination given, and divide the greater by the less.*

Divide:

1. 100 bu. 1 pk. 1 qt. 1 pt. by 7.
2. 46 T. 2 cwt. 2 lb. 8 oz. by 8.
3. 200 gal. 1 qt. by 12.
4. 190 lb. 7 oz. 17 pwt. 12 gr. by 15.
5. 550 yd. by 24.
6. 171 ℔ 10 ℥ 5 ʒ 2 ℈ 12 gr. by 32.
7. 52 wk. 1 da. 5 hr. 48 min. 45 sec. by 25.
8. 125 cords by 30.
9. 708 cu. yd. 9 cu. ft. 1125 cu. in. by 45.
10. 64 A. 10 sq. rd. 26 sq. yd. 3 sq. ft. 108 sq. in. by 50.
11. 15 loads of wheat measured 370 bu. 2 qt. 1 pt. What was the average quantity of the loads?
12. 18 loads of hay weighed 22 T. 13 cwt. 69 lb. What was their average weight?
13. How many times could a vessel holding 2 gal. 3 qt. 2 gi. be emptied into a cistern whose capacity is 1350 gal.?
14. A house of 24 rooms required 450 square yards of carpet. What was the average quantity per room?
15. 48 equal loads of wood measured 50 cords. How much did each load contain?
16. How many Exposition medals, each weighing 1 oz. 11 pwt. 6 gr., could be manufactured from 244 lb. 1 oz. 13 pwt. 18 gr. of silver?

17. 36 pieces of cloth of equal length measured 927 yards. What was the length of each piece?

18. A man performed a certain amount of work in 313 days. In what number of days could 48 men have done the same amount of work?

19. 96 packages of medicines of equal size weighed 101 ℔ 3 ℥ 1 ʒ 1 ℈ 16 gr. How much did each package weigh?

20. 5 men dug a ditch 5 ft. wide and 3 ft. deep in 6 days of 10 hours each. If each man averaged 1 cu. yd. 6 cu. ft. per hour, what was the length of the ditch?

LONGITUDE AND TIME

72. The *longitude* of a place is its distance from a given meridian, expressed in degrees, minutes, and seconds.

The earth rotates upon its axis once every day. Consequently, *the sun seems to pass around the earth once in twenty-four hours.*

As there are 360° in the circumference of a circle, the sun apparently passes through $\frac{1}{24}$ of 360°, or 15°, in 1 *hour of time;* and as $15° = 900'$, it passes through $\frac{1}{60}$ of 900′, or 15′, in 1 *minute of time;* and as $15' = 900''$, it passes through $\frac{1}{60}$ of 900″, or 15″, in 1 *second of time.*

Therefore, 360° of *longitude* = 24 *hours of time;* 15° of longitude = 1 hour of time; 15′ of longitude = 1 minute of time; and 15″ of longitude = 1 second of time. Hence,

1st. **To find the difference of time corresponding to any difference of longitude.**

RULE. *Divide the difference of longitude expressed in degrees, minutes, and seconds* (° ′ ″) *by* 15. *The quotient will be the difference of time in hours, minutes, and seconds.*

2d. To find the difference of longitude corresponding to any difference of time.

RULE. *Multiply the difference of time expressed in hours, minutes, and seconds by* 15. *The product will be the difference of longitude in degrees, minutes, and seconds* (° ′ ″).

(*a*) The difference of longitude between two places is 19° 12′ 30″. What is their difference of time?

OPERATION

15) 19° 12′ 30″

1 hr. 16 min. 50 sec.

EXPLANATION. Dividing 19° by 15, the quotient is 1, with a remainder 4°. Reducing 4° to ′, adding 12′ and dividing by 15, the quotient is 16, with a remainder of 12′. Reducing 12′ to ″, adding 30″ and dividing by 15, the quotient is 50. The result is 1 hr. 16 min. 50 sec.

(*b*) The difference of time between two places is 1 hr. 10 min. 30 sec. What is their difference of longitude?

OPERATION

hr.	min.	sec.
1	10	30
		15
17°	37′	30″

EXPLANATION. Multiplying 30 seconds by 15 gives 450″ of longitude = 7′ 30″. Multiplying 10 minutes by 15, and adding 7′ gives 157′ = 2° 37′. Multiplying 1 hour by 15, and adding 2° gives 17°. The result is 17° 37′ 30″.

73. As the earth rotates upon its axis from *west to east*, it makes the sun *appear* to pass from the *east toward the west.* Hence, any place *east* of a given point has *later time* than any place *west* of it. Therefore,

When the difference of longitude between two places, and the time at one place are given, the time at the place farthest east is found by adding their difference of time; and the time at the place farthest west by subtracting their difference of time.

REMARKS. 1. When two places are on the *same side* of a standard meridian, the less longitude must be *subtracted* from the greater, in order to find their difference of time; but if they are on *opposite sides* of it, their longitudes must be *added* to find their difference of time.

2. The *standard meridian* of the English is that of Greenwich, near London. In the United States, both the meridian of Greenwich and that of Washington are used.

1. The difference of longitude between two places is 24°. What is their difference of time?

2. The difference of longitude between two cities is 18° 45′. What is their difference of time?

3. The difference of longitude between two points is 32° 31′ 30″. Find their difference of time.

4. The longitude of Cincinnati is 84° 30′ west. That of Minneapolis 93° 20′ west. What is their difference of time?

5. How much sooner does the sun rise at Philadelphia, longitude 75° 10′ west, than at San Francisco, longitude 122° 26′ 45″ west?

6. How much later does the sun set at St. Louis, longitude 90° 15′ 15″ west, than at New York, 74° 3′ west?

7. If a traveler should set his watch correctly at Boston, longitude 71° 3′ 30″ west, and then journey to Chicago, longitude 87° 37′ 45″ west, how much too fast would his watch be on arriving?

8. Rome is in longitude 12° 27′ east, and San Francisco in longitude 122° 26′ 45″ west. When it is 12 o'clock noon at San Francisco, what is the time at Rome?

9. Montreal is in longitude 73° 25′ west, and Paris in 2° 20′ east. When it is noon at Paris, what is the time at Montreal?

10. The difference of time between two places is 1 hr. 33 min. 20 sec. What is their difference of longitude?

11. The difference of time between Baltimore and Denver is 1 hr. 53 min. 20 sec. Find their difference of longitude.

12. If the sun rises 3 hr. 29 min. 40 sec. later at Portland, Oregon, than at Portland, Maine, what is their difference of longitude?

13. Little Rock is west of Charleston, South Carolina, in longitude 92° 15′ west, and their difference of time is 49 min. 32 sec. What is the longitude of Charleston?

14. Cairo in Egypt is in longitude 31° 16′ east, and the difference of time between Cairo and New Orleans is 8 hr. 5 min. 14 sec. What is the longitude of New Orleans?

15. Stockholm is west of Moscow, in longitude 18° 5′ east. Their difference of time is 1 hr. 17 min. 52 sec. What is the longitude of Moscow?

16. Two travelers started from the same place at the same time, one going 20° 14′ 17″ east, the other 22° 23′ 13″ west. They arrived at their destinations at the same moment, the first at 12 M. At what time did the second arrive?

FACTORING

DEFINITIONS

74. 1. *Factoring* is the process of separating a number into its factors.

2. The *factors* of a number are all of its *exact divisors*.

Thus, 2, 3, 4, and 6 are the factors of 12.

REMARK. Unity (1) and the number itself are not considered factors.

3. An *integer* is a whole number; as 9, 47, 125, etc.

4. Whole numbers are divided into two classes, *prime* numbers and *composite* numbers.

5. *Prime numbers* are those which can not be separated into factors; as 7, 11, 13, etc.

6. *Composite numbers* are those which can be separated into factors; as 15, 18, etc.

Thus, $15 = 3 \times 5$; $18 = 2 \times 9$ or 3×6.

7. The *prime factors* of a composite number are those whose continued product is equal to the number.

Thus, 2, 3, and 7 are the prime factors of 42, because $2 \times 3 \times 7 = 42$.

8. A *common factor* of two or more numbers will exactly divide each of them.

Thus, 5 is a common factor of 10 and 15.

9. Numbers that have no common factor are *prime to each other.*

Thus, 9 and 16 are prime to each other.

75. 1. All integers are either *odd* or *even.*

2. An *even number* can be exactly divided by 2.

An *odd number* can not be exactly divided by 2.

3. 2 is the *only even prime number.* The odd numbers are partly prime and partly composite.

Thus, 11 is a prime odd number, and 21 is a composite odd number.

4. The prime numbers between 1 and 100 are as follows:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37; 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

76. PRINCIPLES. 1. *A factor of a number is a factor of any product of that number.*

Thus, as 7 is a factor of 14, it is also a factor of 14×2, or 28; 14×3, or 42, etc.

2. *A factor common to two or more numbers is also a factor of their sum, their difference, and their product.*

Thus, 3 is a common factor of 6 and 15. It is also a factor of $6 + 15$, or 21; $15 - 6$, or 9; and 6×15, or 90.

3. *Every composite number is divisible by the product of any two or more of its prime factors.*

Thus, the prime factors of 30 are 2, 3, and 5; and 30 is divisible by 2×3, or 6; 2×5, or 10; and 3×5, or 15.

77. The following principles are to be observed when separating numbers into their prime factors.

1. *Every even number is divisible by* 2; as 4, 6, 8, 10, etc.

2. *A number is divisible by* 3 *if the sum of its digits is divisible by* 3.

Thus, 1752 is divisible by 3, because $1 + 7 + 5 + 2$, or 15, is divisible by 3.

3. *A number is divisible by* 5 *if its right-hand figure is* 0 *or* 5.

Thus 35, 50, 65, 80, etc., are divisible by 5.

REMARKS. It will be convenient to remember the following facts respecting the composite factors 4, 6, 8, and 9:

1. A number is divisible by 4, if the number expressed by its two right-hand figures is divisible by 4, or if it ends with two or more ciphers.

2. A number is divisible by 6, if it is *even*, and if the sum of its digits is divisible by 3.

3. A number is divisible by 8, if the number expressed by its three right-hand figures is divisible by 8, or if it ends with three or more ciphers.

4. A number is divisible by 9, if the sum of its digits is divisible by 9.

78. To separate a composite number into its prime factors.

What are the prime factors of 66?

OPERATION

$$\begin{array}{r|l} 2 & 66 \\ 3 & 33 \\ \hline & 11 \end{array}$$

EXPLANATION. As 66 is an *even* number, it is divisible by 2. The quotient is 33. As the sum of the digits of 33 $(3 + 3)$ is divisible by 3, 33 is divisible by 3. The quotient is 11, which is a prime number. Therefore, the prime factors of 66 are 2, 3, and 11.

RULE. *Divide the given number by any prime number that will exactly divide it. Divide the quotient by another prime factor, and so continue until the quotient is a prime number. The several divisors and the last quotient will be the required prime factors.*

Find the prime factors of:

1. 36.	9. 98.	17. 168.	25. 570.	33. 1221.
2. 45.	10. 104.	18. 180.	26. 630.	34. 1386.
3. 56.	11. 108.	19. 196.	27. 729.	35. 1482.
4. 60.	12. 117.	20. 216.	28. 864.	36. 1540.
5. 70.	13. 133.	21. 243.	29. 910.	37. 1666.
6. 72.	14. 143.	22. 289.	30. 987.	38. 1748.
7. 78.	15. 150.	23. 333.	31. 1050.	39. 1870.
8. 84.	16. 156.	24. 450.	32. 1122.	40. 1932.

79. To find the prime factors common to two or more numbers.

What prime factors are common to 56 and 98?

OPERATION

2	56	98
7	28	49
	4	7

EXPLANATION. The numbers are written in a line. As 56 and 98 are *even* numbers, they are divisible by 2. The quotients are 28 and 49. Dividing 28 and 49 by 7, the quotients are 4 and 7, *which are prime to each other*. Therefore, 2 and 7 are the prime factors *common* to 56 and 98.

RULE. 1. *Write the given numbers in a line, and divide them by any prime factor common to all.*

2. *Divide the quotients in the same manner, and so continue until the quotients contain no common factor. The several divisors will be the common prime factors.*

Find the prime factors common to:

1. 18 and 48.
2. 28 and 70.
3. 36 and 54.
4. 45 and 75.
5. 60 and 96.
6. 84 and 126.
7. 120 and 210.
8. 154 and 242.
9. 156 and 195.
10. 210 and 385.
11. 48, 72, and 108.
12. 63, 105, and 147.
13. 126, 210, and 294.
14. 153, 255, and 357.
15. 190, 266, and 342.
16. 252, 462, and 546.
17. 306, 510, and 714.
18. 396, 594, and 858.
19. 552, 966, and 1518.
20. 616, 1078, and 1386.

GREATEST COMMON DIVISOR

80. 1. A *divisor* of a number will exactly divide it.

Thus, 7 is a divisor of 28.

2. A *common divisor* of two or more numbers will exactly divide each of them.

Thus, 7 is a common divisor of 21 and 35.

3. The *greatest common divisor* (G. C. D.) of two or more numbers is the *greatest* number that will exactly divide each of them.

Thus, 15 is the greatest common divisor of 45 and 75.

PRINCIPLE. *The greatest common divisor of two or more numbers is the product of all their common prime factors.*

Thus, the prime factors common to 30 and 45 are 3 and 5.
Then, 3×5, or 15, is the G. C. D. of 30 and 45.

RULE. *Separate the given numbers into their prime factors, and find the product of those factors which are common. The result will be the greatest common divisor.*

When the numbers are too large to be factored readily, the following method should be employed.

Find the G. C. D. of 336 and 735.

OPERATION

336)735(2
672
63)336(5
315
21)63(3
63

EXPLANATION. This method depends upon the principle that the greatest common divisor of two numbers is *a divisor of their difference, and also of the remainder after any division.*

Dividing 735 by 336 leaves a remainder of 63. Dividing 336 by 63 leaves a remainder of 21. Dividing 63 by 21 leaves no remainder. Therefore, 21 is the *greatest common divisor*, for $336 \div 21 = 16$, and $735 \div 21 = 35$. Hence, as 16 and 35 are *prime to each other*, 21 *must contain all the prime factors which are common to* 336 *and* 735.

RULE. *Divide the greater number by the less. Divide the divisor by the remainder, and so continue to divide the last divisor by the last remainder until nothing remains. The last divisor will be the greatest common divisor.*

REMARK. To find the G. C. D. of *more* than two numbers, first find the greatest common divisor of *two* of them, then of that divisor and one of the remaining numbers, and so continue until all have been used. The last divisor will be the G. C. D. required.

Find the greatest common divisor of:

1. 105 and 280.
2. 296 and 407.
3. 387 and 602.
4. 408 and 765.
5. 531 and 826.
6. 603 and 1072.

7. 735 and 1078.
8. 1210 and 1925.
9. 1500 and 2475.
10. 1638 and 2145.
11. 2142 and 2457.
12. 2736 and 3264.
13. 4752 and 5445.
14. 7326 and 10101.
15. 9933 and 11713.
16. 1260, 1848, and 2808.
17. 1596, 2052, and 3591.
18. 4347, 5175, and 6555.
19. 4620, 8624, and 13728.
20. 7560, 11025, and 13440.

21. What is the capacity of the largest vessel that will exactly measure 2464, 3168, or 4356 gallons of petroleum?

22. A has 2592 eggs, B 3024, and C 3744. If the largest boxes possible are made in which each can exactly pack his eggs, how many dozen would each box hold?

23. A railroad company built three side tracks, 8910, 12,474, and 14,877 feet long, using the greatest length of rail possible to admit an exact number. How many rails were required?

24. A party of speculators bought three tracts of land containing respectively 1440, 2112, and 3024 acres, which they divided into farms of the largest possible size to contain an equal number of acres. If the farms were sold at $1200 each, how much was realized?

25. A grain dealer wishes to construct a number of bins of equal capacity that will just contain 1485 bu. of wheat, 2079 bu. of oats, and 1386 bu. of barley. How many bins must he have, and how many bushels will each hold?

LEAST COMMON MULTIPLE

81. 1. A *multiple* of a number is a number that can be exactly divided by it.

Thus, 35 is a multiple of 7, because $35 \div 7 = 5$.

2. A *common multiple* of two or more numbers is a number that can be exactly divided by each of them.

Thus, 36 is a common multiple of 6 and 9.

3. The *least common multiple* (L. C. M.) of two or more numbers is the *least number* that can be exactly divided by all of them.

Thus, 42 is the least common multiple of 6, 14, and 21.

PRINCIPLE. *The least common multiple of two or more numbers is equal to the product of all the prime factors of the numbers.*

First Method

Find the L. C. M. of 12, 28, and 42.

OPERATION

$12 = 2 \times 2 \times 3$
$28 = 2 \times 2 \times 7$
$42 = 2 \times 3 \times 7$
$2 \times 2 \times 3 \times 7 = 84$

EXPLANATION. Resolving the numbers into their prime factors, $12 = 2 \times 2 \times 3$; $28 = 2 \times 2 \times 7$; $42 = 2 \times 3 \times 7$. The least common multiple of the numbers must contain *all* their prime factors, and *no other factors*. These prime factors are 2, 2, 3, and 7. Their product is 84, which is the least common multiple.

RULE. 1. *Resolve the given numbers into their prime factors.*

2. *Multiply together all the different prime factors, taking each the greatest number of times that it occurs in any of the given numbers. Their product will be the least common multiple.*

Second Method

Find the L. C. M. of 36, 63, and 84.

OPERATION.

2	36	63	84
2	18	63	42
3	9	63	21
3	3	21	7
7	1	7	7
	1	1	1

L. C. M. $= 2 \times 2 \times 3 \times 3 \times 7 = 252$.

RULE. 1. *Write the numbers in a line, divide by any prime number that will exactly divide two or more of them, and write the quotients and the undivided numbers in a line beneath.*

2. *Divide the resulting numbers in the same manner, and so continue until no two numbers are divisible by any prime number greater than* 1. *The product of the divisors and the numbers undivided will be the least common multiple.*

REMARKS. 1. To find the L. C. M. of *two large numbers*, divide them by their greatest common divisor. Then multiply together the G. C. D. and the quotients.

2. To find the L. C. M. of *more* than two large numbers, first find the L. C. M. of *two* of them, then of that multiple and another of the numbers, etc.

Find the least common multiple of:

1. 10, 15, 18.
2. 20, 24, 30.
3. 16, 18, 24.
4. 24, 42, 56.
5. 32, 48, 96.
6. 30, 35, 42.
7. 24, 36, 54.
8. 48, 80, 120.

9. 35, 40, 56.
10. 28, 44, 77.
11. 42, 48, 56, 84.
12. 40, 60, 72, 90.
13. 36, 44, 66, 99.
14. 30, 45, 50, 75.
15. 36, 52, 78, 117.
16. 30, 32, 48, 80.
17. 60, 90, 108, 135.
18. 33, 44, 55, 60.
19. 72, 80, 90, 120.
20. 42, 78, 91, 273.
21. 170, 180, 204, 255.
22. 280, 315, 360, 504.
23. 495, 693.
24. 663, 1105.
25. 141, 235, 329.
26. 231, 357, 561.

27. A trader has just enough money to buy horses at $48, mules at $45, cows at $36, or oxen at $32 each. How much money has he?

28. What is the area of the smallest tract of land that can be divided into farms of 60, 64, 80, or 96 acres each?

29. A building company wishing to subdivide a tract of land, finds that it can be laid out into an exact number of lots of equal depth, and 48, 60, 72, 75, or 80 feet in width. If the land is divided into 75-foot lots, what would be their value at $450 apiece?

30. A grain dealer has five bins whose capacities are respectively 135, 144, 180, 216, and 240 bushels. What is the smallest quantity of grain that could be measured exactly by each?

31. Five cyclists start from a line at the same time, and ride in the same direction around a race track one mile in circumference, at the rate of 15, 20, 22, 24, and 32 ft. per second respectively. In what time will all first arrive together at the starting place?

CANCELLATION

82. Cancellation is the process of shortening division by rejecting the factors common to the dividend and the divisor.

The operation of cancellation depends upon the following principle:

PRINCIPLE. *Canceling equal factors in both dividend and divisor does not change the quotient.*

Divide 45 times 56 by 12 times 35.

OPERATION 1. $$\frac{45 \times 56}{12 \times 35} = \frac{\cancel{3} \times 3 \times \cancel{5} \times \cancel{2} \times \cancel{2} \times 2 \times \cancel{7}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{5} \times \cancel{7}} = 6.$$

EXPLANATION. Write the numbers representing the dividend above a horizontal line, and those representing the divisor below it. Resolve all of the numbers into their prime factors, indicating the multiplication. *Cancel* the factors *common* to the dividend and the divisor, by drawing a short line across them. The product of the remaining factors, 2 and 3, is the answer, 6.

OPERATION 2. $$\begin{matrix} 3 & 2 \\ \cancel{9} & \cancel{8} \\ \cancel{45} \times \cancel{56} \\ \cancel{12} \times \cancel{35} \\ \cancel{3} & \cancel{5} \end{matrix} = 6.$$

EXPLANATION. Write the numbers as before. 56 and 35 are each divisible by 7. Cancel 56, and write 8 above it; also, cancel 35, and write 5 below it. 5 and 45 are each divisible by 5. Cancel 5, and also cancel 45, writing 9 above it. 8 and 12 are each divisible by 4. Cancel 8, writing 2 above it; also cancel 12, writing 3 below it. 3 and 9 are each divisible by 3. Cancel 3, and also cancel 9, writing 3 above it. As all the common factors have been canceled, the product of the factors remaining in the dividend, 2×3, is the quotient, 6.

RULE. *Cancel the factors common to the dividend and the divisor, and then divide the product of the factors*

remaining in the dividend, by the product of those remaining in the divisor. The quotient will be the answer.

1. Multiply 35 by 36, and divide the product by 15×14.

2. Divide 24 times 56 times 45 by 21 times 18 times 32.

3. How often is $34 \times 48 \times 57$ contained in $72 \times 76 \times 51$?

4. $81 \times 75 \times 56 \times 65$ is how many times $45 \times 52 \times 54 \times 50$?

5. Divide the product of $38 \times 46 \times 63 \times 91 \times 99$ by the product of $39 \times 57 \times 77 \times 84 \times 69$.

6. A grocer exchanged 15 lb. of tea at 60 cents a pound for potatoes at 50 cents a bushel. How many bushels did he receive?

7. How many sods 18 in. long and 12 in. wide would be required for a lawn 50 ft. long and 45 ft. wide?

8. A pile of wood is 80 ft. long, 8 ft. wide, and 6 ft. high. Find its value at $5 a cord.

9. How many dozen glass jars at 10 cents apiece would be equal in value to 16 boxes of starch, each containing 45 lb., at 4 cents a pound?

10. What would be the cost of carpeting a lodge-room 54 ft. long by 36 ft. wide with ingrain carpet at 55 ct. a square yard?

11. If 45 men can build a bridge in 60 days by working 8 hours a day, in how many days could 48 men build a similar bridge, by working 9 hours a day?

12. What would be the cost of excavating a cellar 24 ft. long, 15 ft. wide, and 6 ft. deep, at 25 ct. a cubic yard?

13. 198 bales of wool, each weighing 216 lb., worth 28 ct. a pound, were exchanged for 144 bales of cotton, each weighing 308 lb. What was the value of the cotton per pound?

14. 50 sheets of pasteboard, each 32 in. long by 24 in. wide, were cut into visiting cards 2 in. wide by 3 in. long, which were sold at 25 cents per hundred. What was their total value?

15. How many dozen oyster cans, each 7 in. long, 5 in. wide, and 2 in. deep, can be packed in a box 36 in. long, 20 in. wide, and 14 in. deep?

16. How many boxes 20 in. long, 14 in. wide, and 8 in. high, would exactly occupy the space in a freight car 30 ft. long, 7 ft. wide, and 8 ft. high inside?

17. 16 logs, each 15 in. square at the ends and 15 ft. long, were sawn into palings 30 in. long, 2 in. wide, and 1 in. thick. What was their value at $1.25 per hundred?

18. A reservoir is 198 ft. long, 105 ft. wide, and 20 ft. deep. In what time could it be filled by 6 pipes, each discharging 6 gal. per second?

19. A kiln of brick is 64 ft. long, 25 ft. wide, and 12 ft. high. If each brick is 8 in. long, 4 in. wide, and 2 in. thick, what is the value of the kiln at $10 per thousand bricks, allowing one sixth for waste?

FRACTIONS

DEFINITIONS

83. A *unit* can be divided into two or more *equal parts*. A *fraction* is one or more of the equal parts of a unit. If a unit is divided into *two* equal parts, one of the parts is called *one half;* if a unit is divided into *three* equal parts, one of the parts is called *one third;* two of the parts are called *two thirds;* if a unit is divided into *four* equal parts, one of the parts is called *one fourth;* two of the parts, *two fourths;* three of the parts, *three fourths*, etc. Hence, *a fraction is named from the number of equal parts into which the unit is divided.*

84. 1. Fractions are divided into two classes, *common* and *decimal.*

2. A *common* fraction is the result of dividing a unit into *any number* of equal parts.

3. A *decimal* fraction is the result of dividing a unit into *tenths*, *hundredths*, *thousandths*, etc.

4. A common fraction is expressed *in figures* by two numbers, one written beneath the other, with a short line between them. Thus, the fraction $\frac{2}{5}$ expressed in words is *two fifths.*

5. The number *beneath* the line is called the *denominator.* It denotes the *number of equal parts* into which the unit is divided. The number *above* the line is called the *numerator.* It shows the *number of parts taken.*

Thus, the fraction $\frac{2}{5}$ denotes that a unit has been divided into *five* equal parts, and that *two* of the parts have been taken.

6. The numerator and denominator are called the *terms* of the fraction.

85. A comparison of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, shows that the *size* of a part depends upon the *number* of parts into which the unit is divided. Therefore, of several fractions, *that having the smallest denominator expresses parts of the largest size.*

86. There is another way to consider a fraction with reference to its numerator and denominator. Suppose it were required to divide four yards of cloth into five equal parts. One of the parts would be *one fifth of four yards*, which is the same as *four fifths of one yard.* Therefore, the fraction $\frac{4}{5}$ may be regarded either as *four fifths of one unit*, or as *one fifth of four units.*

87. 1. Since fractions arise from division, the terms used in division may be applied to them. Thus, to take one fifth of four yards is to divide 4 yards by 5, and the result is $\frac{4}{5}$ yard. Therefore, the *numerator* of a fraction corresponds to the *dividend;* the *denominator* to the *divisor*, and the *entire fraction* to the *quotient.*

2. An *integer* may be expressed in the form of a fraction by writing 1 beneath it for the denominator. Thus, 4 may be written $\frac{4}{1}$.

88. *Write the following fractions in figures:*

1. One third, three fourths, two fifths, five sixths.

2. Seven eighths, four ninths, six tenths, nine elevenths.

3. Five twelfths, eight thirteenths, eleven fourteenths.

4. Thirteen sixteenths, twelve seventeenths, seven eighteenths, two nineteenths.

5. Seventeen twenty-fourths, twenty-five thirty-seconds, forty-four forty-fifths.

6. Thirty-seven fiftieths, fifty-three sixty-eighths.

7. Sixty-nine eightieths, seventy-five ninety-ninths, ninety-two one-hundredths.

Copy and read the following fractions:

$\frac{7}{12}$, $\frac{8}{15}$, $\frac{16}{25}$, $\frac{19}{37}$, $\frac{21}{40}$, $\frac{32}{55}$, $\frac{45}{64}$, $\frac{59}{76}$, $\frac{60}{89}$, $\frac{77}{90}$, $\frac{99}{100}$.

$\frac{68}{101}$, $\frac{47}{256}$, $\frac{81}{325}$, $\frac{75}{444}$, $\frac{132}{509}$, $\frac{206}{617}$, $\frac{333}{758}$, $\frac{512}{831}$, $\frac{729}{999}$, $\frac{1001}{1010}$.

89. 1. The *value* of a fraction is its comparison with a unit.

2. If the numerator of a fraction is *less* than the denominator, the value of the fraction is *less* than 1. Thus, $\frac{3}{5}$ is less than 1.

3. If the numerator of a fraction is *equal* to the denominator, the value of the fraction is *equal* to 1. Thus, $\frac{5}{5}$ is equal to 1.

4. If the numerator of a fraction is *greater* than the denominator, the value of the fraction is *greater* than 1. Thus, $\frac{6}{5}$ is greater than 1.

5. A *proper fraction* is one whose numerator is less than its denominator. Therefore, its *value* is less than 1.

6. An *improper fraction* is one whose numerator is equal to or greater than its denominator. Therefore, its *value* is equal to or greater than 1.

90. 1. Common fractions are divided into three classes, *simple*, *compound*, and *complex*.

2. A *simple* fraction is one whose terms are integers. It may be either proper or improper; as, $\frac{5}{7}$, $\frac{8}{5}$.

3. A *compound* fraction is a fraction of a fraction; as, $\frac{3}{4}$ of $\frac{5}{8}$.

4. A *complex* fraction is one which has a fraction in one or both of its terms; as, $\frac{\frac{3}{4}}{6}$, $\frac{5}{\frac{3}{8}}$, $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{3\frac{1}{3}}{4\frac{1}{2}}$. The first is read $\frac{3}{4}$ *over* 6, or $\frac{3}{4}$ *divided by* 6.

5. A *mixed number* is an integer with a fraction annexed. Thus, $6\frac{2}{3}$, read *six and two thirds*, is a mixed number.

91. PRINCIPLES. 1. *Multiplying or dividing the numerator produces the same change in the value of the fraction.*

For (1), if the numerator of $\frac{2}{9}$ is multiplied by 4, the result is $\frac{8}{9}$.

The number of parts is increased, while their size is the same.

(2) If the numerator of $\frac{9}{10}$ is divided by 3 the result is $\frac{3}{10}$.

The number of parts is diminished, while their size is the same.

2. *Multiplying or dividing the denominator produces an opposite change in the value of the fraction.*

For (1), if the denominator of $\frac{3}{4}$ is multiplied by 2, the result will be $\frac{3}{8}$.

The size of the parts is diminished, while their number is the same.

(2) If the denominator of $\frac{3}{10}$ is divided by 2, the result will be $\frac{3}{5}$.

The size of the parts is increased, while their number is the same.

3. *The value of a fraction is not changed by multiplying or dividing both terms by the same number.*

For (1), if both terms of $\frac{2}{3}$ are multiplied by 3, the result will be $\frac{6}{9}$.

The number of parts is increased, but their size is diminished correspondingly.

(2) If both terms of $\frac{8}{12}$ are divided by 4, the result will be $\frac{2}{3}$.

The number of parts is diminished, but their size is increased correspondingly.

These principles may be briefly stated as follows:

I. *Multiplying the numerator or dividing the denominator multiplies the value of the fraction.*

II. *Dividing the numerator or multiplying the denominator divides the value of the fraction.*

III. *Multiplying or dividing both terms by the same number does not change the value of a fraction.*

REDUCTION OF FRACTIONS

92. *Reduction of fractions* is the process of changing their form without altering their value. There are *six cases.*

Case I

93. To reduce an integer to a fraction having a given denominator.

Reduce 20 to fifteenths.

Operation. $20 \times \frac{1}{15} = \frac{300}{15}$.

Explanation. There are $\frac{1}{15}$ in 1. In 20 there are 20 times $\frac{1}{15} = \frac{300}{15}$.

Rule. *Multiply the integer by the given denominator, and write the denominator under the product.*

Reduce:

1. 15 to twelfths.
2. 19 to twentieths.
3. 17 to seventeenths.
4. 24 to twenty-fifths.
5. 39 to nineteenths.
6. 45 to thirty-seconds.
7. 56 to twenty-sevenths.
8. 63 to forty-eighths.
9. 72 to fifty-fourths.
10. 81 to thirty-ninths.
11. 99 to forty-fourths.
12. 27 to seventy-fifths.
13. 68 to fiftieths.
14. 49 to thirty-thirds.
15. 76 to sixty-sevenths.
16. 50 to forty-ninths.
17. 32 to ninety-eighths.
18. 84 to thirty-sixths.

Case II

94. To reduce a mixed number to an improper fraction.

Reduce $44\frac{4}{9}$ to ninths.

Operation.
$$\begin{array}{r} 44\frac{4}{9} \\ \times\ 9 \\ \hline 396 \\ +\ 4 \\ \hline \frac{400}{9} \end{array}$$

Explanation. There are $\frac{9}{9}$ in 1. In 44 there are $\frac{396}{9}$. $\frac{396}{9} + \frac{4}{9} = \frac{400}{9}$.

RULE. *Multiply the integer by the denominator of the fraction. Add the numerator to the product, and write the denominator under the sum.*

Reduce to improper fractions:

1. $15\frac{5}{8}$.
2. $18\frac{10}{11}$.
3. $24\frac{4}{15}$.
4. $32\frac{16}{25}$.
5. $48\frac{31}{36}$.
6. $56\frac{17}{43}$.
7. $63\frac{37}{51}$.
8. $75\frac{25}{64}$.
9. $87\frac{35}{72}$.
10. $99\frac{87}{88}$.
11. $101\frac{1}{99}$.
12. $21\frac{125}{144}$.
13. $36\frac{209}{256}$.
14. $45\frac{41}{325}$.
15. $50\frac{49}{480}$.
16. $64\frac{124}{125}$.
17. $72\frac{8}{216}$.
18. $84\frac{23}{132}$.
19. $96\frac{16}{175}$.
20. $101\frac{1}{100}$.

CASE III

95. To reduce an improper fraction to an integer or to a mixed number.

Reduce $\frac{375}{8}$ to a mixed number.

OPERATION. $8\overline{)375}$

$46\frac{7}{8}$

EXPLANATION. There are $\frac{1}{8}$ in 1. Therefore, $\frac{375}{8}=375\div 8=46\frac{7}{8}$.

RULE. *Divide the numerator by the denominator. The quotient will be the required number.*

Reduce to integers or to mixed numbers:

1. $\frac{448}{9}$.
2. $\frac{449}{12}$.
3. $\frac{787}{15}$.
4. $\frac{1828}{21}$.
5. $\frac{1774}{25}$.
6. $\frac{1550}{89}$.
7. $\frac{2197}{48}$.
8. $\frac{8249}{57}$.
9. $\frac{4785}{64}$.
10. $\frac{5055}{77}$.
11. $\frac{4321}{86}$.
12. $\frac{9899}{99}$.
13. $\frac{10201}{101}$.
14. $\frac{13000}{203}$.
15. $\frac{24024}{312}$.
16. $\frac{19575}{444}$.

CASE IV

96. To reduce a fraction to higher terms.

REMARK. This operation depends upon the principle that *multiplying both terms of a fraction by the same number does not change its value.*

Reduce $\frac{4}{9}$ to a fraction whose denominator is 63.

OPERATION. $63 \div 9 = 7$. $\frac{4}{9} \times \frac{7}{7} = \frac{28}{63}$.

EXPLANATION. There are 63 sixty-thirds in 1. In $\frac{1}{9}$ there are $\frac{7}{63}$, and in $\frac{4}{9}$ there are $\frac{28}{63}$.

RULE. *Divide the required denominator by the denominator of the fraction, and multiply both terms by the quotient.*

Reduce:

1. $\frac{3}{8}$ to 24ths.
2. $\frac{5}{9}$ to 45ths.
3. $\frac{9}{16}$ to 48ths.
4. $\frac{11}{14}$ to 56ths.
5. $\frac{16}{25}$ to 75ths.
6. $\frac{25}{27}$ to 108ths.
7. $\frac{13}{15}$ to 120ths.
8. $\frac{19}{24}$ to 168ths.
9. $\frac{25}{36}$ to 216ths.
10. $\frac{33}{45}$ to 360ths.
11. $\frac{49}{54}$ to 486ths.
12. $\frac{48}{49}$ to 588ths.
13. $\frac{35}{52}$ to 676ths.
14. $\frac{52}{63}$ to 756ths.
15. $\frac{71}{72}$ to 1080ths.
16. $\frac{64}{75}$ to 1350ths.
17. $\frac{77}{80}$ to 1680ths.
18. $\frac{49}{96}$ to 2016ths.
19. $\frac{99}{112}$ to 2688ths.
20. $\frac{125}{144}$ to 3744ths.

CASE V

97. To reduce a fraction to its lowest terms.

REMARKS. 1. This operation depends upon the principle that *dividing both terms of a fraction by the same number does not change its value.*

2. A fraction is in its *lowest terms* when the terms are *prime to each other.*

First Method

(*a*) Reduce $\frac{63}{84}$ to its lowest terms.

OPERATION. $\frac{63 \div 7}{84 \div 7} = \frac{9}{12}$. $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$.

EXPLANATION. Dividing both terms of $\frac{63}{84}$ by 7, the result is $\frac{9}{12}$. Dividing both terms of $\frac{9}{12}$ by 3, the result is $\frac{3}{4}$. As 3 and 4 are *prime to each other*, the fraction $\frac{3}{4}$ is in its *lowest terms*.

RULE. *Divide both terms of the fraction by any common divisor. Divide the resulting fraction in the same manner, and so continue, until the terms of the final fraction are prime to each other.*

Second Method

(*b*) Reduce $\frac{315}{462}$ to its lowest terms.

OPERATION.
315)462(1
315
147)315(2
294
21)147(7
147

G. C. D. = 21. $\frac{315 \div 21}{462 \div 21} = \frac{15}{22}$.

EXPLANATION. The greatest common divisor of 315 and 462 is 21. Dividing both terms of $\frac{315}{462}$ by 21, the resulting fraction, $\frac{15}{22}$, is in its *lowest terms*.

RULE. *Divide both terms of the fraction by their greatest common divisor. The fraction so obtained will be in its lowest terms.*

Reduce to their lowest terms:

1. $\frac{84}{147}$.
2. $\frac{273}{338}$.
3. $\frac{304}{399}$.
4. $\frac{476}{561}$.
5. $\frac{585}{660}$.
6. $\frac{627}{950}$.
7. $\frac{702}{910}$.
8. $\frac{434}{1023}$.
9. $\frac{253}{644}$.
10. $\frac{838}{1323}$.
11. $\frac{1215}{1512}$.
12. $\frac{1664}{1728}$.
13. $\frac{1001}{1848}$.
14. $\frac{2047}{2581}$.
15. $\frac{1274}{2156}$.
16. $\frac{1507}{3978}$.
17. $\frac{2184}{2592}$.
18. $\frac{2211}{3417}$.
19. $\frac{5994}{9657}$.
20. $\frac{6783}{11172}$.
21. $\frac{8085}{10164}$.
22. $\frac{7344}{11664}$.
23. $\frac{6660}{10545}$.
24. $\frac{12600}{28392}$.
25. $\frac{15834}{36917}$.
26. $\frac{25515}{58212}$.
27. $\frac{36225}{66654}$.
28. $\frac{3876}{5291}$.

CASE VI

98. To reduce two or more fractions to their least common denominator.

1. When two or more fractions have the same denominator, it is called a *common denominator.*

2. A *common denominator* of two or more fractions is a common multiple of their denominators.

3. The *least common denominator* of two or more fractions is the least common multiple of their denominators.

Reduce $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{8}{15}$ to their least common denominator.

OPERATION.

2	6	10	15
3	3	5	15
5	1	5	5
	1	1	1

L. C. M. $= 2 \times 3 \times 5 = 30$.

$30 \div 6 = 5.$ $\quad \frac{5 \times 5}{6 \times 5} = \frac{25}{30}.$

$30 \div 10 = 3.$ $\quad \frac{7 \times 3}{10 \times 3} = \frac{21}{30}.$

$30 \div 15 = 2.$ $\quad \frac{8 \times 2}{15 \times 2} = \frac{16}{30}.$

EXPLANATION. The least common multiple of the denominators of $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{8}{15}$ is 30. Dividing 30 by 6, 10, and 15 successively, gives 5, 3, and 2 for the quotients. Multiplying both terms of $\frac{5}{6}$ by 5, both terms of $\frac{7}{10}$ by 3, and both terms of $\frac{8}{15}$ by 2, gives $\frac{25}{30}$, $\frac{21}{30}$, and $\frac{16}{30}$.

RULE. 1. *Find the least common multiple of the denominators of the fractions.*

2. *Divide the least common multiple by the denominator of each fraction, and multiply both terms by the quotient.*

REMARK. Before commencing the operation, each fraction must be in its *lowest terms*.

Reduce to their least common denominator:

1. $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}.$
2. $\frac{1}{6}, \frac{2}{9}, \frac{5}{18}.$
3. $\frac{3}{4}, \frac{5}{8}, \frac{7}{12}.$
4. $\frac{5}{9}, \frac{5}{12}, \frac{5}{18}.$
5. $\frac{2}{5}, \frac{5}{8}, \frac{3}{10}.$
6. $\frac{3}{5}, \frac{7}{9}, \frac{8}{15}.$
7. $\frac{3}{8}, \frac{5}{12}, \frac{9}{16}, \frac{7}{24}.$
8. $\frac{9}{10}, \frac{7}{12}, \frac{8}{15}, \frac{11}{20}.$
9. $\frac{2}{5}, \frac{4}{15}, \frac{12}{25}, \frac{16}{75}.$
10. $\frac{7}{12}, \frac{11}{18}, \frac{13}{24}, \frac{19}{36}.$
11. $\frac{5}{12}, \frac{5}{14}, \frac{5}{21}, \frac{5}{28}.$
12. $\frac{11}{12}, \frac{9}{16}, \frac{17}{24}, \frac{19}{32}.$
13. $\frac{3}{7}, \frac{14}{15}, \frac{16}{21}, \frac{24}{35}.$
14. $\frac{7}{9}, \frac{7}{12}, \frac{7}{18}, \frac{7}{27}.$
15. $\frac{2}{15}, \frac{7}{24}, \frac{13}{30}, \frac{21}{40}.$
16. $\frac{5}{18}, \frac{17}{21}, \frac{25}{42}, \frac{53}{63}.$

17. $\frac{4}{9}$, $\frac{8}{15}$, $\frac{16}{27}$, $\frac{28}{45}$.

18. $\frac{15}{16}$, $\frac{23}{24}$, $\frac{35}{36}$, $\frac{47}{48}$.

19. $\frac{4}{15}$, $\frac{8}{25}$, $\frac{13}{30}$, $\frac{29}{50}$, $\frac{64}{75}$.

20. $\frac{16}{21}$, $\frac{17}{24}$, $\frac{19}{42}$, $\frac{25}{56}$, $\frac{37}{84}$.

21. $\frac{11}{15}$, $\frac{13}{18}$, $\frac{17}{20}$, $\frac{29}{36}$, $\frac{28}{45}$.

22. $\frac{7}{10}$, $\frac{9}{14}$, $\frac{8}{15}$, $\frac{5}{21}$, $\frac{16}{35}$.

23. $\frac{7}{15}$, $\frac{9}{16}$, $\frac{11}{40}$, $\frac{25}{48}$, $\frac{39}{80}$.

24. $\frac{5}{6}$, $\frac{9}{10}$, $\frac{8}{11}$, $\frac{14}{15}$, $\frac{28}{55}$.

25. $\frac{19}{24}$, $\frac{28}{30}$, $\frac{29}{36}$, $\frac{37}{45}$, $\frac{49}{60}$.

26. $\frac{13}{14}$, $\frac{21}{26}$, $\frac{25}{28}$, $\frac{39}{52}$, $\frac{64}{91}$.

27. $\frac{15}{26}$, $\frac{28}{30}$, $\frac{28}{39}$, $\frac{44}{65}$, $\frac{59}{78}$.

28. $\frac{17}{18}$, $\frac{12}{25}$, $\frac{13}{30}$, $\frac{29}{45}$, $\frac{68}{75}$.

29. $\frac{9}{14}$, $\frac{16}{21}$, $\frac{17}{22}$, $\frac{81}{42}$, $\frac{64}{77}$.

30. $\frac{11}{15}$, $\frac{13}{21}$, $\frac{16}{35}$, $\frac{28}{45}$, $\frac{44}{63}$.

31. $\frac{17}{18}$, $\frac{17}{20}$, $\frac{17}{27}$, $\frac{17}{30}$, $\frac{17}{45}$.

32. $\frac{16}{21}$, $\frac{17}{26}$, $\frac{25}{39}$, $\frac{37}{42}$, $\frac{68}{91}$.

33. $\frac{27}{40}$, $\frac{39}{65}$, $\frac{75}{104}$.

34. $\frac{15}{28}$, $\frac{45}{76}$, $\frac{125}{133}$.

35. $\frac{37}{48}$, $\frac{101}{102}$, $\frac{121}{136}$.

36. $\frac{43}{63}$, $\frac{69}{112}$, $\frac{125}{144}$.

37. $\frac{21}{35}$, $\frac{34}{51}$, $\frac{63}{81}$, $\frac{88}{165}$.

38. $\frac{89}{91}$, $\frac{99}{119}$, $\frac{109}{221}$.

39. $\frac{201}{343}$, $\frac{302}{441}$.

40. $\frac{257}{351}$, $\frac{371}{507}$.

99. Greatest common divisor of fractions.

Find the greatest common divisor of $\frac{5}{9}$, $\frac{20}{27}$, and $\frac{40}{81}$.

SOLUTION. The greatest common divisor of the numerators 5, 20, and 40 is 5. The least common multiple of the denominators 9, 27, 81, is 81. $5 \div 81 = \frac{5}{81}$, which is the greatest common divisor.

RULE. *Find the greatest common divisor of the numerators for the numerator of the required fraction, and the least common multiple of the denominators for its denominator.*

REMARK. Before commencing the operation, mixed numbers must be reduced to improper fractions, and all fractions must be in their lowest terms.

Find the greatest common divisor of:

1. $\frac{4}{9}$, $\frac{8}{15}$, $\frac{16}{45}$.
2. $\frac{5}{6}$, $\frac{10}{21}$, $\frac{25}{42}$.
3. $\frac{14}{15}$, $\frac{21}{25}$, $\frac{49}{75}$.
4. $\frac{65}{72}$, $\frac{91}{96}$, $\frac{143}{144}$, $\frac{169}{192}$.
5. $\frac{64}{77}$, $\frac{96}{91}$, $\frac{128}{143}$, $\frac{544}{1001}$.
6. $29\frac{7}{10}$, $33\frac{8}{4}$, $37\frac{4}{5}$, $40\frac{1}{2}$.
7. $17\frac{1}{15}$, $18\frac{26}{27}$, $19\frac{41}{45}$, $20\frac{52}{185}$.
8. $41\frac{13}{20}$, $54\frac{13}{24}$, $51\frac{17}{30}$, $50\frac{23}{40}$.
9. $39\frac{16}{45}$, $40\frac{23}{36}$, $43\frac{19}{30}$, $47\frac{1}{18}$, $50\frac{1}{20}$.
10. $4\frac{11}{16}$, $9\frac{13}{18}$, $17\frac{17}{24}$, $24\frac{11}{36}$, $33\frac{41}{48}$.

11. Four tracts of land containing respectively $234\frac{3}{4}$, $365\frac{1}{6}$, $443\frac{5}{12}$, and $521\frac{2}{3}$ acres, were divided into farms of the largest size to contain an equal number of acres. If all were sold at $500 apiece, what sum was realized?

12. A railroad company built four side tracks $5922\frac{1}{2}$, $6351\frac{2}{3}$, $7124\frac{1}{6}$, and $8182\frac{7}{9}$ ft. long, using rails of the greatest length possible to allow an exact number of equal size. How many rails were required?

100. Least common multiple of fractions.

Find the least common multiple of $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{9}$.

SOLUTION. The least common multiple of the numerators 2, 5, 8, is 40. The greatest common divisor of the denominators 3, 6, and 9, is 3. $40 \div 3 = 13\frac{1}{3}$, which is the least common multiple.

RULE. *Find the least common multiple of the numerators for the numerator of the required fraction, and the greatest common divisor of the denominators for its denominator.*

REMARK. Before commencing the operation, mixed numbers must be reduced to improper fractions, and all fractions should be in their lowest terms.

Find the least common multiple of:

1. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{8}{15}$, $\frac{32}{45}$.
2. $\frac{7}{16}$, $\frac{15}{32}$, $\frac{21}{64}$, $\frac{35}{128}$.
3. $\frac{10}{27}$, $\frac{25}{36}$, $\frac{20}{63}$, $\frac{50}{81}$.
4. $\frac{16}{35}$, $\frac{18}{49}$, $\frac{24}{77}$, $\frac{36}{91}$, $\frac{48}{119}$.
5. $\frac{21}{34}$, $\frac{24}{85}$, $\frac{28}{119}$, $\frac{42}{187}$, $\frac{56}{221}$.
6. $3\frac{1}{2}$, $3\frac{3}{4}$, $2\frac{5}{8}$, $2\frac{3}{16}$.
7. $1\frac{11}{15}$, $1\frac{32}{45}$, $1\frac{16}{75}$, $1\frac{8}{135}$.
8. $4\frac{8}{9}$, $2\frac{23}{27}$, $1\frac{43}{45}$, $2\frac{74}{117}$.
9. $1\frac{17}{25}$, $1\frac{31}{35}$, $1\frac{12}{65}$, $1\frac{69}{85}$, $2\frac{41}{95}$.
10. $2\frac{3}{65}$, $\frac{19}{91}$, $1\frac{92}{117}$, $\frac{77}{195}$, $8\frac{111}{169}$.

11. What is the smallest quantity of land that could be divided into lots of $13\frac{13}{20}$, $17\frac{4}{25}$, $23\frac{1}{10}$, or $25\frac{1}{40}$ acres each, and what would be the value of the number of the last kind at $1250 apiece?

12. Five cyclists whose rates of traveling are respectively $6\frac{2}{3}$, $7\frac{1}{2}$, $8\frac{1}{3}$, 10, and $12\frac{1}{2}$ miles per hour, start together, and ride in the same direction around an island $93\frac{3}{4}$ miles in circumference. If they travel $7\frac{1}{2}$ hours per day, in how many days will all first arrive at the starting place?

ADDITION OF FRACTIONS

101. *Addition of fractions* is the process of finding the sum of two or more fractions, or fractional numbers.

(*a*) Find the sum of $\frac{5}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$.

SOLUTION. $\frac{5}{9} + \frac{7}{9} + \frac{8}{9} = \frac{20}{9} = 2\frac{2}{9}$.

EXPLANATION. As all the given fractions have the same denominator, the numerators express parts of the *same size*. The sum of $\frac{5}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$ is $\frac{20}{9} = 2\frac{2}{9}$. Therefore, to add fractions having a *common denominator, find the sum of the numerators, and write it over the common denominator.*

(*b*) Find the sum of $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{4}{15}$.

OPERATION.

2	$\frac{5}{6}$	$\frac{7}{10}$	$\frac{4}{15}$
3	3	5	15
5	1	5	5
	1	1	1

$2 \times 3 \times 5 = 30$; $\frac{5}{6} = \frac{25}{30}$; $\frac{7}{10} = \frac{21}{30}$; $\frac{4}{15} = \frac{8}{30}$. $\frac{25}{30} + \frac{21}{30} + \frac{8}{30} = \frac{54}{30} = 1\frac{24}{30} = 1\frac{4}{5}$.

EXPLANATION. As the denominators are *different*, the numerators *do not express parts of the same size.* Therefore, the fractions must be reduced to a common denominator. The least common multiple of the denominators is 30. Reducing each fraction to this denominator, and adding the numerators, the sum is $\frac{54}{30} = 1\frac{24}{30}$. But the fraction $\frac{24}{30}$ is not in the lowest terms. Reducing it, and annexing it to the integer, the result is $1\frac{4}{5}$.

RULE. 1. *Reduce the fractions to their least common denominator.*

2. *Find the sum of the numerators, and write it over the common denominator.*

REMARKS. 1. Compound fractions must be reduced to *simple fractions*, and each fraction must be in its *lowest terms* before finding the least common multiple of the denominators.

2. In adding *mixed numbers*, find the sum of the integers and of the fractions separately, and unite the sums.

Find the sum of:

1. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$.
2. $\frac{5}{6}$, $\frac{9}{10}$, $\frac{4}{15}$.
3. $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.
4. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{9}{10}$.
5. $\frac{9}{10}$, $\frac{11}{12}$, $\frac{8}{15}$, $\frac{13}{20}$.
6. $\frac{5}{9}$, $\frac{5}{12}$, $\frac{5}{18}$, $\frac{5}{24}$.
7. $\frac{7}{12}$, $\frac{9}{14}$, $\frac{11}{21}$, $\frac{13}{28}$.
8. $\frac{2}{9}$, $\frac{3}{10}$, $\frac{4}{15}$, $\frac{5}{18}$, $\frac{7}{30}$.
9. $\frac{11}{12}$, $\frac{17}{18}$, $\frac{26}{27}$, $\frac{35}{36}$, $\frac{53}{54}$.
10. $\frac{7}{12}$, $\frac{11}{15}$, $\frac{19}{24}$, $\frac{29}{30}$, $\frac{37}{40}$.

11. $1\frac{8}{9}$, $2\frac{11}{12}$, $3\frac{15}{16}$, $4\frac{17}{18}$, $5\frac{7}{24}$.

12. $1\frac{9}{10}$, $1\frac{11}{15}$, $1\frac{12}{25}$, $1\frac{17}{30}$, $1\frac{17}{50}$.

13. $2\frac{5}{12}$, $3\frac{13}{15}$, $4\frac{17}{18}$, $5\frac{19}{20}$, $6\frac{37}{45}$.

14. $4\frac{5}{14}$, $5\frac{11}{21}$, $6\frac{13}{30}$, $7\frac{16}{35}$, $8\frac{25}{42}$.

15. $11\frac{9}{10}$, $13\frac{11}{12}$, $15\frac{14}{15}$, $17\frac{15}{16}$, $19\frac{23}{24}$, $20\frac{17}{48}$.

16. $\frac{9}{16}$ of $\frac{32}{45}$, $\frac{24}{25}$ of $\frac{35}{36}$, $\frac{52}{57}$ of $\frac{76}{91}$, $\frac{21}{50}$ of $\frac{20}{49}$.

17. $100\frac{19}{20}$, $125\frac{23}{24}$, $150\frac{29}{30}$, $175\frac{35}{36}$, $200\frac{7}{40}$, $245\frac{44}{45}$.

18. $\frac{62}{93}$, $\frac{105}{147}$, $\frac{88}{165}$, $\frac{72}{189}$, $\frac{56}{245}$.

19. $\frac{5}{28}$, $\frac{6}{35}$, $\frac{7}{36}$, $\frac{5}{42}$, $\frac{8}{45}$, $\frac{10}{63}$.

20. $1\frac{7}{40}$, $2\frac{11}{60}$, $1\frac{13}{72}$, $2\frac{17}{90}$, $1\frac{23}{108}$, $2\frac{8}{185}$.

21. Five loads of hay weighed respectively $1\frac{1}{4}$, $1\frac{2}{5}$, $1\frac{3}{8}$, $1\frac{3}{10}$, and $1\frac{11}{20}$ tons. What was their total weight?

22. By selling a lot of groceries for $\$38\frac{13}{50}$, I lost $\$5\frac{6}{25}$. For how much should I have sold them to gain $\$6\frac{2}{5}$?

23. The floors of a six-room house measure $272\frac{2}{3}$, $260\frac{3}{4}$, $243\frac{5}{6}$, $228\frac{8}{9}$, $219\frac{11}{12}$, and $213\frac{17}{18}$ square feet respectively. What would be the cost of carpeting them at 62 ct. 5 m. a square yard?

24. A man sold $22\frac{4}{15}$ acres of land, and then had $55\frac{4}{9}$ acres more than he sold. How many acres had he at first?

25. A gentleman's expenses for seven days were: $\$2\frac{1}{2}$, $\$3\frac{3}{4}$, $\$2\frac{4}{5}$, $\$4\frac{9}{10}$, $\$2\frac{19}{20}$, $\$4\frac{12}{25}$, and $\$3\frac{31}{50}$. What amount did he expend?

26. The contents of six cisterns are: $144\frac{5}{9}$, $150\frac{11}{14}$, $163\frac{7}{18}$, $175\frac{10}{21}$, $180\frac{25}{42}$, and $186\frac{41}{68}$ barrels. What are the total contents?

27. A vessel made seven trips across the Atlantic in $10\frac{3}{8}$, $8\frac{11}{24}$, $10\frac{5}{18}$, $8\frac{9}{16}$, $9\frac{7}{12}$, $10\frac{5}{36}$, and $9\frac{13}{48}$ days respectively. How many days were required for all?

28. Six stacks of hay weighed respectively $4\frac{9}{16}$, $4\frac{11}{20}$, $5\frac{16}{25}$, $4\frac{31}{40}$, $4\frac{43}{50}$, and $5\frac{49}{80}$ tons. How much did all weigh?

29. Seven blocks of marble measured: $25\frac{7}{12}$, $26\frac{11}{18}$, $27\frac{13}{24}$, $28\frac{14}{27}$, $29\frac{17}{36}$, $30\frac{29}{54}$, and $31\frac{35}{72}$ cubic feet. What was the total measurement?

30. A owns $66\frac{2}{3}$ acres of land. B owns $8\frac{3}{4}$ acres more than A, C owns $9\frac{5}{8}$ acres more than B, and D owns $7\frac{5}{6}$ acres more than C. How many acres do all own?

SUBTRACTION OF FRACTIONS

102. *Subtraction of fractions* is the process of finding the difference between two fractions or fractional numbers.

(a) Find the difference between $\frac{5}{7}$ and $\frac{2}{7}$.

SOLUTION. $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$.

EXPLANATION. As the given fractions have the same denominator, the numerators express parts of the *same size*. Then, the difference between $\frac{5}{7}$ and $\frac{2}{7}$ is $\frac{3}{7}$. Therefore, to subtract fractions having a *common denominator, find the difference of the numerators, and write it over the common denominator.*

(b) What is the difference between $\frac{5}{6}$ and $\frac{9}{14}$?

SOLUTION. $\frac{5}{6} = \frac{35}{42}$; $\frac{9}{14} = \frac{27}{42}$; $\frac{35}{42} - \frac{27}{42} = \frac{8}{42} = \frac{4}{21}$.

EXPLANATION. The least common denominator of $\frac{5}{6}$ and $\frac{9}{14}$ is 42. Reducing both fractions to this denominator, and subtracting, the difference is $\frac{8}{42}$, which, reduced to its lowest terms, is $\frac{4}{21}$.

RULE. *Reduce the fractions to their least common denominator. Find the difference of the numerators, and write it over the common denominator.*

SUGGESTION. In finding the difference between two *mixed numbers*, reduce the fractional parts to a common denominator. Then, if the numerator of the fraction in the subtrahend is greater than the numerator of the fraction in the minuend, add together the numerator and denominator of the fraction in the minuend, subtract the numerator of the other fraction from the sum, and write the remainder over the common denominator in the result. This is the same as borrowing one from the minuend, and reducing it to the common denominator of the fractions.

(*c*) From $7\frac{2}{5}$ subtract $4\frac{1}{2}$.

OPERATION

$7\frac{2}{5} = 7\frac{4}{10}$

$4\frac{1}{2} = 4\frac{5}{10}$

$2\frac{9}{10}$

EXPLANATION. The least common denominator of the fractions is 10. $\frac{2}{5} = \frac{4}{10}$; $\frac{1}{2} = \frac{5}{10}$. $\frac{5}{10}$ can not be subtracted from $\frac{4}{10}$. Borrow 1 (or $\frac{10}{10}$) from 7. $\frac{4}{10} + \frac{10}{10} = \frac{14}{10}$. $\frac{5}{10}$ from $\frac{14}{10}$ leaves $\frac{9}{10}$. Then, 5 from 7 leaves 2. The result is $2\frac{9}{10}$.

Find the difference between:

1. $\frac{5}{6}$ and $\frac{4}{9}$.
2. $\frac{7}{8}$ and $\frac{5}{12}$.
3. $\frac{13}{18}$ and $\frac{7}{12}$.
4. $\frac{8}{9}$ and $\frac{8}{15}$.
5. $\frac{11}{16}$ and $\frac{5}{12}$.
6. $\frac{14}{15}$ and $\frac{17}{20}$.
7. $\frac{17}{30}$ and $\frac{8}{45}$.
8. $\frac{13}{15}$ and $\frac{11}{21}$.
9. $\frac{19}{28}$ and $\frac{8}{35}$.
10. $\frac{17}{18}$ and $\frac{11}{14}$.
11. $8\frac{7}{12}$ and $12\frac{17}{20}$.
12. $15\frac{9}{14}$ and $21\frac{17}{21}$.
13. $20\frac{11}{24}$ and $30\frac{21}{40}$.
14. $31\frac{27}{50}$ and $50\frac{29}{30}$.
15. $50\frac{11}{36}$ and $75\frac{16}{45}$.
16. $150\frac{17}{33}$ and $99\frac{32}{55}$.
17. $213\frac{11}{42}$ and $113\frac{19}{30}$.
18. $456\frac{25}{63}$ and $345\frac{15}{28}$.
19. $555\frac{19}{39}$ and $320\frac{16}{21}$.
20. $750\frac{16}{35}$ and $624\frac{37}{45}$.
21. $\frac{96}{138}$ and $\frac{140}{209}$.
22. $100 - 10\frac{2}{3} - 15\frac{9}{14} - 20\frac{16}{21} - 26\frac{27}{28} = \text{what?}$
23. $246\frac{1}{4} - 45\frac{5}{12} - (50\frac{9}{10} + 66\frac{8}{15} + 76\frac{13}{20}) = \text{what?}$
24. $108\frac{8}{9} - (49\frac{7}{12} - 27\frac{17}{18} - 12\frac{11}{16}) = \text{what?}$

25. $161\frac{3}{5} - 10\frac{5}{6} - (35\frac{7}{10} - 20\frac{16}{21} + 25\frac{13}{14} - 30\frac{8}{15} + 15\frac{16}{35}) = ?$

26. What fraction added to $\frac{45}{259}$ will equal $\frac{48}{185}$?

27. The greater of two numbers is $150\frac{302}{441}$, and their difference is $105\frac{146}{245}$. What is the smaller number?

28. A certain number is as much less than $20\frac{5}{18}$ as $8\frac{19}{20}$ is less than $18\frac{8}{15}$. What is the number?

29. $49\frac{16}{45}$ is as much greater than a certain number as $17\frac{5}{18}$ is less than $39\frac{7}{30}$. What is the number?

30. Find a number which is as much less than $121\frac{7}{18}$ as $72\frac{7}{24}$ is greater than $50\frac{19}{36}$.

31. A speculator sold a cargo of corn for $\$581\frac{2}{5}$, thereby gaining $\$100\frac{9}{10}$. Had he sold it for $\$601\frac{28}{50}$, how much would he have gained?

32. One cistern contains $241\frac{13}{21}$ barrels of water, and another contains $39\frac{31}{35}$ barrels less. How many barrels are in the second?

33. The difference in weight of two stacks of hay is $1\frac{9}{20}$ tons. If the larger stack weighs $5\frac{25}{32}$ tons, what is their total weight?

34. In a bicycle race A rode $31\frac{3}{40}$ miles, B $3\frac{13}{24}$ miles less, and C rode $2\frac{7}{10}$ miles less than B. How far did B and C each ride?

35. A lady bought some trimmings for $\$2\frac{1}{4}$, lace for $\$3\frac{3}{5}$, gloves for $\$1\frac{9}{10}$, curtains for $\$4\frac{9}{20}$, and a shawl for $\$5\frac{27}{50}$. She gave the clerk a twenty-dollar bill, and received $\$2\frac{19}{25}$ change. How much was this incorrect?

36. A laborer worked $24\frac{7}{10}$ days in March, $1\frac{8}{15}$ days less in April, and $2\frac{7}{12}$ days less in May than in April. How many days did he work altogether?

37. A built $63\frac{5}{12}$ rods of fence, B $9\frac{1}{18}$ rods less, and C $10\frac{29}{36}$ rods less than B. How long was the fence?

38. Smith has $83\frac{9}{16}$ acres of land, which is $10\frac{17}{20}$ acres more than Jones has. Brown owns $62\frac{11}{20}$ acres less than Smith and Jones together. How many acres do all own?

39. A gentleman lost $\$15\frac{9}{10}$. He then borrowed $\$60\frac{3}{5}$, spent $\$75\frac{1}{20}$, earned $\$49\frac{23}{25}$, lent $\$18\frac{27}{100}$, and had $\$156\frac{1}{4}$ remaining. How much money had he at first? Prove the answer.

40. A dug $55\frac{5}{12}$ oz. of gold which was $10\frac{9}{16}$ oz. more than B dug, while C dug $50\frac{13}{24}$ oz. less than A and B together. What was the value of all the gold at $\$19.20$ an ounce?

MULTIPLICATION OF FRACTIONS

103. *Multiplication of fractions* is the process of finding the product of two or more fractions or fractional numbers. It includes *three operations.*

1st. To multiply a fraction by an integer.

Multiply $\frac{5}{7}$ by 9.

OPERATION. $\frac{5}{7} \times \frac{9}{1} = \frac{45}{7} = 6\frac{3}{7}$.

EXPLANATION. In this example the fraction $\frac{5}{7}$ is taken as many times as there are units in the multiplier 9.

2d. To multiply an integer by a fraction.

Multiply 8 by $\frac{4}{5}$.

OPERATION. $\frac{8}{1} \times \frac{4}{5} = \frac{32}{5} = 6\frac{2}{5}$. In this example the integer 8 is taken *such a part of one time as the multiplier is part of a unit.*

EXPLANATION. To multiply a number by $\frac{1}{5}$ is *to take one fifth of that number.* $\frac{1}{5}$ of 8 is $\frac{8}{5}$. Then $\frac{4}{5}$ of 8 are $\frac{32}{5}$, which reduced to a mixed number is $6\frac{2}{5}$.

3d. To multiply a fraction by a fraction.

(*a*) Multiply $\frac{5}{8}$ by $\frac{3}{4}$.

OPERATION. $\frac{5}{8} \times \frac{3}{4} = \frac{5 \times 3}{8 \times 4} = \frac{15}{32}$

EXPLANATION. In this example both factors are fractional, and $\frac{5}{8}$ of a unit is to be taken $\frac{3}{4}$ of one time. $\frac{1}{4}$ of $\frac{1}{8}$ is $\frac{1}{32}$; then $\frac{1}{4}$ of $\frac{5}{8}$ is $\frac{5}{32}$, and $\frac{3}{4}$ of $\frac{5}{8}$ are $\frac{15}{32}$.

REMARK. When common factors are found in the terms of the fractions to be multiplied they should be canceled in order to shorten the work.

(*b*) Multiply $\frac{5}{8}$ of $\frac{16}{21}$ by $\frac{6}{5}$ of $\frac{7}{9}$.

OPERATION. $\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{21}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{9}}} = \frac{4}{9}$

EXPLANATION. The terms of the fractions contain the common factors 3, 5, 7, and 8. Canceling these (Art. 82, Rule) the product of the factors remaining in the numerator is 4, and of those remaining in the denominators is 9. The result is $\frac{4}{9}$.

RULE. 1. *Indicate the operation and cancel all the common factors.*

2. *Write the product of the factors remaining in the numerators over the product of those remaining in the denominators.*

REMARKS. 1. When integers or mixed numbers are included they should be written in the form of improper fractions.

2. In multiplying an integer by a mixed number, or a mixed number by an integer, it is often convenient to multiply the parts separately.

(*a*) Multiply 16 by $5\frac{3}{5}$.

SOLUTION. $16 \times 5 = 80$; $16 \times \frac{3}{5} = \frac{48}{5} = 9\frac{3}{5}$. $80 + 9\frac{3}{5} = 89\frac{3}{5}$.

(*b*) Multiply $18\frac{3}{4}$ by 5.

SOLUTION. $18 \times 5 = 90$; $\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$. $90 + 3\frac{3}{4} = 93\frac{3}{4}$.

Multiply:

1. $\frac{7}{8}$ by 5.
2. $\frac{5}{9}$ by 6.
3. 8 by $\frac{3}{5}$.
4. 9 by $\frac{3}{4}$.
5. $\frac{7}{12}$ by 10.
6. $\frac{5}{6}$ by 15.
7. 12 by $\frac{9}{14}$.
8. 14 by $\frac{7}{9}$.
9. $\frac{11}{18}$ by 21.
10. $\frac{11}{20}$ by 18.
11. 24 by $\frac{9}{16}$.
12. 39 by $\frac{12}{13}$.
13. $\frac{16}{45}$ by 27.
14. $\frac{25}{49}$ by 28.
15. 45 by $\frac{37}{50}$.
16. 48 by $\frac{15}{32}$.
17. $\frac{50}{63}$ by 42.
18. $\frac{19}{36}$ by 72.
19. 84 by $\frac{48}{77}$.
20. 99 by $\frac{49}{81}$.
21. $8\frac{1}{3}$ by 8.
22. $6\frac{1}{4}$ by 12.
23. 15 by $5\frac{5}{6}$.
24. 18 by $8\frac{8}{9}$.
25. $7\frac{1}{7}$ by $2\frac{5}{8}$.
26. $8\frac{4}{5}$ by $4\frac{1}{11}$.
27. $9\frac{3}{8}$ by $3\frac{7}{15}$.
28. $8\frac{7}{10}$ by $2\frac{7}{9}$.
29. $12\frac{4}{5}$ by $5\frac{5}{8}$.
30. $44\frac{4}{9}$ by $4\frac{19}{20}$.

Find the product of:

31. $\frac{3}{4}$, $\frac{8}{9}$, $\frac{5}{12}$.
32. $\frac{7}{8}$, $\frac{9}{10}$, $\frac{16}{21}$.
33. $\frac{9}{16}$, $\frac{12}{25}$, $\frac{10}{27}$.
34. $3\frac{1}{3}$, $2\frac{1}{4}$, $2\frac{2}{5}$.
35. $2\frac{5}{8}$, $3\frac{3}{7}$, $4\frac{4}{9}$.
36. $2\frac{1}{10}$, $1\frac{1}{14}$, $3\frac{1}{5}$, $2\frac{1}{12}$.
37. $\frac{9}{20}$, 16, $1\frac{7}{18}$, $1\frac{2}{3}$.
38. $9\frac{3}{8}$, $1\frac{7}{20}$, $2\frac{2}{15}$, $1\frac{7}{9}$, $1\frac{1}{24}$.
39. $\frac{16}{21}$, $3\frac{3}{10}$, 18, $\frac{15}{22}$, $\frac{35}{48}$, $\frac{2}{45}$.
40. $\frac{24}{25}$, $\frac{27}{40}$, $\frac{13}{42}$, $\frac{28}{51}$, $\frac{35}{52}$, $\frac{34}{68}$.
41. $\frac{14}{27}$, $\frac{45}{56}$, $\frac{57}{58}$, $\frac{52}{75}$, $\frac{87}{91}$, $\frac{84}{95}$.
42. $2\frac{1}{2}$, $1\frac{3}{5}$, $1\frac{7}{8}$, $2\frac{2}{9}$, $2\frac{1}{10}$, $1\frac{1}{14}$.
43. $3\frac{3}{4}$, $\frac{16}{45}$, $4\frac{4}{9}$, $\frac{51}{64}$, $16\frac{2}{3}$, $\frac{36}{85}$.
44. $\frac{35}{24}$, $\frac{85}{39}$, $\frac{105}{34}$, $\frac{46}{63}$, $\frac{78}{69}$, $\frac{108}{125}$.
45. $2\frac{14}{33}$, $5\frac{23}{45}$, $2\frac{46}{75}$, $1\frac{23}{32}$, $2\frac{11}{62}$, $1\frac{47}{49}$.
46. $\frac{3}{32}$, $(9\frac{11}{12} - 5\frac{13}{20})$, $\frac{25}{52}$, $(7\frac{1}{2} - 3\frac{3}{5})$.
47. $1\frac{37}{63}$, $(9\frac{5}{6} - 4\frac{5}{24})$, $\frac{56}{75}$, $(12\frac{4}{9} - 8\frac{25}{36})$.
48. $\frac{25}{96}$, $(2\frac{5}{12} + 2\frac{2}{15})$, $\frac{32}{39}$, $(5\frac{1}{21} - 2\frac{11}{15})$.
49. $\frac{72}{77}$, $(10\frac{7}{18} - 7\frac{16}{45})$, $(3\frac{11}{42} + 1\frac{9}{28})$, $(\frac{14}{39} - \frac{1}{6})$.
50. $(30\frac{19}{24} - 29\frac{13}{40})$, $(21\frac{17}{45} - 19\frac{5}{18})$, $(10\frac{32}{33} - 9\frac{11}{15})$, $(12\frac{5}{6} - 11\frac{26}{51})$, $(\frac{11}{14} - \frac{7}{18})$.

104. Compound fractions (Art. 90, 3) are reduced to simple fractions by the rule for multiplication. (Art. 103, Rule.)

Reduce $\frac{5}{8}$ of $\frac{9}{10}$ of $\frac{4}{15}$ to a simple fraction.

OPERATION. $\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{9}}}{10} \times \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{3}{20}$

EXPLANATION. Indicating the operation and canceling the common factors the result is $\frac{3}{20}$.

Reduce to simple fractions:

1. $\frac{9}{10}$ of $\frac{16}{21}$.
2. $\frac{7}{15}$ of $\frac{9}{14}$.
3. $\frac{3}{5}$ of $\frac{7}{8}$.
4. $\frac{4}{7}$ of $\frac{5}{9}$.
5. $\frac{7}{9}$ of $\frac{18}{35}$.
6. $\frac{5}{12}$ of $4\frac{4}{15}$.
7. $\frac{9}{20}$ of $5\frac{5}{6}$.
8. $\frac{14}{15}$ of $3\frac{17}{21}$.
9. $\frac{25}{27}$ of $2\frac{2}{35}$.
10. $\frac{39}{40}$ of $3\frac{11}{15}$.
11. $\frac{11}{12}$ of $\frac{15}{28}$ of $\frac{35}{44}$.
12. $\frac{15}{16}$ of $\frac{18}{25}$ of $\frac{40}{63}$.
13. $\frac{22}{39}$ of $\frac{12}{35}$ of $2\frac{25}{33}$.
14. $\frac{28}{45}$ of $\frac{48}{49}$ of $1\frac{41}{64}$.
15. $\frac{63}{80}$ of $\frac{75}{91}$ of $3\frac{23}{27}$.
16. $\frac{57}{68}$ of $\frac{64}{75}$ of $\frac{51}{38}$ of $3\frac{17}{36}$.
17. $\frac{69}{84}$ of $\frac{56}{115}$ of $\frac{87}{133}$ of $2\frac{53}{116}$.
18. $\frac{65}{72}$ of $\frac{88}{105}$ of $\frac{108}{143}$ of $2\frac{47}{50}$.
19. $\frac{95}{126}$ of $\frac{135}{203}$ of $\frac{174}{209}$ of $\frac{231}{243}$ of $10\frac{2}{25}$.
20. $\frac{117}{161}$ of $\frac{187}{259}$ of $1\frac{164}{273}$ of $1\frac{146}{297}$ of $1\frac{110}{323}$.

105. *Find the cost of:*

1. $3\frac{3}{8}$ lb. of tea at $66\frac{2}{3}$ ct. a pound.
2. $6\frac{3}{5}$ oz. of silver at $83\frac{1}{3}$ ct. an ounce.
3. $55\frac{5}{9}$ rods of fence at $56\frac{1}{4}$ ct. a rod.

4. $18\frac{3}{4}$ lb. of butter at $16\frac{2}{3}$ ct. a pound.

5. $21\frac{7}{8}$ yd. of cloth at $\$1\frac{11}{25}$ a yard.

6. $6\frac{9}{16}$ bbl. of apples at $\$2\frac{2}{5}$ a barrel.

7. $26\frac{2}{3}$ acres of land at $\$56\frac{1}{4}$ an acre.

8. $9\frac{3}{5}$ oz. of opium at $68\frac{3}{4}$ ct. an ounce.

9. $14\frac{1}{16}$ tons of hay at $\$8\frac{4}{5}$ a ton.

10. $341\frac{3}{18}$ cu. ft. of marble at $\$3\frac{24}{25}$ a cubic foot.

11. What number divided by $5\frac{5}{8}$ will give $14\frac{2}{5}$ for the quotient?

12. A room is $15\frac{3}{4}$ ft. long and $13\frac{1}{3}$ ft. wide. Find the cost of carpeting the floor at $\$\frac{9}{10}$ a square yard.

13. What distance could a man walk in $6\frac{2}{3}$ days of $9\frac{3}{8}$ hours each, at the rate of $3\frac{3}{5}$ miles an hour?

14. $14\frac{2}{9}$ acres of land produced $93\frac{3}{4}$ bu. of potatoes per acre. What was the value of the crop at $\$\frac{18}{25}$ a bushel?

15. A trader having $42\frac{3}{16}$ cords of wood exchanged $\frac{32}{45}$ of it for apples, at the rate of $3\frac{1}{3}$ barrels for each cord. How many barrels were required?

16. A dealer sold $17\frac{7}{9}$ casks of wine, each containing $39\frac{3}{8}$ gal., at $\$1\frac{11}{25}$ a gallon. What was its value?

17. Hay worth $\$14\frac{2}{5}$ a ton was sold for $\frac{25}{27}$ of its value. What was received for $\frac{15}{16}$ of $9\frac{9}{10}$ tons?

18. A clerk earned $\frac{24}{25}$ of $\frac{32}{45}$ of $\$168\frac{3}{4}$ a month for $8\frac{1}{3}$ months, and saved $\frac{5}{16}$ of it. How much did he save?

19. Multiply the sum of $2\frac{7}{10}$ and $1\frac{7}{15}$ by their difference.

20. A speculator bought $56\frac{1}{4}$ acres of land at $\$83\frac{1}{3}$ an acre, but was obliged to sell $\frac{12}{25}$ of it for $\frac{8}{9}$ of the cost. How much did he realize?

21. Find the value of a block of granite 5 ft. 3 in. long, 4 ft. 2 in. wide, and 3 ft. 4 in. high, at $\$4\frac{4}{5}$ a cubic foot.

22. A merchant imported 18 cases of silk, each containing $16\frac{2}{3}$ bolts of $55\frac{5}{9}$ yards per bolt, at a cost of $\$\frac{15}{16}$ a yard. What was the total cost?

23. Find the cost of excavating a cellar 33 ft. 9 in. long, 26 ft. 8 in. wide, and 7 ft. 6 in. deep, at $\$\frac{6}{25}$ a cubic yard.

24. A has $56\frac{1}{4}$ acres of land; B owns $\frac{14}{15}$ as much as A; C has $\frac{25}{42}$ as much as B, and D $\frac{16}{25}$ as much as C. How many acres have all together?

25. A owns $43\frac{3}{4}$ acres, and B owns $\frac{5}{7}$ as much. If each should sell $\frac{33}{50}$ of his land to the other, how much more would B then have than A?

DIVISION OF FRACTIONS

106. *Division of fractions* is the process of finding the quotient when the dividend, the divisor, or both, are fractions or fractional numbers. It includes three operations.

1st. To divide a fraction by an integer.

Divide $\frac{9}{10}$ by 3.

OPERATION. $\frac{9}{10} \div 3 = \frac{9}{10} \times \frac{1}{3} = \frac{3}{10}$.

EXPLANATION. To divide a number by 3 is *to take one third of that number*. Hence, $\frac{9}{10}$ divided by $3 = \frac{3}{10}$.

2d. To divide an integer by a fraction.

Divide 5 by $\frac{3}{4}$.

OPERATION. $5 \div \frac{3}{4} = 5 \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3}$.

EXPLANATION. $\frac{3}{4}$ are contained in 1 ($\frac{4}{4}$) $\frac{4}{3}$ times. Hence, $\frac{4}{4}$ are contained in 5, $\frac{4}{3} \times 5 = \frac{20}{3} = 6\frac{2}{3}$ times.

Or, 5 reduced to fourths = 20 fourths; *1 fourth* is contained in 20 fourths *20 times; 3 fourths* are contained in 20 fourths *one third* of 20 times, $= 6\frac{2}{3}$ times.

3d. To divide a fraction by a fraction.

(*a*) Divide $\frac{5}{8}$ by $\frac{4}{5}$.

OPERATION. $\frac{5}{8} \div \frac{4}{5} = \frac{5}{8} \times \frac{5}{4} = \frac{25}{32}$.

EXPLANATION. $\frac{1}{5}$ are contained in 1 ($\frac{5}{5}$) $\frac{5}{4}$ times. Hence, $\frac{4}{5}$ are contained in $\frac{5}{8}$, $\frac{5}{8}$ of $\frac{5}{4}$ times, $= \frac{25}{32}$ times.

REMARK. When common factors are found in the terms of the dividend and divisor, they should be canceled.

(*b*) Divide $\frac{8}{15}$ by $\frac{4}{5}$.

OPERATION. $\frac{8}{15} \div \frac{4}{5} = \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{15}}} \times \frac{\cancel{5}}{\cancel{4}} = \frac{2}{3}$.

(*c*) Divide $7\frac{1}{2}$ by $8\frac{1}{3}$.

OPERATION. $7\frac{1}{2} = \frac{15}{2}$; $8\frac{1}{3} = \frac{25}{3}$; $\frac{15}{2} \div \frac{25}{3} = \frac{\cancel{15}}{2} \times \frac{3}{\underset{5}{\cancel{25}}} = \frac{3 \times 3}{2 \times 5} = \frac{9}{10}$.

EXPLANATION. Reducing the mixed numbers to improper fractions, and proceeding as before, the result is $\frac{9}{10}$.

RULE. *Indicate the operation, invert the divisor, and proceed according to the rule for multiplication of fractions.*

REMARKS. 1. When integers or mixed numbers are included, they should be expressed in the form of improper fractions.

2. When the divisor is a compound fraction, all of the fractions included in it should be inverted before multiplying.

NOTE. When the divisor is inverted, it indicates *how many times it is contained in a unit*; this, multiplied by the dividend, *shows how many times the dividend contains the divisor.* (Art. 106, 3d explanation.)

Divide:

1. $\frac{18}{25}$ by 6.
2. $\frac{32}{39}$ by 8.
3. 10 by $\frac{5}{9}$.
4. 18 by $\frac{12}{25}$.
5. $9\frac{3}{5}$ by 12.
6. $13\frac{1}{3}$ by 16.
7. 15 by $6\frac{2}{3}$.
8. 27 by $3\frac{3}{5}$.
9. $9\frac{1}{11}$ by $11\frac{1}{9}$.
10. $15\frac{5}{8}$ by $4\frac{11}{16}$.
11. $7\frac{7}{18}$ by $3\frac{14}{27}$.
12. $10\frac{5}{16}$ by $22\frac{11}{12}$.
13. $2\frac{37}{49}$ by $8\frac{1}{28}$.
14. $55\frac{1}{8}$ by $4\frac{19}{32}$.
15. $7\frac{35}{39}$ by $8\frac{28}{26}$.
16. $\frac{7}{12}$ of $\frac{13}{15}$ by $\frac{28}{45}$ of $\frac{39}{40}$.
17. $\frac{4}{9}$ of $\frac{12}{25}$ by $\frac{8}{21}$ of $\frac{49}{50}$.
18. $\frac{35}{48}$ of $\frac{26}{51}$ by $\frac{21}{34}$ of $\frac{52}{81}$.
19. $\frac{33}{50}$ of $\frac{32}{63}$ by $\frac{56}{75}$ of $\frac{44}{49}$.
20. $\frac{38}{45}$ of $\frac{72}{95}$ by $\frac{21}{80}$ of $\frac{64}{105}$.
21. $\frac{14}{69}$ of $\frac{18}{77}$ of $\frac{23}{48}$ by $\frac{35}{99}$ of $\frac{27}{28}$ of $\frac{16}{75}$.
22. $\frac{65}{102} \times \frac{114}{135} \times \frac{145}{288}$ by $\frac{91}{108} \times \frac{87}{126} \times \frac{95}{136}$.
23. $3\frac{16}{33} \times 3\frac{21}{32}$ by $2\frac{11}{144} \times 1\frac{67}{176}$.
24. $1\frac{19}{56} \times 1\frac{84}{171} \times 1\frac{127}{225}$ by $2\frac{35}{76} \times 1\frac{67}{189}$.
25. $2\frac{61}{85} \times 6\frac{18}{49} \times 1\frac{85}{143}$ by $1\frac{73}{119} \times 3\frac{21}{50}$.

26. If 18 bbl. of flour are worth $\$67\frac{1}{2}$, what is the value of 1 barrel?

27. How much sugar at $6\frac{2}{3}$ cents a pound would cost 85 cents?

28. One man can build a wall in $43\frac{3}{4}$ days. In what time could 14 men build it?

29. If $78\frac{1}{8}$ bu. of potatoes are worth \$50, what is the value of 1 bushel?

30. How many acres can a man plow in one day, if he can plow $29\frac{1}{6}$ acres in $11\frac{1}{9}$ days?

31. If a vessel sails $104\frac{1}{6}$ mi. in $5\frac{5}{9}$ hr., what is its rate per hour?

32. In how many days could a workman earn $\$36\frac{1}{4}$ at the rate of $\$1\frac{37}{50}$ per day?

33. If $6\frac{9}{16}$ tons of hay cost $\$89\frac{1}{4}$, what was the cost per ton?

34. By what must $24\frac{8}{9}$ be multiplied to make the product $13\frac{1}{3}$?

35. When coal-oil is $\$3\frac{9}{25}$ a barrel, how many barrels will cost $\$117\frac{3}{5}$?

36. $\$82\frac{1}{2}$ was paid for $114\frac{7}{12}$ rods of fence. What was the cost per rod?

37. If $27\frac{7}{9}$ acres produce $1562\frac{1}{2}$ bu. of corn, what is the yield per acre?

38. The product of two numbers is $\frac{91}{99}$. One of them is $\frac{221}{260}$. What is the other?

39. If $21\frac{7}{8}$ yards of cloth cost $\$31\frac{1}{2}$, what would be the cost of $13\frac{8}{9}$ yards?

40. How many rods of fence can a man build in $9\frac{8}{15}$ days if he builds $43\frac{3}{4}$ rods in $6\frac{5}{12}$ days?

41. How many acres of land should produce $20\frac{1}{20}$ tons of clover, if $16\frac{2}{3}$ acres produce 35 tons?

42. When $8\frac{7}{16}$ bbl. of cider sell for $\$48\frac{3}{5}$, what is the value of $5\frac{5}{8}$ bbl.?

43. If a machine can saw $5\frac{5}{24}$ ft. of boards per minute, in what time could it saw 250 boards, each $12\frac{1}{2}$ ft. long?

44. 15 men can do a piece of work in $13\frac{2}{3}$ days. How many men could do as much work in $7\frac{1}{12}$ days?

45. If an engine consumes $18\frac{1}{8}$ bu. of coal in running $25\frac{7}{9}$ mi., what distance could it run with $33\frac{3}{4}$ bu.?

46. If a steamer travels $118\frac{1}{8}$ mi. in $12\frac{3}{5}$ hr., how far could it run in $10\frac{2}{3}$ hr.?

47. If $86\frac{5}{8}$ is $9\frac{9}{10}$ times a certain number, what is $8\frac{4}{7}$ times that number?

48. The product of three numbers is 1000. Two of them are $9\frac{3}{8}$ and $5\frac{3}{4}\frac{1}{5}$. What is the third number?

49. Divide the product of $4\frac{1}{8}$ times $11\frac{1}{9}$ by the product of $19\frac{4}{5}$ multiplied by $4\frac{1}{6}$.

50. How many boxes of eggs, each containing $12\frac{1}{2}$ dozen, worth $\$\frac{3}{16}$ a dozen, should be exchanged for 5 bbl. of cider, each holding $31\frac{1}{4}$ gal., at $\$\frac{9}{20}$ a gallon?

107. To find what part one number is of another.

(*a*) 3 is what part of 5?

SOLUTION. It is 3 divided by 5, $= \frac{3}{5}$.

REMARK. To find *what part* one number is of another always involves division. The preceding question may be illustrated thus: "A boy had five apples, and gave away three. *What part* of the apples did he give?"

The five apples may be divided into five equal parts, and three of the parts taken, just as the same opera-

tion can be performed upon a unit. *One* of the parts is *one fifth of 5 apples;* then, *three* of the parts are *three fifths of 5 apples.*

(*b*) $\frac{2}{3}$ is what part of $\frac{3}{4}$?

OPERATION. $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.

EXPLANATION. One unit is divided into 3 equal parts and 2 of the parts are taken; another unit is divided into 4 equal parts, and 3 of the parts taken. The question is, *what part is the first part of the second part?* Reducing the fractions to a common denominator, $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{1}{12}$ is $\frac{1}{9}$ of $\frac{9}{12}$. Therefore, $\frac{8}{12}$ are $\frac{8}{9}$ of $\frac{9}{12}$. But the inconvenience of reducing the fractions to a common denominator and finding the quotient of the numerators is avoided by applying the rule for division of fractions. (Art. 106.)

REMARK. Questions like the above may have *three forms.* 1st, $\frac{2}{3}$ is what part *of* $\frac{3}{4}$? 2d, what part *of* $\frac{3}{4}$ is $\frac{2}{3}$? 3d, what part is $\frac{2}{3}$ *of* $\frac{3}{4}$? The answer is the same in each case, and it is convenient to notice that *the number or quantity following the phrase "part of," or the preposition "of," is the divisor in every instance.*

1. $\frac{5}{12}$ is what part of $\frac{8}{9}$?
2. $1\frac{2}{5}$ is what part of $\frac{8}{15}$?
3. 10 is what part of $15\frac{5}{8}$?
4. 18 is what part of $33\frac{3}{4}$?
5. $16\frac{2}{3}$ is what part of 40?
6. $12\frac{3}{5}$ is what part of 42?
7. What part of $11\frac{11}{14}$ is $5\frac{5}{8}$?
8. What part of $18\frac{3}{4}$ is $18\frac{1}{18}$?
9. What part of $16\frac{4}{21}$ is $11\frac{23}{35}$?
10. What part of $13\frac{1}{8}$ is $8\frac{1}{28}$?
11. What part of $17\frac{13}{16}$ is $15\frac{1}{24}$?

12. What part of $19\frac{1}{26}$ is $3\frac{3}{65}$?

13. What part is $19\frac{14}{15}$ of $20\frac{4}{9}$?

14. What part is $13\frac{13}{27}$ of $25\frac{5}{18}$?

15. What part is $5\frac{1}{57}$ of $11\frac{11}{38}$?

16. $\frac{9}{40}$ of $\frac{16}{27}$ is what part of $\frac{18}{49}$ of $\frac{28}{45}$?

17. $\frac{20}{39}$ of $\frac{52}{75}$ is what part of $\frac{82}{63}$ of $\frac{85}{86}$?

18. $\frac{88}{105} \times 1\frac{23}{33}$ is what part of $\frac{68}{69} \times 3\frac{31}{51}$?

19. $\frac{36}{65} \div \frac{54}{91}$ is what part of $\frac{32}{57} \div \frac{48}{133}$?

20. $\frac{50}{63} \div \frac{40}{81}$ is what part of $1\frac{31}{95} \div \frac{49}{76}$?

21. If a man can do a certain work in $19\frac{4}{9}$ days, what part of it could he do in $11\frac{2}{3}$ days?

22. A cyclist rode $40\frac{5}{18}$ mi. at the rate of $9\frac{2}{3}$ mi. per hour. What part of the distance was traveled per hour.

23. A and B sold a lot of grain for $\$103\frac{17}{25}$, of which A received $\$43\frac{1}{5}$. What part of the grain did he own?

24. A farmer owning $54\frac{11}{16}$ acres of land, sold $34\frac{1}{36}$ acres. What part of his land did he sell?

25. A 50-gallon cask is $\frac{15}{16}$ full. If $24\frac{3}{8}$ gal. be drawn off, what part of the cask will remain full?

108. A complex fraction (Art. 90, 4) is an indicated division, the numerator being the *dividend*, and the denominator the *divisor* (Art. 87). The terms farthest apart are sometimes called the *extremes*, and those nearest together the *means*. Thus, in the complex fraction $\frac{\frac{2}{5}}{\frac{3}{4}}$, the *extreme* terms are 2 and 4, and the *mean* terms 5 and 3. There are *three methods* of reducing a complex fraction to a simple fraction.

FIRST. *By inverting the terms of the divisor, and then multiplying.* (Art. 106, Rule.)

Reduce $\frac{4\frac{1}{2}}{6\frac{3}{4}}$ to a simple fraction.

OPERATION. $4\frac{1}{2} = \frac{9}{2}$; $6\frac{3}{4} = \frac{27}{4}$. $\frac{9}{2} \div \frac{27}{4} = \frac{\cancel{9}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{27}}} = \frac{2}{3}$.

EXPLANATION. Both terms of the complex fraction are reduced to improper fractions. Then the terms of the divisor are inverted, the common factors canceled, and the result is $\frac{2}{3}$.

SECOND. *By multiplying together the extreme terms for the numerator, and the mean terms for the denominator of the required fraction.*

Reduce $\frac{4\frac{1}{2}}{6\frac{3}{4}}$ to a simple fraction.

OPERATION. $\frac{4\frac{1}{2}}{6\frac{3}{4}} = \frac{\frac{9}{2}}{\frac{27}{4}} = \frac{\cancel{9} \times \overset{2}{\cancel{4}}}{\cancel{2} \times \underset{3}{\cancel{27}}} = \frac{2}{3}$.

EXPLANATION. The mixed numbers are reduced to improper fractions. Then the *extreme terms* 9 and 4 are multiplied together for the numerator of the required fraction, and the *mean terms* 2 and 27 for its denominator.

THIRD. *By multiplying both terms of the complex fraction by the least common multiple of the denominators.*

Reduce $\frac{4\frac{1}{2}}{6\frac{3}{4}}$ to a simple fraction.

OPERATION. $\frac{4\frac{1}{2} \times 4}{6\frac{3}{4} \times 4} = \frac{18}{27} = \frac{2}{3}$.

EXPLANATION. The least common multiple of the denominators 2 and 4 is 4. Multiplying both terms of the complex fraction by 4, the result is $\frac{18}{27} = \frac{2}{3}$.

Reduce the following to simple fractions, or to mixed numbers:

1. $\frac{\frac{3}{5}}{\frac{9}{10}}$.
2. $\frac{\frac{5}{6}}{\frac{15}{16}}$.
3. $\frac{6\frac{3}{4}}{7\frac{1}{2}}$.
4. $\frac{3\frac{3}{4}}{5\frac{5}{8}}$.
5. $\frac{4\frac{4}{9}}{5\frac{5}{6}}$.
6. $\frac{\frac{14}{15}}{21}$.
7. $\frac{21\frac{1}{12}}{28}$.
8. $\frac{18}{\frac{27}{40}}$.
9. $\frac{48}{\frac{32}{33}}$.
10. $\frac{9\frac{3}{8}}{10\frac{5}{12}}$.
11. $\frac{22\frac{2}{7}}{24}$.
12. $\frac{14\frac{7}{12}}{28\frac{1}{8}}$.
13. $\frac{4\frac{4}{35}}{5\frac{1}{25}}$.
14. $\frac{3\frac{3}{65}}{6\frac{9}{26}}$.
15. $\frac{3\frac{2}{95}}{4\frac{4}{57}}$.
16. $\frac{11\frac{1}{48}}{7\frac{7}{32}}$.
17. $\frac{\frac{9}{16}\times\frac{24}{35}}{\frac{18}{25}\times\frac{15}{56}}$.
18. $\frac{\frac{28}{45}\div\frac{16}{27}}{\frac{35}{48}\div\frac{25}{108}}$.
19. $\frac{\frac{8}{15}+\frac{8}{25}}{\frac{1}{6}+\frac{9}{10}}$.
20. $\frac{12\frac{5}{12}-4\frac{3}{20}}{15\frac{1}{18}-6\frac{14}{45}}$.
21. $\frac{9\frac{9}{13}}{4\frac{4}{35}}\times\frac{\frac{26}{105}}{\frac{7}{12}}$.
22. $\frac{9\frac{3}{8}+2\frac{7}{40}}{12\frac{8}{15}-4\frac{5}{6}}$.

109. 1. The product is $13\frac{1}{3}$, and the multiplier $3\frac{3}{8}$. What is the multiplicand?

2. The quotient is $2\frac{7}{10}$, and the dividend $11\frac{1}{4}$. What is the divisor?

3. The multiplicand is $\frac{25}{84}$, and the product $\frac{15}{16}$. What is the multiplier?

4. The dividend is 14, and the divisor $39\frac{3}{8}$. What is the quotient?

5. A farmer having $14\frac{1}{16}$ tons of hay, sold $6\frac{3}{4}$ tons. What part of the hay was sold?

6. A steamer made a trip of $126\frac{7}{18}$ miles, at the rate of $7\frac{7}{12}$ miles per hour. How long was she occupied?

7. $20\frac{5}{6}$ acres of land cost $\$693\frac{3}{4}$. What was the cost per acre?

8. A dealer paid \$300 for $58\frac{19}{32}$ cords of wood. What was the cost per cord?

9. A and B dug a ditch for $\$40\frac{14}{25}$, of which A's share was $\$16\frac{9}{10}$. What part of the work did he do?

10. $243\frac{5}{6}$ yards of cloth were made into suits, each requiring $7\frac{7}{18}$ yards. How many suits were made?

11. 8 men can do a certain work in $10\frac{5}{12}$ days. How many men could do the same work in $5\frac{5}{9}$ days?

12. A man owning $\frac{9}{10}$ of a farm sold $\frac{5}{8}$ of his share for \$1440. At that rate, what was the value of the farm?

13. If a machine can cut $2\frac{3}{16}$ sq. rd. of grass per minute, in what time could it mow 6 lots, each measuring $253\frac{1}{8}$ sq. rd.?

14. A man paid $\$165\frac{3}{5}$ for forty-eight sheep. At that rate, how many could have been bought for $\$172\frac{1}{2}$?

15. I gave \$55 for $45\frac{5}{6}$ yd. of carpeting. Had I bought $56\frac{1}{4}$ yd. at the same rate, what would the bill have been?

16. Owning $64\frac{7}{20}$ acres of land, I sold $\frac{1}{2}\frac{5}{2}$ of it for \$2925. What was the selling price per acre?

17. A merchant paid $\$3\frac{3}{4}$ a box for cigars, and sold them for $1\frac{1}{15}$ times the cost. If he received \$100, how many boxes were sold?

18. If $12\frac{3}{8}$ bbl. of coal-oil cost $\$44\frac{11}{20}$, how many barrels could be bought for $\$112\frac{1}{2}$?

19. I paid $\$73\frac{1}{2}$ for $76\frac{9}{16}$ gal. of wine, and sold $43\frac{3}{4}$ gal. for $\$52\frac{1}{2}$. How much was gained on the part sold?

20. A and B built a fence for $\$33\frac{3}{4}$. A worked $8\frac{1}{3}$ days, and built $55\frac{5}{9}$ rods. B worked as many days as A built rods per day. How much did each earn?

21. A dealer sold $\frac{3}{4}\frac{2}{5}$ of $16\frac{7}{80}$ tons of hay for $\frac{4}{7}\frac{4}{5}$ of $\$263\frac{1}{4}$. At what price per ton was it sold?

22. A drover paid $\$806\frac{2}{5}$ for hogs at $\$5\frac{3}{5}$ a head. Having lost 19, at what price per head should he have sold the others to gain $\$37\frac{7}{20}$ on the cost of all?

23. I sold $109\frac{3}{8}$ bu. of apples, which was $\frac{5}{9}$ of what I had remaining. What was the value of the entire crop at $\$\frac{1}{2}\frac{3}{5}$ a bushel?

24. A jeweler paid $\$225$ for gold, at $\$\frac{9}{10}$ a pennyweight, and made it into rings weighing $3\frac{1}{3}$ pwt. each, which he sold at $\$3\frac{3}{4}$ apiece. Find his entire gain.

25. $\frac{3}{4}\frac{9}{0}$ of $\dfrac{9\frac{9}{14}}{10\frac{1}{3}\frac{1}{5}}$ divided by $\frac{1}{2}\frac{6}{5}$ of $\dfrac{18\frac{3}{4}}{8\frac{8}{15}}$ = what?

REDUCTION OF FRACTIONAL CONCRETE NUMBERS

110. To reduce a fraction to a lower denomination.

Fractional concrete numbers are reduced to lower denominations by multiplication. (Art. 51, Rule 1.)

EXAMPLE. Reduce $\frac{1}{2}\frac{}{0}$ of a gallon to the fraction of a pint.

OPERATION. $\dfrac{1}{2\cancel{0}} \times \dfrac{\cancel{4}}{1} \times \dfrac{2}{1} = \dfrac{2}{5}$ pint.
$\quad 5$

EXPLANATION. Gallons are reduced to quarts by multiplying by 4; quarts are reduced to pints by multiplying by 2. Therefore, $\frac{1}{2}\frac{}{0}$ gal. $= \frac{2}{5}$ pint.

Reduce:

1. $\frac{5}{512}$ bu. to the fraction of a pint.
2. $\frac{3}{256}$ gal. to the fraction of a gill.
3. $\frac{1}{1920}$ cwt. to the fraction of an ounce.
4. $\frac{4}{99}$ rd. to the fraction of a foot.
5. $\frac{1}{7260}$ acre to the fraction of a square yard.
6. $\frac{15}{4096}$ cord to the fraction of a cubic foot.
7. $\frac{1}{1920}$ day to the fraction of a minute.
8. $\frac{1}{6000}$ lb. Troy to the fraction of a grain.
9. $\frac{1}{640}$ ℔ Apoth. to the fraction of a scruple.
10. $\frac{1}{9504}$ mi. to the fraction of a foot.
11. $\frac{1}{2016}$ bbl. of wine to the fraction of a gill.
12. $\frac{1}{653440}$ acre to the fraction of a square foot.

111. To find the value of a fraction in integers.

Fractional concrete numbers are reduced to integers of lower denominations by multiplication. When the multiplication produces a mixed number, the integer is reserved as a part of the answer, and the operation is continued with the fractional part only. This process is termed *finding the value of a fraction in integers.*

Find the value of $\frac{4}{9}$ of a week in integers.

OPERATION. $\frac{4}{9} \times \frac{7}{1} = \frac{28}{9} = 3\frac{1}{9}$ da.

$$\frac{1}{\cancel{9}_{3}} \times \frac{\overset{8}{\cancel{24}}}{1} = \frac{8}{3} = 2\tfrac{2}{3} \text{ hr.} \qquad \frac{2}{\cancel{3}_{1}} \times \frac{\overset{20}{\cancel{60}}}{1} = 40 \text{ min.}$$

EXPLANATION. Multiplying $\frac{4}{9}$ wk. by 7 reduces it to $3\frac{1}{9}$ days. Multiplying $\frac{1}{9}$ da. by 24 reduces it to $2\frac{2}{3}$ hr. Multiplying $\frac{2}{3}$ hr. by 60 reduces it to 40 min. Therefore, $\frac{4}{9}$ wk. = 3 da. 2 hr. 40 min.

Find the value in integers of:

1. $\frac{47}{64}$ bu.
2. $\frac{21}{32}$ gal.
3. $\frac{33}{160}$ T.
4. $\frac{85}{66}$ mi.
5. $\frac{25}{36}$ cord.
6. $\frac{39}{50}$ wk.
7. $\frac{99}{160}$ lb. Troy.
8. $\frac{125}{216}$ lb. Apoth.
9. $\frac{10}{33}$ perch.
10. $\frac{75}{256}$ T.
11. $\frac{125}{363}$ sq. mi.
12. $\frac{605}{756}$ bbl.

112. To reduce a fraction to a higher denomination.

Fractional concrete numbers are reduced to higher denominations by division. (Art. 51, Rule 2.)

Reduce $\frac{4}{5}$ of an inch to the fraction of a yard.

OPERATION. $\frac{4}{5} \div \frac{12}{1} \div \frac{3}{1} = \frac{4}{5} \times \frac{1}{12} \times \frac{1}{3} = \frac{1}{45}$ yd.

EXPLANATION. Inches are reduced to feet by dividing by 12; feet are reduced to yards by dividing by 3. Therefore, $\frac{4}{5}$ inch $= \frac{1}{45}$ yard.

Reduce:

1. $\frac{5}{8}$ pt. to the fraction of a bushel.
2. $\frac{3}{4}$ gi. to the fraction of a gallon.
3. $\frac{16}{25}$ lb. to the fraction of a ton.
4. $\frac{33}{50}$ in. to the fraction of a rod.
5. $\frac{99}{100}$ sq. foot to the fraction of a square rod.
6. $\frac{28}{45}$ hr. to the fraction of a week.
7. $\frac{24}{25}$ gr. Troy to the fraction of a pound.
8. $\frac{9}{10}$ gr. Apoth. to the fraction of a pound.
9. $\frac{14}{15}$ pt. to the fraction of a hogshead.
10. $\frac{220}{221}$ yd. to the fraction of a mile.
11. $\frac{484}{485}$ sq. yd. to the fraction of an acre.
12. $\frac{126}{135}$ gi. to the fraction of a barrel.

113. To reduce one compound number to the fraction of another compound number.

Reduce 2 qt. 1 pt. 2 gi. to the fraction of a gallon.

OPERATION.

$$\begin{array}{cccc} \text{gal.} & \text{qt.} & \text{pt.} & \text{gi.} \\ \hline 1 & 0 & 0 & 0 \end{array} \quad \begin{array}{cccc} & 2 & 1 & 2 \\ \hline & & & \end{array} = \frac{22}{32} = \frac{11}{16} \text{ gal.}$$

EXPLANATION. Write the given numbers in the form of a fraction, the less for the numerator. Reducing both to the same denomination, 2 qt. 1 pt. 2 gi. = 22 gi.; 1 gal. = 32 gi. $22 \div 32 = \frac{22}{32} = \frac{11}{16}$ gallon.

REMARK. In all problems of this kind, the number following the preposition *of* is the divisor.

1. Reduce 1 pk. 4 qt. 1 pt. to the fraction of a bushel.
2. Reduce 4 cwt. 87 lb. 8 oz. to the fraction of a ton.
3. Reduce 1 qt. 1 pt. 3 gi. to the fraction of a gallon.
4. Reduce 4 yd. 6 in. to the fraction of a rod.
5. Reduce 116 sq. rd. 11 sq. yd. to part of an acre.
6. Reduce 15 hr. 21 min. 36 sec. to part of a day.
7. What part of 7 lb. 9 oz. 5 pwt. 6 gr. is 4 lb. 3 oz. 16 pwt. 6 gr.?
8. What part of 10 bu. 3 pk. 2 qt. 1 pt. is 6 bu. 3 pk. 4 qt. 1 pt.?
9. What part of 5 cwt. 95 lb. 8 oz. is 2 cwt. 48 lb. 2 oz.?
10. What part of 5 A. 20 sq. rd. 30 sq. yd. is 4 A. 16 sq. rd. 24 sq. yd.?
11. 291 cu. yd. 18 cu. ft. is what part of 68 cords?
12. 6 rd. 2 yd. 1 ft. 3 in. is what part of 6 rd. 4 yd. 2 ft. 4 in.?

114. Addition and subtraction of fractional compound numbers.

In finding the sum or the difference of fractional compound numbers, reduce them to integers, and proceed according to the rules for addition and subtraction of compound numbers. (Arts. 66, 67.)

(*a*) Add $\frac{3}{5}$ da., $\frac{3}{8}$ hr., and $\frac{3}{4}$ min.

OPERATION. $\frac{3}{5} \times \frac{24}{1} = 14\frac{4}{5}$ hr. $\frac{2}{5} \times \frac{\overset{12}{60}}{1} = 24$ min.

$\frac{3}{8} \times \frac{\overset{15}{60}}{1} = 22\frac{1}{2}$ min. $\frac{1}{2} \times \frac{\overset{30}{60}}{1} = 30$ sec.

$\frac{3}{4} \times \frac{\overset{15}{60}}{1} = 45$ sec.

hr.	min.	sec.
14	24	0
	22	30
		45
14	47	15

EXPLANATION. $\frac{4}{5}$ da. = 14 hr. 24 min.; $\frac{1}{2}$ hr. = 22 min. 30 sec.; $\frac{3}{4}$ min. = 45 sec.

The sum is 14 hr. 47 min. 15 sec.

(*b*) From $2\frac{5}{32}$ bu. subtract $\frac{9}{16}$ pk.

OPERATION. $\frac{25}{\overset{8}{32}} \times \frac{4}{1} = 3\frac{1}{8}$ pk. $\frac{1}{8} \times \frac{8}{1} = 1$ qt.

$\frac{9}{\overset{2}{16}} \times \frac{8}{1} = 4\frac{1}{2}$ qt. $\frac{1}{2} \times \frac{2}{1} = 1$ pt.

pk.	qt.	pt.
3	1	0
	4	1
2	4	1

EXPLANATION. $2\frac{5}{32}$ bu. = 3 pk. 1 qt.; $\frac{9}{16}$ pk. = 4 qt. 1 pt. The difference is [illegible] pt.

1. $\frac{3}{8}$ [illegible] $\frac{3}{4}$ pt. = what?
2. [illegible]t. + $\frac{11}{16}$ lb. = what?
3. [illegible]. rd. + $\frac{8}{27}$ sq. yd. + $\frac{1}{3}$ sq. ft.
4. [illegible] da. + $\frac{5}{18}$ hr. + $\frac{5}{6}$ min. = what?

5. $\frac{7}{8}\frac{3}{0}$ ℔ + $\frac{1}{2}\frac{9}{0}$ ℥ + $\frac{1}{2}\frac{3}{0}$ 3 + $\frac{9}{2}\frac{}{0}$ ℈ = what?

6. $\frac{2}{4}\frac{5}{8}$ bbl. + $\frac{2}{6}\frac{5}{4}$ gal. + $\frac{2}{3}\frac{3}{2}$ qt. + $\frac{3}{16}$ pt. = what?

7. $\frac{6}{7}\frac{5}{2}$ lb. Troy − $\frac{2}{3}\frac{5}{2}$ oz. = what?

8. $\frac{1}{2}\frac{3}{6}\frac{}{4}$ mi. − $\frac{6}{9}\frac{4}{9}$ rd. = what?

REVIEW OF FRACTIONS

115. 1. A man owning $\frac{2}{3}\frac{5}{2}$ of a vessel sold $\frac{8}{15}$ of his share for $\$1562\frac{1}{2}$. At that rate, what was the value of the vessel?

2. A pile of wood is $93\frac{3}{4}$ ft. long, 8 ft. wide, and $5\frac{1}{3}$ ft. high. Find its value at $\$3\frac{1}{5}$ a cord.

3. $26\frac{1}{3}\frac{1}{6} - 9\frac{9}{2}\frac{}{0} - (12\frac{2}{3}\frac{9}{0} - 8\frac{7}{18}) =$ what?

4. If $81\frac{1}{4}$ bbl. of flour cost $\$292\frac{1}{2}$, how much would $109\frac{3}{8}$ bbl. cost?

5. Six stacks of hay together weighed 20 tons. The first five weighed, respectively, $2\frac{3}{4}$, $3\frac{5}{8}$, $3\frac{7}{10}$, $3\frac{9}{16}$, and $2\frac{2}{4}\frac{7}{0}$ tons. How much did the sixth stack weigh?

6. $\frac{25\frac{5}{18}}{32\frac{1}{12}} \times \frac{20\frac{5}{8}}{10\frac{7}{24}} \div \frac{12\frac{3}{19}}{6\frac{4}{25}} =$ what?

7. A field $33\frac{3}{4}$ rd. wide contains $11\frac{1}{4}$ acres. Find the cost of fencing it at $\$\frac{18}{25}$ per rod.

8. I sold $56\frac{1}{4}$ acres of land at $\$41\frac{2}{3}$ an acre, and invested $\frac{18}{25}$ of the money in village lots at $\$187\frac{1}{2}$ apiece. How many lots were bought?

9. $\frac{3}{6}\frac{9}{8}$ of $15\frac{1}{2}\frac{}{6}$ is what part of $9\frac{7}{12}$ times $1\frac{1}{2}\frac{1}{5}$?

10. A, B, and C built a fence for $\$123\frac{3}{4}$. B built $\frac{5}{8}$ as many rods as A, and C built $\frac{7}{10}$ as many rods as B. How much did each earn?

11. $8\frac{17}{36} + \frac{9}{10} \times 15\frac{5}{18} - 24\frac{5}{9} \div 21\frac{15}{16} \times \frac{4}{27} =$ what?

12. A schoolroom is $33\frac{1}{3}$ ft. long, 24 ft. wide, and $13\frac{1}{2}$ ft. high. Find the cost of painting the walls and ceiling at $\$\frac{9}{50}$ a square yard.

13. If a man can travel a certain distance in $9\frac{3}{8}$ hr. at the rate of $3\frac{1}{3}$ mi. per hour, in what time could he do it by traveling $3\frac{3}{4}$ mi. per hour?

14. Multiply together $\frac{9}{6\frac{3}{4}}$, $\frac{8\frac{4}{5}}{6\frac{1}{9}}$, $\frac{5\frac{5}{8}}{6\frac{3}{10}}$, $\frac{7\frac{1}{2}}{6\frac{2}{3}}$, and $\frac{5\frac{5}{6}}{18}$.

15. From a roll of carpeting containing $56\frac{7}{10}$ yd., a part was sold for $\$31\frac{1}{2}$, at $\$1\frac{1}{4}$ a yard. What part of the roll remained?

16. A owes B $\$207\frac{9}{10}$, which is $2\frac{3}{4}$ times as much as he owes C, or $3\frac{3}{5}$ times as much as he owes D. What amount does he owe?

17. Divide the product of $9\frac{13}{18}$ times $9\frac{9}{14}$ by the difference between $19\frac{7}{15}$ and $9\frac{1}{20}$.

18. A, B, and C dug a ditch. B dug $\frac{8}{9}$ as many rods as A, and C dug $\frac{7}{8}$ as many as B. If C dug $29\frac{1}{6}$ rods, how long was the ditch?

19. A marble base for a monument is 2 ft. 4 in. square, and 15 in. high. Find its value at $\$5\frac{2}{5}$ a cubic foot.

20. A farmer divided $\frac{7}{12}$ of his land between his son and daughter, giving the daughter $40\frac{5}{8}$ acres, which was $\frac{13}{15}$ as much as the son received. How many acres were in the entire farm?

21. Express in its simplest form the quotient of $\frac{172296}{486645}$ divided by $\frac{18750}{506250}$.

22. If a lot $44\frac{4}{9}$ rd. long and $28\frac{4}{5}$ rd. wide produces $37\frac{1}{2}$ bu. of barley per acre, what would be the value of $\frac{8}{15}$ of the crop at $\$1\frac{2}{25}$ a bushel?

23. $\frac{4}{7}\frac{9}{5}$ wk. + $\frac{2}{5}\frac{1}{0}$ da. + $\frac{4}{25}$ hr. = what?

24. If 25 men can do a piece of work in $9\frac{3}{5}$ da., how many men could do $\frac{1}{16}$ of the work in $9\frac{3}{8}$ da.?

25. A 60-gallon cask is $\frac{2}{3}\frac{5}{2}$ full. If $24\frac{3}{8}$ gal. should be drawn off, what part of the cask would remain filled?

26. Multiply $\frac{1}{2}\frac{3}{0} + \frac{5}{6} \times \frac{1}{2}\frac{8}{5}$ by $\frac{1}{2}\frac{3}{0} - \frac{5}{6} \times \frac{1}{2}\frac{8}{5}$.

27. $\frac{1}{2}\frac{3}{0} + \frac{5}{6} \times \frac{1}{2}\frac{8}{5} \times \frac{1}{2}\frac{3}{0} - \frac{5}{6} \times \frac{1}{2}\frac{8}{5}$ = what?

28. I exchanged 11 gal. 2 qt. 1 pt. 3 gi. of wine at $\$\frac{2}{2}\frac{4}{5}$ a gallon for apples at $\$\frac{9}{2}\frac{}{0}$ a bushel. How many bushels were required?

29. The fore wheels of a wagon are $11\frac{1}{4}$ ft. in circumference, and the hind wheels $12\frac{3}{8}$ ft. How many more revolutions will one of the former make than one of the latter in traveling $8\frac{7}{16}$ miles?

30. A speculator owning $\frac{3}{4}\frac{5}{8}$ of a cargo of grain, sold $\frac{9}{14}$ of his share to one man, and the remainder, which was $584\frac{15}{16}$ bu. to another. How many bushels did the cargo contain?

31. Multiply the sum of $7\frac{1}{2}$ and $3\frac{3}{5}$ by their difference, and divide the product by $4\frac{1}{15}$ times $5\frac{1}{2}\frac{7}{0}$.

32. A man lent $\$15\frac{7}{10}$; he then earned $\$29\frac{3}{4}$, lost $\$18\frac{9}{2}\frac{}{0}$, borrowed $\$37\frac{1}{2}$, spent $\$33\frac{7}{5}\frac{}{0}$, and had $\$99\frac{2}{2}\frac{4}{5}$ remaining. How much had he at first?

33. A huckster sold $78\frac{3}{4}$ bu. of apples at $\$\frac{1}{2}\frac{6}{5}$ a bushel, and invested $\frac{1}{2}\frac{1}{1}$ of the money in chickens at $\$\frac{3}{10}$ apiece, and the remainder in ducks at $\$\frac{8}{2}\frac{}{5}$ apiece. How many of each did he buy?

34. A gentleman bought $44\frac{4}{9}$ acres of land at $\$56\frac{1}{4}$ an acre, but was obliged to sell $\frac{27}{40}$ of it for $\frac{8}{9}$ of its value. How much did he receive for what he sold?

35. $\dfrac{10\frac{9}{16}}{13\frac{3}{4}} - \dfrac{11\frac{11}{20}}{39\frac{3}{8}} - \dfrac{8\frac{4}{45}}{25\frac{5}{18}} =$ what?

36. If 8 men can build $106\frac{2}{3}$ rods of fence in $3\frac{5}{9}$ da., how many men could build $93\frac{3}{4}$ rods in $4\frac{1}{6}$ days?

37. Find the cost of polishing five faces of a cubical block of granite whose edge is 1 ft. 8 in. long, at $\$1\frac{2}{25}$ a square foot.

38. In a partnership, A furnished $\frac{4}{15}$ of the capital, B $\frac{5}{18}$ of it, C $\frac{3}{10}$, and D the remainder, which was $\$787\frac{1}{2}$. What was the entire capital?

39. Divide the sum of $9\frac{8}{15}$ and $3\frac{3}{10}$ by their difference, and multiply the quotient by $\frac{3}{4}$ of $\frac{34}{105}$.

40. A and B are $72\frac{1}{2}$ mi. apart, and travel toward each other, A at the rate of $5\frac{5}{8}$ mi. an hour, B $4\frac{4}{9}$ mi. per hour. In how many hours will they meet, and how far will each have traveled?

41. If 8 men can reap a field in $10\frac{5}{12}$ da., in how many days could 15 men reap 3 fields, each $3\frac{3}{5}$ times as large as the first?

42. What was the cost of $68\frac{4}{15}$ acres of farm land, if $\$237\frac{1}{2}$ was gained by selling $27\frac{7}{9}$ acres for $\$1800$?

43. Find the difference in the cost of carpeting two rooms, one $20\frac{1}{4}$ ft. long by $16\frac{2}{3}$ ft. wide, the other $16\frac{7}{8}$ ft. long by $13\frac{1}{3}$ ft. wide, with Brussels carpet $\frac{3}{4}$ yd. wide, at $\$\frac{3}{4}$ a yard.

44. $(10\frac{5}{12} + 8\frac{1}{3}) \times (10\frac{5}{12} - 8\frac{1}{3}) \div 15\frac{5}{8} =$ what?

45. Two men built a fence for $\$56\frac{1}{4}$. A worked $6\frac{2}{3}$ days, and built 25 rods. B worked as many days as A built rods per day. How much did each earn?

46. How many tiles 16 in. long by 15 in. wide would pave an area $53\frac{1}{3}$ ft. long by $31\frac{1}{4}$ ft. wide?

47. Divide the L. C. M. of 91, 143, and 231 by the G. C. D. of 3432, 4719, and 5434, and divide the quotient by $12\frac{3}{5} \div \frac{9}{20}$.

48. If $18\frac{1}{18}$ acres produce $609\frac{3}{8}$ bu. of barley, how many bushels should $33\frac{1}{3}$ acres produce?

49. If $17\frac{1}{9}$ acres produce $481\frac{1}{4}$ bu. of wheat, how many acres should produce 750 bu.?

50. The fencing of a rectangular field cost $\$159\frac{1}{2}$. If its length is $67\frac{1}{2}$ rods, and its area $22\frac{1}{2}$ acres, what was the cost per rod?

51. Divide the product of $6\frac{2}{3}$ and $5\frac{5}{8}$ by their difference, and multiply the quotient by their sum.

52. A can do a piece of work in $5\frac{5}{8}$ da., B a piece $4\frac{1}{6}$ times as large in $18\frac{3}{4}$ da., and C a piece $3\frac{5}{9}$ times as large as A's in $26\frac{2}{3}$ da. In what time can all do a piece $3\frac{1}{5}$ times as large as the first?

53. A stock of goods worth $\$4218\frac{3}{4}$ caught fire. $\frac{2}{15}$ of it was destroyed; $\$233\frac{2}{5}$ worth was stolen; $\$2410\frac{7}{20}$ worth was uninjured, and the remainder, being damaged, was sold for $\frac{5}{9}$ of its cost. How much was paid for it?

54. How many rugs 6 ft. 8 in. long by 3 ft. 9 in. wide would cover the floor of a parlor 33 ft. 4 in. long by 22 ft. 6 in. wide?

55. A drover paid $\$562\frac{1}{2}$ for sheep at $\$3\frac{3}{4}$ a head. If 10 died, at what price per head must he sell the others to gain $\frac{3}{25}$ of the cost of all?

56. I bought a lot of lumber for $\$191\frac{1}{4}$, which was $\frac{3}{20}$ less than its value, and sold it for $\frac{1}{12}$ more than its value. How much was gained?

57. A trader exchanged $15\frac{5}{18}$ acres of land at $\$56\frac{7}{10}$ an acre, for horses at $\$57\frac{3}{4}$ apiece. How many horses were required?

58. I lent A $\frac{11}{35}$ of my money, and B $\frac{7}{12}$ of the remainder. If B received $\$56\frac{1}{4}$ more than A, how much money had I left?

59. I paid $\$52\frac{1}{2}$ an acre for a farm, and then sold $\frac{9}{10}$ of it at cost for $\$1968\frac{3}{4}$. How many acres were in the entire farm?

60. The fence surrounding a rectangular lot cost $\$148\frac{4}{5}$ at $\$\frac{9}{10}$ a rod. If the lot was $42\frac{2}{3}$ rd. long, what was its value at $\$67\frac{1}{2}$ an acre?

61. A man divided $\frac{25}{48}$ of his farm between his two sons, giving to the younger $\frac{11}{14}$ as much as to the elder. If the elder received $9\frac{3}{8}$ acres more than the younger, how many acres were in the farm?

62. What number added to $\frac{16}{45}$ of $16\frac{17}{28}$ will equal $\frac{63}{80}$ of $15\frac{11}{27}$?

63. Sound travels at the rate of 1090 feet per second. If $4\frac{1}{2}$ seconds elapse between seeing the flash of a cannon, and hearing its report, in how many minutes could a man walk the distance at the rate of $7\frac{1}{2}$ feet per second?

64. A dealer sold $13\frac{1}{8}$ tons of hay for $\$136\frac{1}{2}$, thereby losing $\frac{3}{16}$ of the cost. At what price per ton should he have sold it to gain $\frac{3}{16}$ of the cost?

65. In an orchard, $\frac{2}{9}$ of the trees are apple trees, $\frac{3}{14}$ pear trees, $\frac{5}{18}$ peach trees, $\frac{4}{21}$ plum trees, and the remainder, which is 24, are cherry trees. What is the number of peach trees?

66. $$\frac{8\frac{4}{7}+5\frac{1}{3}}{26\frac{1}{9}-5\frac{5}{6}} \times \frac{20\frac{5}{8}-10\frac{5}{12}}{6\frac{3}{10}+4\frac{8}{15}} \div \frac{9\frac{9}{13}\times 1\frac{11}{25}}{44\frac{3}{50}\div\frac{3}{10}} = \text{what?}$$

67. A owns $51\frac{5}{32}$ acres of land which is $2\frac{7}{20}$ acres less than B owns. If C owns $49\frac{3}{40}$ acres less than A and B together, how many acres have all?

68. Two cities are situated upon the same road, $187\frac{1}{2}$ mi. apart. An express train running $31\frac{1}{4}$ mi. per hour, and a freight train traveling $\frac{8}{15}$ as fast, start at the same time from one city to the other. How far behind will the freight train be when the express arrives?

69. The length of a wall measured by a ten-foot pole was supposed to be 270 feet, but the pole was found to be $1\frac{1}{2}$ inches too long. What was the true length of the wall?

70. The sum of two numbers is $32\frac{13}{15}$ and their difference is $\frac{8}{25}$ of the less number. What are the numbers?

71. A stick of timber was 26 ft. 8 in. long, and 15 in. square. $\frac{1}{16}$ of it was sawn off, and the remainder sold at $\$\frac{8}{25}$ a cubic foot. What was paid for it?

72. Divide the product of $\frac{13\frac{1}{8}}{20\frac{5}{12}}$ and $\frac{7\frac{7}{15}}{6\frac{3}{10}}$ by the quotient of $\frac{10\frac{1}{2}}{5\frac{5}{6}}$ divided by $\frac{9\frac{3}{4}}{6\frac{6}{7}}$.

73. A has $33\frac{3}{4}$ acres of land, which is $\frac{9}{11}$ as much as B owns. If each should sell $\frac{7}{15}$ of his land to the other, how much more than A would B then have?

74. In each of three successive years, a speculator gained $\frac{4}{15}$ of the money he had at the beginning of the year. If his entire gain was $\$11497\frac{1}{5}$, what was his original capital?

75. A dealer paid $\$510\frac{3}{10}$ for corn at $\$\frac{14}{25}$ a bushel, and sold $\frac{38}{45}$ of it for $\frac{20}{21}$ of its cost. How much did he receive for the part sold?

76. What fraction of 7 lb. 8 oz. 9 pwt. 3 gr. is 4 lb. 3 oz. 7 pwt. 7 gr.?

77. On counting their money, it was found that B's equaled $\frac{13}{15}$ of A's, and that C had $\frac{5}{12}$ as much as both of the others. If C had $\$15$ less than B, how much had all?

78. A pasture is $33\frac{1}{3}$ rd. wide, and its area is 7 A. 130 sq. rd. It is surrounded by a fence 4 boards high, each board being 6 in. wide. Find the value of the lumber at $\$1\frac{7}{25}$ per 100 sq. ft.

79. A and B are $56\frac{8}{9}$ mi. apart, and travel toward each other, A at the rate of $4\frac{7}{10}$ mi. and B at the rate of $3\frac{5}{6}$ mi. per hour. When they meet, how far will each have traveled?

80. A man can do a piece of work in $11\frac{1}{4}$ days. If his son works $\frac{9}{16}$ as fast, in what time could they complete it by working together?

81. I paid $\$236\frac{1}{4}$ for corn at $\$\frac{9}{20}$ a bushel, and sold $\frac{16}{35}$ of it for $\$144$. At what price per bushel was it sold?

82. $\frac{2}{4}\frac{5}{8}$ of $43\frac{1}{5} \times \dfrac{6\frac{3}{4} - 2\frac{7}{10}}{2\frac{7}{10} + 2\frac{3}{8}} \div \dfrac{11\frac{1}{2}\frac{9}{1} \times 4\frac{8}{2}\frac{}{5}}{1\frac{4}{3}\frac{}{5} \div 9\frac{9}{14}} = \text{what?}$

83. A lot $41\frac{2}{3}$ rd. long by $28\frac{4}{5}$ rd. wide produced a crop of potatoes that was sold for $\$ 475\frac{1}{5}$, at $\$ \frac{1}{2}\frac{8}{5}$ a bushel. What was the yield per acre?

84. A trader gained $\frac{7}{1}\frac{}{8}$ of his capital in one speculation, and $\frac{4}{15}$ of the amount in another, but in a third he lost $\frac{1}{2}\frac{2}{5}$ of all his money, and had $\$ 1543\frac{3}{4}$ remaining. How much had he at first?

85. A travels at the rate of $259\frac{7}{8}$ mi. in $6\frac{3}{10}$ da., B at the rate of $273\frac{3}{8}$ mi. in $6\frac{3}{4}$ da. If they start from the same place, and travel in opposite directions, how far apart will they be at the end of $16\frac{2}{3}$ days?

86. By selling a lot of grain for $\$ 243\frac{1}{5}$, I made $\frac{8}{16}$ of the cost. I paid for it with flour at $\$ 3\frac{3}{4}$ a barrel, and $\$ 17\frac{3}{10}$ in money. How many barrels of flour were given?

87. A farmer sold $\frac{1}{2}\frac{3}{5}$ of his land to A for $\$ 3250$, at $\$ 66\frac{2}{3}$ per acre, and the remainder to B for $\$ 2812\frac{1}{2}$. What price per acre did B pay?

88. If $\$ 18\frac{3}{4}$ is added to my money, the sum multiplied by $\frac{2}{3}\frac{6}{3}$, $\$ 6\frac{1}{4}$ subtracted from the product, and the remainder divided by $1\frac{9}{16}$, the result will be $\$ 100$. How much money have I?

89. A wholesale dealer sold a part of his flour at $\$ 3\frac{3}{5}$ a barrel for $\$ 202\frac{1}{2}$, which was $\frac{9}{2}\frac{}{0}$ of what he received for the remainder at $\$ 4\frac{4}{5}$ a barrel. How many barrels were sold?

90. Two cities, A and B, are situated upon the same stream. A steamer leaves A for B, sailing at the rate

of $8\frac{5}{12}$ mi. per hour, while another leaves B for A, at the rate of $10\frac{1}{3}$ mi. per hour. If they meet in $13\frac{1}{3}$ hr., what is the distance from A to B?

91. $\dfrac{\frac{3}{4}\{19\frac{1}{18} - 2\frac{11}{12} \times 4\frac{6}{7} + 29\frac{1}{6} \div 2\frac{5}{8}\}}{\frac{5}{6}\{30\frac{1}{45} + 5\frac{3}{28} \div 3\frac{12}{35} - 5\frac{13}{15} \times 2\frac{2}{11}\}} = \text{what?}$

92. A has $\$87\frac{1}{2}$, which is $\frac{7}{17}$ of the sum of B's and C's money. If B has $\frac{5}{11}$ as much as A and C together, how much more has C than B?

93. What was the cost of $343\frac{3}{4}$ bu. of wheat, if $\$11\frac{1}{4}$ was lost by selling $187\frac{1}{2}$ bu. for $\$168\frac{3}{4}$?

94. An improvement company wishing to subdivide a tract of land, finds that it can be laid out into an exact number of lots of equal length, and $43\frac{1}{3}$, $40\frac{5}{6}$, $38\frac{8}{9}$, or $37\frac{11}{12}$ ft. in width. If they divide it into $40\frac{5}{6}$-foot lots, what will be their total value at $\$168\frac{3}{4}$ each?

95. A lot is $66\frac{2}{3}$ rd. long, and its area is $15\frac{5}{8}$ acres. It is surrounded by a fence five boards high, each board being 6 in. wide. Find the value of the lumber at $\$14\frac{2}{5}$ per thousand feet.

96. A speculator bought a quantity of grain for $\frac{19}{20}$ of its value, and sold it for $\frac{13}{12}$ of its value, thereby gaining $\$100$. How much did he pay for it?

97. A dealer paid $\$337\frac{1}{2}$ for apples at $\$2\frac{1}{4}$ a bbl., and sold $\frac{8}{15}$ of them at a profit of $\$\frac{3}{4}$ a barrel, and the remainder at a loss of $\$\frac{1}{2}$ a barrel. What was his net gain?

98. A room is 33 ft. 9 in. long, 22 ft. 6 in. wide, and 11 ft. 3 in. high. Find the entire cost of plastering it at $33\frac{1}{3}¢$ a square yard, and of covering the floor with Brussels carpet $\frac{3}{4}$ yd. wide, at $66\frac{2}{3}¢$ a yard.

DECIMAL FRACTIONS

DEFINITIONS

116. When a unit is divided into 10 equal parts, the parts are called *tenths;* when each tenth is divided into 10 equal parts, the parts are called *hundredths;* when each hundredth is divided into 10 equal parts, the parts are called *thousandths*, etc.

These divisions of a unit are called *decimal fractions*, because they decrease in value by the *scale of ten.* Hence, a *decimal fraction* is one or more *tenths, hundredths, thousandths*, etc., *of a unit.*

REMARK. The chief difference between common fractions and decimal fractions is this: The denominator of a common fraction may be *any number*, as 7, 12, 245, etc., and it is *always written*, while the denominator of a decimal fraction *is always* 1 *with one or more ciphers annexed*, as 10, 100, 1000, etc., and it is seldom written.

117. As the orders of decimal fractions decrease in value from left to right by the *scale of ten*, they are written like the orders of integers. To distinguish a decimal from an integer, a period (.), called the *decimal point*, is placed before it. Thus, in the expression 12.25, the number on the *left* of the point is an *integer*, and that on the *right* a *decimal.*

The first order on the right of the decimal point is *tenths;* the second, *hundredths;* the third, *thousandths;*

the fourth, *ten-thousandths;* the fifth, *hundred-thousandths;* the sixth, *millionths*, etc. That is, *each successive order toward the right is one tenth of that directly preceding it.* Thus, the decimal $.1111 = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000}$. Hence, the orders of decimals decrease from left to right, and increase from right to left, the same as the orders of integers.

A figure in the *first* decimal order represents a fraction whose denominator is 1 with *one cipher* annexed. $.3 = \frac{3}{10}$. A figure in the *second* decimal order represents a fraction whose denominator is 1 with *two ciphers* annexed. $.03 = \frac{3}{100}$. A figure in the *third* decimal order represents a fraction whose denominator is 1 with *three ciphers* annexed. $.003 = \frac{3}{1000}$, etc.

Hence, 1st. *The denominator of a decimal fraction is* 1, *with as many ciphers annexed as are equal to the number of figures on the right of the decimal point.*

2d. *The value of any decimal figure depends upon the place from units' order which it occupies.*

The *numerator* of a decimal expresses the number of parts taken. *It must contain as many figures as there are ciphers in the denominator.* When vacant orders occur in the numerator, *their places must be occupied by ciphers.* Thus, $\frac{4}{100}$ written as a decimal is .04; $\frac{4}{1000}$ is .004, etc.

118. 1. A *pure decimal* is one which stands alone; as .0125.

2. A *mixed decimal* is an integer with a decimal annexed; as 6.7, which is the same as $6\frac{7}{10}$.

3. A *complex decimal* is a decimal with a common fraction annexed; as $.8\frac{3}{4}$, which is the same as $\frac{8\frac{3}{4}}{10}$.

119. As the orders of decimals decrease from left to right by the scale of ten, *prefixing* a cipher to a decimal moves it *one place toward the right*, and therefore *divides its value by* 10. Thus, a cipher prefixed to .1 changes it to .01; a cipher prefixed to .01 changes it to .001, etc., each successive fraction being *one tenth* as great as the one preceding it.

NOTATION AND NUMERATION OF DECIMALS

To write decimals.

120. Express decimally six hundred twenty-five *ten-thousandths.*

The number written in figures is .0625.

EXPLANATION. A unit is divided into 10,000 equal parts, and 625 parts are taken. The *numerator*, then, is 625, and the *denominator*, which is not written, is 10,000. As the denominator contains *four ciphers*, there must be *four decimal orders in the numerator*. (Art. 117, 2d.) Therefore, a cipher must be written in the tenths' order which is vacant, and the decimal point prefixed.

RULE. *Write the number of parts expressed by the numerator as an integer; then, if necessary, prefix ciphers until the right-hand figure stands in the required order, and place the decimal point before tenths.*

REMARKS. 1. When the decimal to be written is an improper fraction, the number of ciphers in the denominator determines the position of the decimal point. Thus, the improper fraction, four hundred fifteen *hundredths* has *two ciphers* in its denominator. Therefore, *there are two decimal orders* in the numerator, and it is written 4.15.

2. In writing decimal numbers, observe that the *italicized words* indicate the denominator of the fraction.

Write the following decimal numbers:

1. Eight *hundredths;* forty-nine *hundredths.*
2. Seven hundred and twelve *hundredths.*
3. Six *thousandths;* thirty-two *thousandths.*
4. Five hundred and one *thousandths.*
5. Four hundred and fifty-six *thousandths.*
6. Three thousand two hundred and nine *thousandths.*
7. Nineteen *ten-thousandths.*
8. Seven hundred and six *ten-thousandths.*
9. Four thousand and eleven *ten-thousandths.*
10. Eight *ten-thousandths.*
11. Thirty-nine thousand and sixteen *ten-thousandths.*
12. Two hundred and fifty-six *hundred-thousandths.*
13. Ninety-three *hundred-thousandths.*
14. Seven thousand and four *hundred-thousandths.*
15. 60581 *hundred-thousandths.*
16. 123456 *hundred-thousandths.*
17. Sixty-four *millionths;* two *millionths.*
18. Nine hundred and seven *millionths.*
19. Ten thousand and one *millionths.*
20. Eight thousand seven hundred and six *millionths.*
21. 765432 *millionths;* 3040506 *millionths.*
22. Four hundred and thirty-seven *ten-millionths.*
23. Six thousand and forty-eight *ten-millionths.*
24. One million ten thousand and one *ten-millionths.*
25. Seventy-eight thousand and ninety-six *ten-millionths.*

26. 31645279 *ten-millionths.*

27. One hundred and twenty-three *hundred-millionths.*

28. Sixty-nine thousand and seven *hundred-millionths.*

29. 5417082 *hundred-millionths.*

30. 90340608 *hundred-millionths.*

31. 123456789 *hundred-millionths.*

32. Eighty thousand four hundred and 12 *billionths.*

33. 9876453 *billionths.*

34. One million ten thousand and one *billionths.*

35. Sixty-four units and seven *tenths.*

36. Thirty-nine units and eight *hundredths.*

37. One hundred units and one *thousandth.*

38. 512 units and 624 *ten-thousandths.*

39. Ten units and 10101 *hundred-thousandths.*

40. One thousand units and one *ten-thousandth.*

41. 8410 units and 841 *millionths.*

42. Four hundred units and four *hundred-thousandths.*

43. 1001 units and 101 *ten-thousandths.*

44. 927 units and 729 *millionths.*

45. 405 units 683201 *ten-millionths.*

Express the following common fractions as decimals:

46. $\frac{8}{10}$, $\frac{5}{10}$, $\frac{16}{1000}$, $\frac{49}{100}$, $\frac{75}{100}$, $\frac{84}{100}$, $\frac{92}{100}$.

47. $\frac{7}{1000}$, $\frac{19}{1000}$, $\frac{28}{1000}$, $\frac{101}{1000}$, $\frac{144}{1000}$, $\frac{456}{1000}$.

48. $\frac{6}{10000}$, $\frac{32}{10000}$, $\frac{75}{10000}$, $\frac{128}{10000}$, $\frac{1001}{10000}$, $\frac{8756}{10000}$.

49. $\frac{625}{1000}$, $\frac{15237}{100000}$, $\frac{12}{100}$, $\frac{1728}{10000}$, $\frac{15625}{1000000}$.

50. $4\frac{314}{10000}$, $32\frac{15}{1000}$, $45\frac{64}{100000}$, $100\frac{101101}{1000000}$.

To read decimals.

121. Read the decimal .0325.

The number is read, "Three hundred and twenty-five *ten-thousandths*."

EXPLANATION. The numerator of the decimal is 325; the right-hand order is *ten-thousandths*. Therefore, the *name* of the decimal is three hundred and twenty-five *ten-thousandths*.

RULE. *Read the decimal as an integer; then, add the name of the right-hand order.*

REMARK. A *mixed decimal* is usually read as an integer with a fraction annexed, but it may be read as an improper fraction. Thus 4.05 is read, "four units and five *hundredths*," but may be read, "four hundred and five *hundredths*."

Read the following decimal numbers:

1. .09; .016; .324; .008; 9.1; 3.7.
2. 5.26; 8.04; 13.75; 17.089; 21.006.
3. .1872; .0319; .0064; .0007; 40.0108.
4. 16.5002; 23.0001; 39.0111; 50.0704; 68.1234.
5. .04567; .31625; .00028; .86002; .00005.
6. 43.10101; 79.02064; 51.00897; 36.00025.
7. .001492; .654321; .070605; .000101; .047016.
8. 84.00072; 62.005003; 75.075075; 90.001001.
9. .4354657; .0068012; .0007436; .0391625.
10. .12345678; .02030405; .00960047; .00003061.
11. 19.0625; 28.349; 36.10101; 45.54; 72.01096.
12. 89.6; 64.00216; 57.425; 72.101001; 54.0204.
13. 48.037126; 39.00082743; 99.03125; 24.004016.
14. 76.00567894; 87.010203; 100.1234567.
15. .60; .750; .8010; .9600; .41200; .375000.

Express the following decimals as common fractions:

16. .4; .09; .32; .75; .016; .512; .008.

17. .069; .333; .0017; .1562; .1001; .0324.

18. .0005; .0625; .00731; .10101; .01011.

19. .020304; .006875; .101625; .000437.

20. .049; .6048; .00752; .091125; .010101.

REDUCTION OF DECIMALS

122. *Reduction of decimals* is the process of changing their form without altering their value. There are *four cases.*

CASE I

123. To reduce an integer to an improper decimal fraction.

This is done by annexing one or more decimal ciphers to the integer. Thus, annexing *one* decimal cipher to 9 reduces it to $9.0 = \frac{90}{10}$; annexing *two* decimal ciphers to 9 reduces it to $9.00 = \frac{900}{100}$, etc.

Hence, (1) *Annexing decimal ciphers to an integer does not change its value;* and conversely, (2) *Omitting decimal ciphers from the right of an integer does not change its value.*

CASE II

124. To reduce a decimal to higher terms.

This is done by annexing one or more ciphers to the decimal fraction. Thus, annexing *one* cipher to the decimal .9 reduces it to $.90 = \frac{90}{100}$; annexing *two* ciphers to .9 reduces it to $.900 = \frac{900}{1000}$, etc. This is the same

as multiplying both terms of a fraction by the same number.

Hence, (1) *Annexing ciphers to a decimal does not change its value;* and conversely, (2) *Omitting ciphers from the right of a decimal does not change its value.*

REMARKS. 1. Annexing or prefixing ciphers to a decimal has an *opposite effect* from the same operation performed upon an integer. Thus, annexing a cipher to the decimal .7 does not change its value; but annexing a cipher to the integer 7 changes it to 70, and therefore *multiplies it by* 10.

Also, prefixing a cipher to the decimal .7 changes it to .07, and therefore *divides it by* 10, while prefixing a cipher to the integer 7 does not affect its value.

2. Pupils should also understand the effect of *removing* ciphers. Let the teacher explain the effect of removing the ciphers from 320, .320, .032, 032.

CASE III

125. To reduce a decimal to a common fraction.

Reduce .375 to a common fraction.

OPERATION. $.375 = \frac{375}{1000} = \frac{3}{8}$.

EXPLANATION. .375 is expressed as a common fraction by writing its denominator. It then becomes $\frac{375}{1000}$, which, reduced to its lowest terms $= \frac{3}{8}$.

RULE. *Omitting the decimal point, write the denominator beneath the numerator and reduce the fraction to its lowest terms.*

Reduce the following to common fractions, or to mixed numbers:

1. .36.	4. .95.	7. .225.	10. .0512.
2. .94.	5. .048.	8. .245.	11. .7875.
3. .75.	6. .875.	9. .0625.	12. .78125.

13. .63125.	**18.** .978125.	**23.** 15.025.	**28.** .32$\frac{2}{5}$.
14. .6225.	**19.** .9765625.	**24.** 40.4375.	**29.** 2.54$\frac{1}{6}$.
15. .91125.	**20.** 6.4.	**25.** .62$\frac{1}{2}$.	**30.** 4.685$\frac{5}{7}$.
16. .703125.	**21.** 8.75.	**26.** .41$\frac{2}{3}$.	**31.** 6.046$\frac{7}{8}$.
17. .04096.	**22.** 9.48.	**27.** .56$\frac{1}{4}$.	**32.** 8.444$\frac{4}{9}$.

CASE IV

126. To reduce a common fraction to a decimal.

Reduce $\frac{7}{8}$ to a decimal.

OPERATION

$$8\overline{)7.000}\quad .875$$

EXPLANATION. $\frac{7}{8}$ is 7 divided by 8. As 8 is not contained in 7, annex a decimal cipher to 7, which reduces it to 7.0, or 70 *tenths*. 70 tenths divided by $8 = 8$ tenths, with a remainder of 6 tenths. Annex a cipher to .6, which reduces it to .60 or 60 *hundredths*. .60 divided by $8 = 7$ hundredths, with a remainder of 4 hundredths. Annex a cipher to .04, which reduces it to .040, or 40 *thousandths*. This, divided by $8 = 5$ thousandths. Therefore, $\frac{7}{8}$ reduced to a decimal = .875.

RULE. *Annex decimal ciphers to the numerator, divide by the denominator, and point off as many figures from the right of the quotient as will equal the number of ciphers annexed to the numerator.*

REMARKS. 1. If the number of figures in the quotient is *less* than the number of decimal ciphers annexed to the dividend, *prefix* a sufficient number of ciphers to make them equal.

2. When the given fraction is in its lowest terms the division will never terminate if the denominator contains any prime factor besides 2 or 5 (the factors of 10). In such cases, after extending the division sufficiently far, it is customary either to write the sign + after the last quotient figure, or to write the remainder over the divisor, as in division of simple numbers.

Reduce the following to pure or to mixed decimals:

1. $\frac{17}{20}$. 7. $\frac{39}{200}$. 13. $\frac{47}{80}$. 19. $\frac{45}{128}$. 25. $\frac{121}{32}$.

2. $\frac{18}{25}$. 8. $\frac{121}{250}$. 14. $\frac{11}{16}$. 20. $\frac{99}{8125}$. 26. $\frac{25}{36}$.

3. $\frac{49}{50}$. 9. $\frac{847}{500}$. 15. $\frac{3}{32}$. 21. $\frac{89}{40}$. 27. $\frac{82}{49}$.

4. $\frac{13}{40}$. 10. $\frac{128}{625}$. 16. $\frac{9}{160}$. 22. $\frac{512}{125}$. 28. $\frac{4}{85}$.

5. $\frac{5}{8}$. 11. $\frac{297}{400}$. 17. $\frac{25}{64}$. 23. $\frac{729}{80}$. 29. $\frac{119}{120}$.

6. $\frac{64}{125}$. 12. $\frac{999}{1250}$. 18. $\frac{19}{320}$. 24. $\frac{75}{16}$. 30. $\frac{125}{144}$.

ADDITION OF DECIMALS

127. *Addition of decimals* is the process of finding the sum of two or more decimals or decimal numbers.

Add 6.5, 17.32, 254.609, 85.4307, and 1287.04905.

OPERATION

6.5
17.32
254.609
85.4307
1287.04905
1650.90875

EXPLANATION. The numbers are written so that units of the same order stand in the same column. The addition is performed as in addition of simple numbers, and the decimal point in the sum is placed under the column of decimal points.

RULE. 1. *Write the numbers to be added so that units of the same order shall stand in the same column.*

2. *Add as in addition of simple numbers, and place the decimal point in the sum under the column of decimal points.*

REMARK. When a complex decimal does not contain as many decimal orders as any of the other numbers, reduce the common fraction to a decimal and extend the division to the required number of decimal orders.

1. Add 4.28, 5.019, 7.8, 6.57, 8.956.

2. .5+.25+.375+.0625+.91125+.004375=what?

3. Find the sum of 25.39, 39.4, 47.851, 51.0206, 63.78, 72.5584.

4. Add 17.014, 34.1078, 68.39, 136.18625, 272.8, 544.25195.

5. .476 + .00689 + .019472 + .1384 + .356 + .00027 + .002968 = what?

6. Find the sum of 4.0567, 9.16, 15.24856, 22.392, 30.7, 41.899529.

7. Add 9.016524, 15.3247, 24.15, 16.437, 21.00419, 14.067586.

8. Add 36 and 49 *thousandths*, 68 and 17052 *hundred-thousandths*, 132 and 126 *ten-thousandths*, 240 and 985436 *millionths*, 125 and 6 *hundredths*, 201 and 18625 *hundred-thousandths*, and 184 and 36194 *millionths*.

9. Find the sum of 21 *tenths*, 326 *hundredths*, 4015 *thousandths*, 63169 *ten-thousandths*, 700247 *hundred-thousandths*, 8283054 *millionths*, 90007152 *ten-millionths*, and 1007236585 *hundred-millionths*.

10. Reduce to mixed decimals and add: $1\frac{7}{8}$, $2\frac{7}{20}$, $3\frac{9}{40}$, $4\frac{7}{16}$, $5\frac{9}{80}$.

11. An ocean steamer registered the following distances for six days: 400.5 mi., 417.225 mi., 420.05625 mi., 415.75 mi., 430.0175 mi., and 416.45125 mi. How far did it sail?

12. The output of a coal mine for seven months was as follows: 1040.825, 983.71875, 1131.0125, 1056.25, 974.375, 1104.75125, and 1209.0675 tons. What was the total output?

13. Eight farms measured respectively 91.126, 78.6, 84.0172, 62.74359, 76.405, 98.03241, 80.2568, and 68.819 acres. What was their total area?

14. Seven loads of wheat contained 38.3828125, 50.265625, 46.09375, 37.6875, 51.125, 43.75, and 39.5 bushels. What was the entire quantity?

15. Six car loads of broken stone measured 10.168459, 11.0032, 9.87, 12.05628, 11.735, and 9.252061 perches. What was the total amount?

16. The contents of eight oak logs were: 48.024, 51.3169, 42.007168, 39.64, 50.07283, 46.2, 48.365802, and 49.4983 cu. ft. What were the total contents?

17. Seven shipments of wine were: 302.00625, 256.5, 419.078125, 185.75, 357.0875, 291.0546875, and 189.125 gallons. How much was shipped altogether?

18. The taxes of a district for ten years were: \$3096.72094, \$3289.14308, \$3528.01632, \$3752.60146, \$4117.08254, \$4361.23578, \$4630.37482, \$4905.46726, \$5473.89814, and \$5804.20966. What amount was collected?

19. The floors of a house of ten rooms measured $230\frac{3}{4}$, $247\frac{3}{16}$, $306\frac{27}{40}$, $264\frac{25}{32}$, $340\frac{3}{8}$, $296\frac{13}{20}$, $242\frac{29}{80}$, $365\frac{1}{2}$, $256\frac{121}{160}$, and 373.7125 sq. ft. What was the cost of carpeting them at \$1.25 a square yard?

20. A gold miner's gatherings for eight months were respectively: $15.54\frac{7}{8}$ oz., $12.10\frac{1}{2}$ oz., $14.4\frac{5}{16}$ oz., $13.32\frac{3}{4}$ oz., $10.8\frac{7}{10}$ oz., $11.20\frac{9}{40}$ oz., $12.29\frac{13}{20}$ oz., and 10.21875 oz. If he sold it all at \$19.875 an ounce, how much did he realize?

SUBTRACTION OF DECIMALS

128. *Subtraction of decimals* is the process of finding the difference between two decimals or decimal numbers.

Subtract 5409.6825 from 9374.865.

OPERATION

9374.8650
5409.6825
3965.1825

EXPLANATION. The numbers are written so that units of the same order stand in the same column. The subtraction is performed as in subtraction of simple numbers, and the decimal point in the remainder is placed under the decimal points above.

RULE. 1. *Write the less number under the greater, so that units of the same order shall stand in the same column.*

2. *Subtract as in subtraction of simple numbers and place the decimal point in the remainder under the decimal points above.*

1. From 387.26 take 196.84.
2. From 512.98 take 245.945.
3. From 618.0372 take 459.106.
4. From 4612 *ten-thousandths* take 4612 *hundred-thousandths.*
5. From 101 and 101 *ten-thousandths* take 10 and 101 *thousandths.*
6. Take 57498 *millionths* from 49 and 125 *hundred-thousandths.*
7. Take 24 and 75 *ten-thousandths* from 247 and 5 *tenths.*
8. Take 98.7654321 from 123.456789.
9. Subtract 1001 *thousandths* from 1001 *hundredths.*
10. Subtract 125 *ten-thousandths* from 125 *tenths.*

11. Subtract the difference between 418.03125 and 314.21875 from their sum.

12. 123.45 − 12.345 − 1.2345 − .12345 = what?

13. 1000 − (432.015 + 324.4225) = what?

14. 3750.0375 − (729.36 + 816.419 + 954.246) = what?

15. .6832 − .09564 + .136 − .45982 + .37523 − .04751 + .40854 = ?

16. The greater of two numbers is 180.342, and their sum is 256.1. What is their difference?

17. A certain number is as much less than 64.52 as 127.513 is greater than 92.041. What is the number?

18. The sum of the ages of A, B, and C is 125 years. A's age is 43.3125 years, which is 2.125 years greater than B's. How old is C?

19. A man traveled 250.625 miles in 3 days. He went 93.6875 mi. the first day, and 11.75 mi. less on the second. How far did he travel the third day?

20. Owning 160 acres of land, I sold 43.875 acres to A, and a certain number to B, after which I had 62.5 acres remaining. How much more did B buy than A?

21. C raised 50.46875 tons of hay, which was 2.3125 tons more than D raised. If E raised 47.25 tons less than C and D combined, how many tons did all raise?

22. Four men earned $1000. The first earned $287.25, which was $25.6 more than the second earned. If the third earned $27.3 less than the second, how much did the fourth man earn?

23. A merchant having 2160.8125 bu. of coal sold 1089.9375 bu., and afterward bought 660.8125 bu. less than he sold. How many bushels had he then?

24. Four casks contain 250 gal. of wine. There are 63.875 gal. in the first, which is 1.1875 gal. more than the second contains. If the third contains 65.625 gal. less than the first and second combined, what are the contents of the fourth cask?

25. A owned 120.6875 acres of land, and B 50.75 acres less. A sold 36.4375 acres to B, who afterward sold 12.625 acres to A. How much more had A than B then?

MULTIPLICATION OF DECIMALS

129. *Multiplication of decimals* is the process of finding the product, when decimals are included in either or both of the factors.

PRINCIPLE. *The product of two decimals must contain as many decimal orders as there are decimal orders in both factors.*

(a) Multiply .15 by .5.

OPERATION

.15
.5
.075

EXPLANATION. The product of the numerators (15 and 5) is 75. The product of the denominators (100 and 10) is 1000. The result is $\frac{75}{1000} = .075$. That is, $\frac{15}{100} \times \frac{5}{10} = \frac{75}{1000}$.

REMARK. From this solution it is evident that *the denominator of the product of two decimals is* 1, *with as many ciphers annexed as are equal to the number of ciphers in both denominators.*

(b) Multiply .325 by .26.

OPERATION

.325
.26
1950
650
.08450

EXPLANATION. Multiplying as in simple numbers, the product is 8450. But as there are *three* decimal orders in the multiplicand, and *two* in the multiplier, there must be *five* in the product. Prefixing a cipher to 8450, the result is .08450; omitting the cipher from the right, the answer is .0845.

RULE. *Multiply as in multiplication of simple numbers, and point off from the right of the product as many decimal orders as there are decimal orders in both factors.*

REMARKS. 1. When the product does not contain as many figures as are equal to the number of decimal orders in both factors, prefix the necessary number of ciphers.

2. Omit ciphers from the right of the decimal in the product.

3. To multiply a decimal by 1 with one or more ciphers annexed, as 10, 100, 1000, etc., *remove the decimal point as many orders toward the right as there are ciphers in the multiplier.* Thus, $.0625 \times 1000 = 62.5$.

Multiply:

1. 37.26 by 49.
2. 256.4 by 5.8.
3. 4.37 by 30.5.
4. .596 by 6.25.
5. .0375 by 51.2.
6. 8.125 by .432.
7. 7.8125 by 1.728.
8. .003125 by 320.
9. .1024 by .9375.
10. 3906.25 by .0256.
11. 20 thousand $\times$ 20 *thousandths* = what?
12. 24 and 156 *thousandths* $\times$ 156 and 24 *hundredths* = ?
13. 1 and 5625 *ten-thousandths* $\times$ 2 and 48 *thousandths* = ?
14. 46875 *hundred-thousandths* $\times$ 512 *ten-thousandths* = ?
15. $1.953125 \times .004096$ = what?
16. $43\frac{3}{4}$ times $21\frac{3}{5}$ = what?
17. $28\frac{1}{8}$ times $.44\frac{4}{9}$ = what?
18. $1\frac{7}{25}$ times $7\frac{13}{16}$ = what?
19. $10.4\frac{1}{6}$ times $4\frac{76}{125}$ = what?
20. $11\frac{23}{32}$ times $8.53\frac{1}{3}$ = what?

21. What is the value of 12.8 tons of hay at $12.50 per ton?

22. A man traveled a certain distance in 18.75 days, at the rate of 32.56 miles per day. What was the distance?

23. A field is 51.2 rods long by 31.25 rods wide. How much is it worth at $68.75 an acre?

24. A dealer having 2343.75 bu. of coal, sold .25 of it at $.16 a bushel. How much did he receive for it?

25. $390\frac{5}{8} \times 1\frac{9}{16} \times .08192 =$ what?

26. A merchant having 140.625 bbl. of flour exchanged .75 of it for wheat, at the rate of 3.2 bu. for one barrel. How many bushels were required?

27. What would be the value of a block of marble 6.4 ft. long, 4.8 ft. wide, and 3.2 ft. thick, at $3.75 a cubic foot?

28. A salesman earned .4 of .3125 of $819.2 a month for 8.75 months, and saved .625 of it. How much did he save?

29. A stock dealer sold 375 sheep at $3.6 each, and invested .875 of the money in cattle at $15.75 a head. How many cattle did he buy?

30. Find the cost of excavating a cellar 22.5 ft. long, 18.75 ft. wide, and 6.4 ft. deep, at $.25 a cubic yard.

31. A speculator having $4687.5 invested .24 of it in town lots, .36 of the remainder in grain, and the balance in hay at $14.25 a ton. How many tons were required?

32. A dealer bought 337.5 gallons of wine at 1.66\frac{2}{3}$ a gallon, and sold .768 of it for 1.25 times the cost. How much did he receive for it?

33. A field 31.25 rd. long by 25.6 rd. wide produced 87.5 bu. of potatoes per acre. What was the value of the crop at $.48 per bushel?

34. A, B, and C bought 937.5 bu. of wheat, of which A took .316, B .324, and C the remainder. What was C's share?

35. A farmer bought 204.8 acres of land at $50 an acre. He sold .375 of it at $56.25 per acre, .4375 of it at $62.5 per acre, and the remainder at $68.75 per acre. What was his gain on the transaction?

DIVISION OF DECIMALS

130. *Division of decimals* is the process of finding the quotient when the divisor, the dividend, or both are decimals or decimal numbers.

As the dividend is the product of the divisor and quotient, and as *the product of two factors divided by either of them must equal the other*, placing the decimal point in the quotient depends upon the following:

PRINCIPLE. *There must be as many decimal orders in the quotient as the number of decimal orders in the dividend exceeds the number of those in the divisor.*

(*a*) Divide .375 by .5.

OPERATION

$$.5 \overline{)\ .375\ } \quad .75$$

EXPLANATION. Dividing as in simple numbers the quotient is 75. As the dividend is the product of the divisor and quotient, it contains as many decimal orders as both of those factors. There are *three* decimal orders in the *dividend*, and *one* in the *divisor*, so the *quotient* must contain *three minus one* or *two* decimal orders. Pointing off two decimal orders in the quotient, the result is .75.

(*b*) Divide .312 by .012.

OPERATION

.012 | .312
26

EXPLANATION. Dividing as before, the quotient is 26. As there are *three* decimal orders in the *dividend*, and *three* in the *divisor*, there are *no* decimal orders in the *quotient*.

Therefore, the result is 26, *an integer*.

(*c*) Divide 10 by .08.

OPERATION

.08 | 10.00
125

EXPLANATION. As .08 is one of the factors, and 10 is the product, *two decimal ciphers were omitted from the product*. Supplying these, and dividing as before, the result is 125, *an integer*.

(*d*) Divide 93.7 by 8.

OPERATION

8 | 93.7000
11.7125

EXPLANATION. Annexing three ciphers to the decimal (Art. 123), and dividing, the quotient is 117,125. As there are *four* decimal orders in the dividend, and *none* in the divisor, there must be *four* in the quotient. Therefore, the result is 11.7125.

RULE. *Divide as in division of simple numbers, and point off from the right of the quotient as many decimal orders as the number of decimal orders in the dividend exceeds those in the divisor.*

REMARKS. 1. When the divisor contains *more* decimal orders than the dividend, annex a sufficient number of ciphers to the dividend to make the number of decimal orders equal to the number of those in the divisor. The quotient will be an integer.

2. When the quotient does not contain as many figures as the number of decimal orders in the dividend exceeds the number of those in the divisor, *prefix the required number of ciphers*.

3. If there should be a remainder after dividing all the figures of the dividend, annex decimal ciphers to the dividend, and continue to divide until there is no remainder, or until the division is sufficiently exact.

4. To divide by 1 with one or more ciphers annexed, as 10, 100, 1000, etc., *remove the decimal point as many orders toward the left as there are ciphers in the divisor. If there are not so many figures on the left of the point, prefix the required number of ciphers.* Thus, $62.5 \div 1000 = .0625$.

Divide:

1. 753.6 by 24.
2. 416.48 by .76.
3. 2040.78 by 8.6.
4. 372.372 by 4.03.
5. 312.2232 by 68.47.
6. 1792.91 by 245.
7. 1.219744 by 1.296.
8. .05616 by .0325.
9. .2512125 by .5742.
10. 11.33 by 164.8.
11. 2.27448 by 729.
12. .488469597 by .853.
13. 105.325 by .08426.
14. 32 by .3125.
15. 75 by 160.
16. 144 by 28.125.
17. 1 by 12.8.
18. 4.86 by 1000.
19. $(.972 \div 3.375) \div .45 =$ what?
20. $1728 \div (50.625 \div 4.6875) =$ what?
21. If 13.8 acres produce 1293.75 bu. of potatoes, what is the average yield per acre?
22. If 11.25 cords of wood cost \$ 51.75, what was the cost per cord?
23. A carpenter earned \$ 72.80 in 26 days of 8 hours each. How much was that per hour?
24. If a regiment of 1280 men consume 30,000 lb. of provisions in 7.5 da., what is the average daily allowance per man?
25. An express train ran 960 miles in 25.6 hr., what was its rate per hour?

26. 48 sacks of coffee, each containing 156.25 lb., were sold for $1687.50. What was the price per pound?

27. A cyclist rode 60 mi. in 4.5875 hr. At that rate, in what time could he have ridden 160 mi.?

28. I bought 43.75 yards of carpet for $47.15. At the same rate, how many yards would have cost $60.75?

29. A farmer paid $1072.50 for land at $48.75 an acre. If the farm was 68.75 rd. long, what was its width?

30. I bought a lot of grain for $781.25, and sold a part of it at cost for $593.75. What decimal part of the entire quantity was sold?

31. A dealer sold .625 of his stock of wood for $562.50, at $4.80 a cord. How many cords had he at first?

32. I sold .84375 of a cask of wine for $64.80, which was at the rate of $1.60 a gallon. How many gallons did the cask contain?

33. Express decimally $\frac{18\frac{3}{4}}{15\frac{5}{8}}$, and $\frac{12\frac{4}{5}}{5\frac{3}{25}}$, and divide the former by the latter.

34. If 16.5 bu. of wheat are worth 37.125 bu. of corn, how many bushels of wheat should be exchanged for 172.8 bu. of corn?

35. I paid $135 for coal at $3.125 a ton, and sold .875 of it for $141.75; at what price per ton was it sold?

36. If 12 men can husk 3375 bu. of corn in 7.5 da., in what time could 8 men husk 1350 bu.?

37. A man owning .4375 of a vessel, sold .48 of his share for $8925. At that rate, what was the value of the vessel?

38. I lent A .45 of my money, and B .6 of the remainder. If A received $15 more than B, how much money had I left?

39. Multiply the sum of 20.825 and 12.425 by their difference, and divide the product by 22.8.

40. $27.125 - .3 \times 20.8\frac{1}{3} + 9\frac{1}{6} \times .45 =$ what?

REDUCTION OF DECIMAL CONCRETE NUMBERS

CASE I

131. To reduce a decimal to a lower denomination.

Decimal concrete numbers are reduced to lower denominations by multiplication. (Art. 51, Rule 1.)

Reduce .0125 bu. to the decimal of a pint.

OPERATION

.0125 bu.

4

.05ø̸ø̸ pk.

8

.4ø̸ qt.

2

.8 pt.

EXPLANATION. Multiplying .0125 bu. by 4 reduces it to .05 pk.

Multiplying .05 pk. by 8 reduces it to .4 qt.

Multiplying .4 qt. by 2 reduces it to .8 pt.

Reduce:

1. .0025 bu. to the decimal of a pint.
2. .0225 gal. to the decimal of a gill.
3. .00055 cwt. to the decimal of an ounce.
4. .000375 mi. to the decimal of a yard.
5. .002 acre to the decimal of a square yard.
6. .00125 cord to the decimal of a cubic foot.

7. .000075 week to the decimal of a minute.
8. .004125 lb. Troy to the decimal of a pwt.
9. .00015 lb. Apoth. to the decimal of a grain.
10. .00075 bbl. of wine to the decimal of a gill.
11. .0000625 mi. to the decimal of a foot.
12. .0001125 lb. Troy to the decimal of a grain.
13. .0000025 sq. mi. to the decimal of a square rod.
14. .00765625 bu. to the decimal of a pint.
15. .000015625 ton to the decimal of an ounce.

CASE II

132. To find the value of a decimal in integers.

Decimal concrete numbers are reduced to integers of lower denominations by multiplication. When the product is a mixed decimal, the integer is reserved as a part of the answer, and the operation is continued with the decimal part only. This process is termed *finding the value of a decimal in integers.*

Find the value in integers of .46875 gallon.

OPERATION

.46875 gal.
4
1.875ØØ qt.
2
1.75Ø pt.
4
3.ØØ gi.

EXPLANATION. Multiplying .46875 gal. by 4 reduces it to 1.875 qt. Multiplying .875 qt. by 2 reduces it to 1.75 pt. Multiplying .75 pt. by 4 reduces it to 3 gi. The result is 1 qt. 1 pt. 3 gi.

Find the value in integers of:

1. .390625 bushel.
2. .65625 gallon.
3. .234375 ton.
4. .5859375 mile.
5. .703125 acre.
6. .03515625 cord.
7. .1953125 day.
8. .546875 lb. Troy.
9. .859375 lb. Apoth.
10. .3125 bbl. of wine.
11. .953125 cu. yd.
12. .0903125 sq. mi.
13. .625 rod.
14. .1546875 lb. Apoth.
15. .3375 year.
16. .527777$\frac{7}{9}$ yard.

CASE III

133. To reduce a decimal to a higher denomination.

Decimal concrete numbers are reduced to higher denominations by division. (Art. 51, Rule 2.)

Reduce .96 pt. to the decimal of a bushel.

OPERATION

2 | .96 pt.
8 | .48 qt.
4 | .060 pk.
.015 bu.

EXPLANATION. Dividing .96 pt. by 2 reduces it to .48 qt. Dividing .48 qt. by 8 reduces it to .06 pk. Dividing .06 pk. by 4 reduces it to .015 bu.

Reduce:

1. .864 pt. to the decimal of a bushel.
2. .36 gi. to the decimal of a gallon.
3. .768 oz. to the decimal of a cwt.
4. .99 yd. to the decimal of a mile.
5. .726 sq. yd. to the decimal of an acre.
6. .88 cu. ft. to the decimal of a cord.
7. .735 hr. to the decimal of a week.

8. .81 pwt. to the decimal of a pound.

9. .9 gr. to the decimal of a pound Apoth.

10. .252 gi. to the decimal of a barrel.

11. 1 pk. 4 qt. 1 pt. to the decimal of a bushel.

12. 1 qt. 1 pt. 3 gi. to the decimal of a gallon.

13. 1 da. 7 hr. 30 min. to the decimal of a week.

14. 9 oz. 3 pwt. 18 gr. to the decimal of a pound.

15. 8 ℥ 3 ʒ 1 ℈ 10 gr. to the decimal of a pound.

REVIEW OF DECIMALS

134. 1. Find the value of a pile of wood 102.4 ft. long, 16 ft. wide, and 7.5 ft. high, at $5.55 a cord.

2. If a man can walk 234.375 miles in 7.5 days, in how many days could he walk 300 miles?

3. If a lot 44.8 rd. long and 37.5 rd. wide produces 45 bu. of oats per acre, what would the crop be worth at $.48 per bushel?

4. Reduce to mixed decimals and add: $2\frac{3}{4}$, $3\frac{7}{8}$, $4\frac{15}{16}$, $5\frac{17}{20}$, $6\frac{12}{25}$, $7\frac{29}{40}$, $8\frac{21}{50}$, $9\frac{77}{80}$.

5. Find the value in integers of .045 year.

6. If 68.75 cords of wood cost $302.50, how much would 56.25 cords cost?

7. The slate blackboards for a school building of 16 rooms cost $1125 at $.3125 a square foot. How many square yards were required for each room?

8. Multiply the sum of 15.93 and 4.32 by their difference, and divide the product by 1.935.

9. Find the cost of 9 bu. 3 pk. 1 pt. of berries at $3.20 a bushel.

10. $\frac{6\frac{2}{5}}{7\frac{1}{9}} - \frac{9\frac{3}{8}}{16\frac{2}{3}} + \frac{8\frac{7}{16}}{12\frac{1}{2}}$ reduced to a decimal = what?

11. Find the value of a lot 37.5 rods square at $40.96 an acre.

12. The rent of my house this year is $270, which is 1.125 times as great as it was last year. The rent last year was .064 of the value of the property. What is the property worth?

13. I sold 15,625 ft. of lumber at $16.80 per M; 10,240 laths, at $.25 per C; and 10,250 lb. of hay, at $14.40 per T. What was the amount of the bill?

14. .96 ℔ + .4 ℥ + .6 ʒ + .1 ℈ + .4 gr. = what?

15. A man owning .64 of a vessel sold .4375 of his share for $3500. At that rate, what was his remaining interest worth?

16. Two freight trains are 675 miles apart, and travel toward each other, running 15.1 and 13.025 miles per hour, respectively. In what time will they meet?

17. B has .625 as much money as A, and C has .625 as much as B. If C has $292.5 less than A, how much has B?

18. Owning 75 acres of land, I sold a part of it for $2475, at $68.75 an acre. What decimal part of the land was sold?

19. Multipy $\frac{15}{16}$ of .384 by $8\frac{8}{9}$ times .3125.

20. If 39.0625 yd. of cloth cost $62.50, how many yards should cost $50?

21. Find the cost of carpeting a room 15.75 ft. square with Brussels carpet .75 yd. wide, at $.80 a yard.

22. A, B, and C paid $1440 for sheep at $3.75 a head. B paid .96 as much as A, and C paid .625 as much as B. How many belonged to each?

23. A laborer charged $15 for sawing a pile of wood 31.25 ft. long, 8 ft. wide, and 6.4 ft. high. What was his price per cord?

24. A's property is worth $5280, which is 1.28 times the value of B's. If B's is worth 1.32 times as much as C's, what is the entire value of their property?

25. Two men start from the same place, at the same time, and travel in opposite directions, one at the rate of 44.53125 mi. per day, the other 39.0625 mi. per day. How far apart will they be at the end of 12.8 days?

26. Find the value of a granite base 40 in. square and 18 in. high, at $6.75 a cubic foot.

27. I paid $1.875 for berries at $1.60 a bushel. What quantity was bought?

28. A man owning .45 of a tract of land, sold .52 of his share to A, and the remainder to B for $1822.50 at $56.25 per acre. How many acres were in the entire tract?

29. 19 of a farmer's sheep were killed. If these were .152 of the number remaining, how much did the entire flock cost at 3.33\frac{1}{3}$ a head?

30. A dealer bought 156.25 bbl. of apples at $2.50 a barrel, and then sold .625 of them for 1.024 times their cost. How much did he realize?

31. Multiply .30$\frac{6}{7}$ by 2.91$\frac{2}{3}$.

32. I paid $1560 for wheat at $.8125 a bushel, and sold .78125 of it for $1312.50. At what price per bushel was it sold?

33. What would be the cost of a cement sidewalk 437.5 yd. long, by 2.5 yd. wide, at $14.4 per 100 sq. ft?

34. A sold B a farm, and gained .28 of the cost. B sold it to C, and gained .125 of what it cost him. C sold it to D for $2970, thereby losing .25 of what he paid for it. How much did the farm cost A?

35. A farmer exchanged 78 bu. 4 qt. of wheat worth $.88 a bushel, for pigs at $2.75 apiece. How many pigs did he receive?

36. A man traveling from C to D went .175 of the distance the first day, .25 of it the second, .2625 of it the third, and the remainder, which was 50 miles, on the fourth day. What is the distance from C to D?

37. A dealer paid $1392 for wheat at $.725 a bushel, and sold .5625 of it for $945. At what price per bushel was it sold?

38. The fencing of a field whose area is 7.8 acres cost $106.35. If the field is in the form of a rectangle 32.5 rods wide, how much did the fence cost per rod?

39. The fore wheel of a carriage is 9.625 feet in circumference, and the hind wheel 12.375 ft. How many more revolutions will the former make than the latter in traveling 5.25 miles?

40. A tank is 8.8 ft. long, 4.375 ft. wide, and 3.5 ft. deep inside. How many barrels of water will it hold, allowing 231 cu. in. to a gallon, and 31.5 gal. to a barrel?

41. A owns 68.75 acres of land, and B owns 81.25 acres. If each should sell .48 of his land to the other, how much more would B then have than A?

42. If 25 men can do a piece of work in 16.8 da. by working 7.5 hr. a day, how many men could accomplish as much in 9.375 da. by working 10.5 hr. a day?

43. Divide the product of 20.25 and 16.5 by their difference, and multiply the quotient by $.44\frac{4}{9}$.

44. If a certain number is multiplied by 24.8, if 25.92 is added to the product, if the sum is divided by 3.456, and 240 is subtracted from the quotient, the result will be zero. What is the number?

45. $94.50 was gained by selling a quantity of wine for $567 at $.375 a pint. If each barrel contained 31.5 gal., what was the cost per barrel?

46. I bought a tract of land at $37.50 an acre, and sold .46875 of it at cost for $2531.25. How many acres were in the tract?

47. A, B, and C built 160 rods of fence. B built .78125 as many rods as A, and C built .92 as many as B. How many rods did each build?

48. Multiply $\frac{1.95}{13.6}$ by $\frac{.425}{1.56}$, and divide the product by $\frac{1.875}{76.8}$.

49. The fence surrounding a rectangular lot cost $125.10 at $.75 a rod. If the lot was 38.4 rd. wide, what was its value at $125 an acre?

50. I bought a lot of grain for $457.25, which was .18 more than its value, and sold it for .14 less than its value. Find my loss.

51. A owning 204.8 acres of land sold .78125 of it to B, who sold .704 of his part to C, who sold .46875 of what he bought to D, who disposed of it at $87.50 an acre. How much did he receive for it?

52. 28.8 acres of barley yielded 37.5 bu. per acre, and .66⅔ of the crop was sold for $360. At what price per bushel was it sold?

53. In a sack of gold coins there are 7 quarter-eagles and 4 half-eagles for each gold dollar. If their total value is $962.5, how many coins are in the sack?

54. At the rate of .0012 mi. per second, in how many hours can a man walk 17.28 miles?

55. A earns $1.87½ a day, and B $1.66⅔. How much will B have earned when A has earned $40.50?

56. If a lot 57.6 rd. long and 37.5 rd. wide produces 40 bu. of corn per acre, what would be the value of .3125 of the crop at $.80 a bushel?

57. A speculator paid $19,531.25 for land at $50 an acre. He sold .66 of it for 1.28 times its cost, and the remainder for .88 of its cost. What was his net gain on the transaction?

58. A druggist bought a cask of wine containing 39 gal. 2 gi. at $1.60 a gallon, upon which he intended to make a profit of .3 of the cost. By mistake he sold .36 of it at $1.44 a gallon. At what price per gallon should he have sold the remainder to realize the desired profit?

59. On counting their money, it was found that James had 1.32 times as much as Robert, and that William had .5 as much as both. If William had $108.75, how much belonged to James?

60. What decimal of 2 bu. 4 qt. is 1 bu. 3 pk. 3 qt. 1 pt.?

61. A man owning .4875 of a farm sold .375 of his share and had 48.75 A. left. How many acres had he at first, and how many were in the farm?

62. A steamer started from New Orleans to Pittsburg, sailing at the rate of 150 mi. per day. Two days later another steamer followed the first, at the rate of 187.5 mi. per day. At what distance from New Orleans was the first overtaken?

63. A bin is 8.75 ft. long, 4.5 ft. wide, and 3.2 ft. deep inside. How much grain will fill it, allowing 2150.4 cu. in. to a bushel?

64. $\frac{10.24}{3.125} \div \left[\frac{3.375}{28.125} \times \frac{15.36}{.5625}\right] =$ what?

65. A man divided .74 of his farm between his two sons, giving to the younger .85 as much as to the elder. If the elder received 7.2 acres more than the younger, how many acres were in the farm?

66. Out of a shipment of merchandise valued at $4687.5, .28 was destroyed by fire, .232 of the remainder was thrown overboard, and the balance being damaged was sold for .625 of its value. How much was received for it?

67. What was the cost of 76.8 acres of pasture land, if $504 was lost by selling 43.2 acres for $1440?

68. A's property is worth $2160, which is 1.2 times the value of B's, and also .96 of the value of C's. How much more is C's worth than B's?

69. I paid $1440 for wheat at $.75 a bushel, and sold 78,125 of it at a profit of .12 of its cost. What was the selling price?

THE METRIC SYSTEM

DEFINITIONS

135. 1. The *metric system* of weights and measures is based upon the decimal scale. It derives its name from the *meter*, which is the primary unit of the system.

2. The *meter* is the unit of *length*. It is regarded as one ten-millionth of the distance from the equator to the pole, its estimated length being 39.37 inches, or 3.28 feet.

3. The *are* is the unit of *square measure*. It is a square whose side is 10 meters, and whose area is 100 square meters.

4. The *stere*, or *cubic meter*, is the unit of *volume*. It is a cube whose edge is one meter.

5. The *liter* is the unit of *capacity*. Its contents are equal to the volume of a cube whose edge is .1 meter.

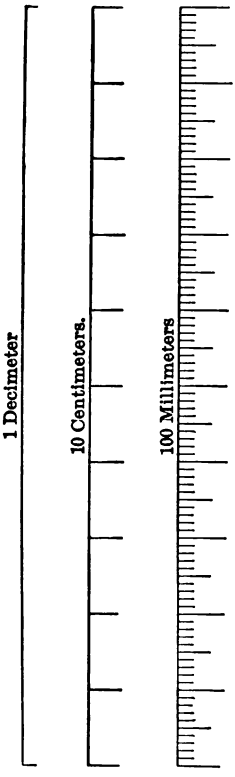
6. The *gram* is the unit of *weight*. It is the weight of pure water, at its greatest density, contained in a cube whose edge is .01 meter.

REMARKS. 1 From these primary units the higher and lower denominations are derived decimally:

1st. The *higher denominations* are expressed by prefixing to the name of the unit the *Greek* numerals, *deka* (10), *hekto* (100), *kilo* (1000), and *myria* (10,000).

Thus, a *dekaliter* is ten liters; a *hektare* is one hundred ares; a *kilogram* is one thousand grams, and a *myriameter*, ten thousand meters.

2d. The *lower denominations* are expressed by prefixing to the name of the unit the *Latin* numerals, *deci* (.1), *centi* (.01), and *milli* (.001). Thus, a *decimeter* is one tenth of a meter; a *centiliter* is one hundredth of a liter, and a *milligram*, one thousandth of a gram.



2. In the metric system, the decimal point is placed between the unit and its divisions, and the entire quantity may be considered as a mixed decimal. Thus, 2 dekameters 5 meters 4 decimeters 8 centimers may be written 25.48 meters.

7. As the metric system is based upon the decimal notation, its denominations increase and decrease by the *scale of ten.* Therefore, 1st. *In reduction descending, remove the decimal point as many places to the right as there are ciphers in the multiplier.*

2d. *In reduction ascending, remove the decimal point as many places to the left as there are ciphers in the divisor.*

Measures of Length

136. The *meter* is the unit of length. It is estimated at 39.37 inches.

TABLE

			EQUIVALENTS
10 millimeters (mm.)	= 1 centimeter . . .	cm.	.3937 in.
10 centimeters	= 1 decimeter	dm.	3.937 in.
10 decimeters	= 1 *Meter*	m.	39.37 in.
10 meters	= 1 dekameter . . .	Dm.	32.81 ft.
10 dekameters	= 1 hektometer . . .	Hm.	328.1 ft.
10 hektometers	= 1 kilometer	Km.	.62137 mi.
10 kilometers	= 1 myriameter . . .	Mm.	6.2137 mi.

REMARK. The measures most commonly used are the meter, kilometer, and millimeter. Cloth and short distances are measured by the meter, long distances by the kilometer, and very short distances by the millimeter.

(*a*) Reduce 6.324 m. to decimeters.

OPERATION. $6.324 \times 10 = 63.24$ dm.

EXPLANATION. Meters are reduced to decimeters by multiplying by 10. Write 6324, and set the decimal point one place farther toward the right than it is in 6.234. (Art. 135, 7.)

(*b*) Reduce 946.8 meters to hektometers.

OPERATION. $946.8 \div 100 = 9.468$ Hm.

EXPLANATION. Meters are reduced to hektometers by dividing by 100. Write 9468, and set the decimal point two places farther toward the left than it is in 946.8. (Art. 135, 7.)

Reduce:

1. 12.456 m. to centimeters.
2. 16.8 Hm. to meters.
3. .432 m. to millimeters.
4. .00576 Km. to decimeters.
5. .03125 Dm. to centimeters.
6. 39.724 cm. to decimeters.
7. 41832 m. to Hm.
8. 28.296 mm. to m.
9. 43.75 dm. to Km.
10. 5280 cm. to Dm.

Measure of Surface

137. The *are* is the unit of land measure. It is equal to 119.6 square yards, or .02471 acre.

TABLE

		Equivalents
1 centare (ca.)	= 1 square meter . . . m^2	1.196 sq. yd.
100 centares	= 1 are . . . . . a.	119.6 sq. yd.
100 ares	= 1 hektare . . . . . Ha.	2.471 acres.

Remarks. 1. The centare is a square whose side is one meter. The are is a square whose area is 100 square meters.

The hektare is a square whose area is 10,000 square meters, or nearly $2\frac{1}{2}$ acres.

2. All surfaces except land are measured by the square meter, decimeter, centimeter, or millimeter.

Reduce:

1. 18.324 a. to centares.
2. 10.1 Ha. to ares.
3. .256 a. to square meters.
4. .06875 Ha. to centares.
5. 17280 m^2 to hektares.
6. 8.64 ca. to ares.
7. 21.6 a. to hektares.
8. 43.75 ca. to hektares.

Measures of Capacity

138. The *liter* is the unit of capacity. It is equal to 1.0567 quarts Liquid Measure, or .908 quarts Dry Measure.

TABLE

		Equivalents	
		Liquid Measure	Dry Measure
10 centiliters (cl.)	= 1 deciliter . dl.	.845 gills.	6.1022 cu. in.
10 deciliters	= 1 *liter* . . . l.	1.0567 quarts.	.908 qt.
10 liters	= 1 dekaliter . Dl.	2.6417 gallons.	9.08 qt.
10 dekaliters	= 1 hektoliter Hl.	26.417 gallons.	2.8372 bu.
10 hektoliters	= 1 kiloliter . Kl.	264.17 gallons.	28.372 bu.

REMARKS. 1. The liter is commonly used for measuring liquids. The hektoliter is used for measuring grain, fruit, etc.
2. The *cubic meter* (m^3) is the equivalent of the kiloliter. The *stere* (used for measuring wood) is also a cubic meter.

Reduce:

1. 10.8 l. to centiliters.
2. 1.512 Hl. to deciliters.
3. .0625 Dl. to centiliters.
4. .024 Hl. to liters.
5. .07854 Dl. to centiliters.
6. 81.36 cl. to liters.
7. 1760 dl. to Hl.
8. 12.8 l. to Kl.
9. 7812.5 cl. to Dl.
10. 101720 dl. to m^3.

MEASURE OF WEIGHT

139. The *gram* is the unit of weight. It is equal to 15.432 grains.

TABLE

			EQUIVALENTS Avoirdupois Weight
10 milligrams (mg.)	=1 centigram	cg.	.15432 grains.
10 centigrams	=1 decigram	dg.	1.5432 grains.
10 decigrams	=1 *gram*	g.	15.432 grains.
10 grams	=1 dekagram	Dg.	.3527 ounces.
10 dekagrams	=1 hektogram	Hg.	3.5274 ounces.
10 hektograms	=1 kilogram	Kg.	2.2046 pounds.
10 kilograms	=1 myriagram	Mg.	22.046 pounds.
10 myriagrams or 100 kilograms	=1 quintal	Q.	220.46 pounds.
10 quintals or 1000 kilograms	=1 tonneau	T.	2204.6 pounds.

REMARK. The gram, kilogram, and metric ton, or *tonneau*, are most commonly used — the gram in compounding medicines and weighing the precious metals and all very light articles; the kilogram or "kilo" for weighing coarse or heavy articles, and the metric ton for weighing hay, coal, etc.

Reduce:

1. 38.4 g. to centigrams.
2. .00512 Mg. to grams.
3. .0875 Hg. to decigrams.
4. .001864 Q. to dekagrams.
5. .000456 Kg. to centigrams.
6. 25.6 dg. to Hg.
7. 125000 mg. to Dg.
8. 4.8 g. to Kg.
9. 8.64 Dg. to Mg.
10. 390625 Dg. to M. T.

140. (*a*) Reduce 25 hektares to acres and square rods.

OPERATION

2.471
25
12355
4942
61.775 acres
160
46500
775
124.000 sq. rd.

EXPLANATION. 1 hektare = 2.471 acres. Multiplying 2.471 acres by 25 reduces 25 hektares to 61.775 acres. Multiplying .775 acre by 160 reduces it to 124 square rods. The result is 61 acres 124 square rods.

(*b*) Reduce 8829 cubic feet to cubic meters.

OPERATION. 27)8829(327 cu. yd.
81
72
54
189
189

1.308)327.000(250 m^3.
2616
6540
6540
0

EXPLANATION. Dividing 8829 cubic feet by 27 reduces it to 327 cubic yards. Dividing 327 cubic yards by 1.308 reduces it to 250 cubic meters.

REMARK. In solving the following problems, consult the preceding *Tables of Equivalents*.

1. Find the value in bushels, etc., of 25 hektoliters.

2. How many square yards, etc., in 125 square meters?

3. Reduce 1500 meters to rods, etc.

4. What is the value of 1250 grams Troy?

5. What is the value of 625 grams Apoth.?

6. How many acres, etc., in 31.25 hektares?

7. Reduce 2500 kilograms to ton, cwt., etc.

8. How many square meters are there in a lot 39 yd. long by 11.5 yd. wide?

9. Reduce 127 bu. 2 pk. 6 qt. to hektoliters.

10. How many cubic meters are there in a breakwater 163.5 yd. long, 4.5 yd. deep, and 2 yd. wide.

11. Reduce 40 lb. 2 oz. 5 pwt. to kilograms.

12. How many kilometers are equal to 31 mi. 21.92 rd.?

REVIEW OF METRIC SYSTEM

141. 1. Find the value of 25 hektares of land at \$2.40 an are.

2. A pile of wood is 25 m. long, 2.4 m. wide, and 1.75 m. high. Find its value at \$1.33⅓ a stere.

3. A merchant sold 5 Hl. of wine at \$0.375 a liter. How much did he receive for it?

4. A grocer paid \$57.60 for tea at \$1.20 a kilo, and retailed it at the rate of \$0.15 a hektogram. Find his entire profit.

5. A block of marble .9 m. wide and .64 thick, cost \$126 at \$125 per cubic meter. What was its length?

6. A jeweler paid $150 for gold at $0.66⅔ per g., and manufactured it into rings weighing 4.5 g. each, which he sold at $4 apiece. Find his gain.

7. 25 tonneaux of copper were made into telephone wire weighing 3.2 ounces per meter. What was its value at $50 per Km.?

8. I bought 160 Hl. of wheat at $2.40 a hektoliter, and sold it at $0.90 a bushel. Find my gain.

9. A druggist compounded 1 kilogram of morphine into four-grain pellets, which he sold at the rate of $0.015 apiece. How much did he realize?

10. How many cubical blocks whose edges are 5 cm. in length can be packed in a box 6 dm. long, 4 dm. wide, and 2.5 dm. deep?

11. A crop of barley from a field 375 m. long by 240 m. wide was sold for $540 at $0.50 a bushel. What was the yield per hektare?

12. A dealer bought 15 tonneaux of hay at $16.50 a tonneau, and sold it at $20 a ton. Find his gain.

13. The gravel for a public road cost $1080 at $1.125 a cubic meter. If each load measured .75 $m.^3$, how many loads were used?

14. A merchant imported 450 meters of silk costing $1.33⅓ a meter, and sold it all at $1.60 a yard. What was his profit?

15. 50 casks of French wine, each containing 160 liters, were sold at $3.75 a gallon. How much was realized?

16. If steel rails weigh 27.5 kilograms per meter, how many tonneaux would be required for a railroad track 250 kilometers in length?

PERCENTAGE

DEFINITIONS

142. 1. *Percentage* includes computations in which *one hundred* is assumed as the *unit* or *base*.

2. Any *per cent* of a number is that many *hundredths* of it. Thus, 3 per cent of a number is .03 of it.

REMARK. "Per cent" is an abbreviation of the Latin *per centum* which signifies *by the hundred*.

3. The sign "%" is frequently used instead of the words *per cent*. Thus 6% is read, *six per cent*.

4. Four quantities belong to percentage: (1) the *base*; (2) the *rate*; (3) the *percentage*; and (4) the *sum* or the *difference* of the base and the percentage.

5. The *base* is the number upon which the percentage is calculated.

6. The *rate* (or *per cent*) is so many hundredths to be taken.

7. The *percentage* is the result of taking any per cent of the base.

8. The *sum* or the *difference* of the base and the percentage is the base *plus or minus the percentage*.

REMARK. As 100 is the base of percentage, the *rate* may be expressed either as a common fraction or as a decimal. Thus, *five*

per cent may be written $\frac{5}{100}$, $\frac{1}{20}$, or .05. When the rate is a mixed number, it may be reduced either to a simple fraction or to a decimal. Thus $7\frac{1}{2}\% = \frac{7\frac{1}{2}}{100} = \frac{15}{200} = \frac{3}{40}$, and $7\frac{1}{2}\%$ also = .075. The decimal form is usually employed in written work.

143. *Express the following both as common fractions and as decimals:*

1. 4%	**8.** 75%	**15.** $62\frac{1}{2}\%$	**22.** .6%
2. 5%	**9.** $2\frac{1}{2}\%$	**16.** $87\frac{1}{2}\%$	**23.** .8%
3. 6%	**10.** $6\frac{1}{4}\%$	**17.** $93\frac{3}{4}\%$	**24.** .75%
4. 10%	**11.** $12\frac{1}{2}\%$	**18.** $\frac{1}{4}\%$	**25.** 125%
5. 20%	**12.** $18\frac{3}{4}\%$	**19.** $\frac{2}{5}\%$	**26.** $6\frac{2}{3}\%$
6. 25%	**13.** $37\frac{1}{2}\%$	**20.** $\frac{5}{8}\%$	**27.** $8\frac{1}{3}\%$
7. 50%	**14.** $56\frac{1}{4}\%$	**21.** .3%	**28.** $16\frac{2}{3}\%$

FOUR CASES OF PERCENTAGE

CASE I

144. To find the percentage when the base and the rate are given.

(*a*) What is 18% of \$125?

OPERATION

\$125 base
.18 rate
1000
125
\$22.50 percentage

EXPLANATION. \$125 is the *base*, .18 the *rate*. (Art. 142, 5, 6.) Multiplying \$125 by .18 the result is \$22.50, which is the *percentage*.

(b) What is $6\frac{1}{4}\%$ of 460?

OPERATION

460
$.06\frac{1}{4}$
2760
115
28.75

EXPLANATION. $6\frac{1}{4}\%$ is $.06\frac{1}{4}$. Multiplying 460 by $.06\frac{1}{4}$ the result is 28.75.

(c) What is $\frac{5}{8}\%$ of 720?

OPERATION

720
.01
8)7.20
.90
5
4.50

EXPLANATION. 1% (.01) of 720 is 7.2, then $\frac{5}{8}\%$ of 720 is $7.2 \times \frac{5}{8} = 4.5$.

(d) What is $\frac{3}{4}\%$ of $83\frac{1}{3}$?

OPERATION. $\frac{\cancel{250}^{5}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{400}_{8}} = \frac{5}{8}$.

EXPLANATION. $\frac{3}{4}\% = \frac{3}{400}$. (Art. 142, Rem.) $83\frac{1}{3} = \frac{250}{3}$. Multiplying $\frac{250}{3}$ by $\frac{3}{400}$ the result is $\frac{5}{8}$.

RULE. *Multiply the base by the rate. The product will be the percentage.*

Find the following:

1. 8% of \$625.
2. 12% of 375 bu.
3. 15% of 160 gal.
4. 26% of 1250 lb.
5. 32% of 150 mi.
6. 45% of 240 A.
7. 54% of 50 cords.
8. 68% of 175 da.

9. 75% of 456 bbl.
10. 80% of 825 men.
11. 96% of $12.50.
12. 104% of 275 sheep.
13. 116% of 125 horses.
14. 250% of 64 tons.
15. 384% of 875 bu.
16. 444% of 225 rods.
17. 512% of $750.
18. 1000% of 5.5 years.
19. 1250% of 2.56 mi.
20. .2% of 3750.
21. .3% of 15.
22. .4% of 312.5.
23. .5% of 1024.
24. .6% of 1875.
25. .7% of 50.
26. .8% of 125.
27. .9% of .9.
28. .1% of 100.
29. .16% of 2500.
30. .05% of 2560.
31. .75% of 640.
32. .625% of 4096.
33. 2.5% of 1440.
34. 3.2% of 87.5.
35. 4.8% of 6.25.
36. 5.6% of 1050.
37. 6.4% of 1250.
38. 7.5% of 204.8.
39. 8.8% of 43.75.
40. 10.24% of 31250.
41. $4\frac{1}{2}$% of $16\frac{2}{3}$.
42. $6\frac{2}{3}$% of $9\frac{3}{8}$.
43. $5\frac{5}{9}$% of $28\frac{4}{5}$.
44. $6\frac{3}{4}$% of $77\frac{7}{9}$.
45. $12\frac{4}{5}$% of $31\frac{1}{4}$.
46. $20\frac{5}{6}$% of $53\frac{1}{3}$.
47. $37\frac{1}{2}$% of $44\frac{4}{9}$.
48. $56\frac{1}{4}$% of $83\frac{1}{3}$.
49. $64\frac{4}{5}$% of $62\frac{1}{2}$.
50. $84\frac{3}{8}$% of $118\frac{2}{5}$.
51. $\frac{3}{4}$% of $1000.
52. $\frac{4}{5}$% of 250 bu.
53. $\frac{5}{6}$% of 1920 gal.
54. $\frac{7}{8}$% of 2400 lb.
55. $\frac{5}{9}$% of 1440 min.
56. $\frac{4}{7}$% of 1050 cu. ft.

57. $\frac{9}{10}\%$ of 1250 mi.

58. $\frac{5}{16}\%$ of 320 A.

59. $\frac{11}{20}\%$ of 10000 cu. yd.

60. $\frac{16}{25}\%$ of 3125 men.

61. $6\frac{1}{4}\%$ of \$88.80.

62. $11\frac{1}{9}\%$ of 31.5 gal.

63. $5\frac{5}{8}\%$ of 28 tons.

64. $16\frac{2}{3}\%$ of 45 mi.

65. $18\frac{3}{4}\%$ of 150 A.

66. $31\frac{1}{4}\%$ of 24 cords.

67. $55\frac{5}{9}\%$ of 28 da.

68. $22\frac{1}{2}\%$ of 15 lb. Troy.

69. $\frac{5}{8}\%$ of $257\frac{1}{2}$ bu.

70. $41\frac{2}{3}\%$ of 1 year.

71. A farmer having raised 1050 bu. of corn sold 72% of it at 55 ct. a bu. What sum did he receive?

72. A merchant having \$1000 spent $6\frac{1}{4}\%$ of it for sugar at 5 cents a pound. How many pounds did he buy?

73. An express train in making a trip of 360 miles ran $41\frac{2}{3}\%$ of the distance in $4\frac{1}{2}$ hr. At what rate per hour did it travel?

74. I paid \$259.20 for 9 doz. boxes of canned fruit, and sold all at a profit of 25%. What was the selling price per box?

75. A drover bought 225 sheep at \$3.50 each. $6\frac{2}{3}\%$ having died, at what price per head should he have sold the remainder to sustain no loss?

76. A grocer having \$720 invested $15\frac{5}{8}\%$ of it in sugar, $18\frac{3}{4}\%$ of it in coffee, $22\frac{2}{9}\%$ of it in dry goods, and the remainder in pork at \$12.50 a barrel. How many barrels of pork did he buy?

77. An army of 15,625 men engaged in battle. 24% of them were wounded, and the number killed was 20% of the number wounded. How many were unhurt?

78. I sold 64 A. of land at $68.75 an acre, and invested $68\frac{3}{4}\%$ of the money in town lots at $275 each. How many lots did I buy?

79. A man owning property worth $5760 gave $41\frac{2}{3}\%$ of it to his son, $57\frac{1}{7}\%$ of the remainder to his daughter, and the rest to his wife. What was the value of the wife's share?

80. A trader having $2160 invested $37\frac{1}{2}\%$ of it in horses, $44\frac{4}{9}\%$ of the remainder in cattle, and the balance in mules at $50 each. How many mules did he buy?

81. A owns 87.5 acres, B 62.5 acres. If each should sell 44% of his land to the other, how much more would A then have than B?

82. A stock dealer paid $562.50 for hogs at $7.50 apiece; 4% having been stolen, at what price per head must he sell the remainder to gain 12% on the cost of all?

83. A tract of land is 50 rd. wide, and its length is 128% of its width. If it produces 125 bu. of potatoes per acre, what would the crop be worth at $0.40 a bushel?

84. A speculator having $12,800 invested $22\frac{1}{2}\%$ of it in cattle, $31\frac{1}{4}\%$ of the remainder in horses, $37\frac{1}{2}\%$ of what then remained in sheep, and the balance in land at $62.50 an acre. How many acres did he buy?

85. A grain dealer paid $1050 for corn at 56 ct. a bushel, and sold $66\frac{2}{3}\%$ of it at a profit of $12\frac{1}{2}\%$. How much did he receive for what he sold?

86. A trader began business with $2250. He gained 35% of his capital in one speculation, and $33\frac{1}{3}\%$ of the amount in another; but in a third speculation he lost $44\frac{4}{9}\%$ of all his money. What sum had he then?

87. B is 20 miles ahead of A. If A travels 12.8 mi. per hour, and B .75 of that distance per hour, in what time will A overtake him?

88. A owning property worth $45,000 sold 75% of it to B, who sold 66⅔% of what he bought to C, who sold 50% of what he owned to D. What then was the value of each one's share?

89. A floor is 24.75 ft. wide, and its length is 133⅓% of its width. How many yards of Brussels carpet (.75 yd. wide) would cover it?

90. A has 68.75 acres of land. B has 44% more than A, and C has 24% less than A. How many acres have all?

91. A dealer sold 36 horses at $87.50 each, thereby gaining $450. At what price per head should he have sold them to gain 32% of the cost?

92. 45% of 64% of 83⅓% of $46,875 was paid for 31¼% of 480 acres of farm land. What was the cost per acre?

93. A pile of wood is 51.2 ft. long, its width is 15⅝% of its length, and its height is 125% of its width. What is its value at $5 a cord?

94. A vessel whose capacity is 2016 gal. is 93¾% full. If 83⅓% of its contents be drawn off into barrels containing 31.5 gal. each, how many barrels will be required?

95. A dealer paid $375 for coal at $0.12 a bu., and sold 36% of it for 116⅔% of the cost. How much did he receive for the part sold?

96. Multiply $43\frac{3}{4}\%$ of 20.48 by $46\frac{2}{3}\%$ of 18.75, and divide the product by $35\frac{5}{7}\%$ of 17.92.

97. A stock of goods costing \$4500 caught fire, $26\frac{2}{3}\%$ of it was destroyed, $16\frac{2}{3}\%$ of the remainder was stolen, and the balance being damaged was sold for $67\frac{1}{2}\%$ of its value. How much was received for it?

98. A room is 22.5 ft. long, its width is 80% of its length, and its height is $55\frac{5}{9}\%$ of its width. Find the cost of plastering it at \$0.25 a square yard.

99. A speculator invested \$720 in rye at $56\frac{1}{4}$ cents a bushel, and sold $78\frac{1}{8}\%$ of it at a profit of $33\frac{1}{3}\%$. How much did he receive for the quantity sold?

100. Add $53\frac{1}{3}\%$ of $43\frac{3}{4}$ to $44\frac{4}{9}\%$ of $28\frac{1}{8}$, and divide the sum by $83\frac{1}{3}\%$ of $25\frac{4}{5}$.

CASE II

145. To find the rate when the base and the percentage are given.

(*a*) What per cent of 50 is 8?

1ST OPERATION

$50\overline{)8.00}$

$.16 = 16\%$

EXPLANATION. 8 is $\frac{8}{50}$ of 50 or $8 \div 50$. Dividing 8 by 50, the result is .16, $.16 = 16\%$.

2D OPERATION

50

$.01$

$.50\overline{)8.00}$

16%

EXPLANATION. 1% of 50 is .50, then the *rate* will be as many *per cent* as .50 are contained times in 8, = 16%.

REMARK. In the first operation the rate is an *abstract number*, representing so many *hundredths of the base*. In the second operation the rate is a *concrete number*, its denomination being *per cent*.

(*b*) $\frac{3}{5}$ is what per cent of $\frac{5}{8}$?

OPERATION. $\frac{3}{\cancel{5}} \times \frac{8}{\cancel{5}} \times \frac{\overset{4}{\cancel{100}}}{1} = 96\,\%$.

EXPLANATION. $\frac{3}{5} \div \frac{5}{8} = \frac{3}{5} \times \frac{8}{5} = \frac{24}{25}$. $\frac{24}{25}$ of $100\,\% = 96\,\%$.

RULE. 1. *Divide the percentage by the base; the quotient will be the rate.*

2. *Divide the percentage by* 1 *% of the base; the quotient will be the rate per cent.*

REMARKS. 1. When the given quantities are *fractional, multiply the percentage by the base with its terms inverted, and that product by* 100. *The result will be the* **rate per cent.**

2. Questions in Case II may be asked in three ways: As, (1) What per cent of 12 is 3? (2) What per cent is 3 of 12? (3) 3 is what per cent of 12? The answer, of course, is the same in all, and it is convenient to notice that the number following the preposition "of" is the base in every instance; hence *it is always the divisor.*

1. What % of 60 is 9?
2. What % of 75 is 21?
3. What % of 125 is 10?
4. What % of 32 is 4?
5. What % of 87.5 is 14?
6. What % of 25.6 is 16?
7. What % of 444 is 99.9?
8. What % of 480 is 187.2?
9. What % of 31.25 is 20?
10. What % of 76.8 is 28.8?
11. $195 is what % of $351?
12. 75 bu. is what % of 240 bu.?
13. 210 lb. is what % of 1260 lb.?
14. 60 mi. is what % of 320 mi.?

What per cent is:

15. 99 A. of 225 A.?
16. 20.25 cords of 56.25 cords?
17. 6 days of 96 days?
18. 50 bbl. of 120 bbl.
19. 112 men of 640 men?
20. 1750 gal. of 1875 gal.?
21. $\frac{4}{15}$ of $\frac{5}{12}$?
22. $\frac{6}{25}$ of $7\frac{1}{5}$?
23. 33 of $.68\frac{3}{4}$?
24. $1\frac{1}{4}$ of $16\frac{2}{3}$?
25. $4\frac{1}{2}$ of $20\frac{1}{4}$?
26. $3\frac{1}{5}$ of $\frac{24}{25}$?

27. A clerk's salary is $750 a year, and he saves $240. What per cent of his salary does he save?

28. What per cent of 48 quails are 6 quails?

29. The enrollment of a school is 285 pupils. If 19 were absent, what was the per cent of absence?

30. $\frac{5}{8}$ of a debt of $540 is what per cent of it?

31. A farmer divided 168 A. of land between his two sons, giving 96 A. to the elder, and the remainder to the younger. What % of the elder's share was the younger's?

32. If a druggist should mix 6 qt. of water with 11 gal. of wine, what per cent of the mixture would be water?

33. If a rectangular tract of land 48 rd. long has an area of 12 A. 96 sq. rd., what % of the length is the width?

34. A farmer paid $300 for sheep at $3.75 a head. $6\frac{1}{4}$ % of them having died, he sold the remainder at $4.75 a head. What % did he gain on the whole transaction?

35. If a trader loses $35 on a horse which he sells for $175, what % does he lose?

36. 2 bu. 1 pk. 1 qt. 1 pt. is what % of 8 bu. 6 qt. 1 pt.?

37. A speculator owning 144 acres of land sold a part of it for $6187.50, at $62.50 an acre. What % of the land was sold?

38. A telephone company with a capital stock of $247,500 declared a dividend of $20,625. What per cent was the dividend?

39. Out of an army of 25,600 men, $6\frac{1}{4}\%$ were killed in battle, and the number wounded was 220% of the number killed. What per cent of the army was uninjured?

40. A trader bought a farm of 75 acres for $4950, and sold 60% of it for $3960. What % was gained on what he sold?

41. A rectangular block of marble is 6.25 ft. long, and 3 ft. thick. If its solid contents are 75 cu. ft., what % of the length is the width?

42. A dealer paid $500 for wine at $62\frac{1}{2}$ a barrel, and after drawing off $37\frac{1}{2}\%$ of it into bottles, he sold the remainder for what all cost. Find his gain per cent on the part sold.

43. A poulterer bought 32 doz. chickens at $2.75 a dozen. $8\frac{1}{3}\%$ having died, he sold the remainder at 25 cents apiece. What was the per cent of gain or loss on the transaction?

44. 32 acres of land yielded 48 bu. of corn per acre, and a part of the crop was sold for $264 at 55 ct. a bushel. What % of the corn was sold?

45. A shipment of flour cost $2160 at $4.80 a bushel. In a storm $16\frac{2}{3}\%$ of it was thrown overboard, and the remainder being damaged, was sold for 75% of its value. What was the % of loss on the entire cost?

46. A dealer sold hay at $15 a ton, thereby gaining $2.20 on each ton. Had he paid $6\frac{1}{4}\%$ less per ton, and sold it for 8% more, what % would he have gained?

CASE III

146. To find the base when the percentage and the rate are given.

(*a*) 12 is 16 per cent of what number?

OPERATION

$$.16\overline{)12.00}\quad 75$$

EXPLANATION. 16 % = .16. The required number has been multiplied by .16, and the product is 12. Hence, the number is 12 divided by .16 = 75.

(*b*) $3\frac{3}{4}$ is $8\frac{1}{3}$ % of what number?

OPERATION. $\frac{15}{\cancel{4}} \times \frac{3}{\cancel{25}} \times \frac{\overset{4}{\cancel{100}}}{1} = 45.$

EXPLANATION. $3\frac{3}{4}$ is $8\frac{1}{3}$ % of the required number. Then 1 % of it is

$$\frac{15}{4} \div \frac{25}{3} = \frac{\overset{3}{\cancel{15}}}{4} \times \frac{3}{\underset{5}{\cancel{25}}} = \frac{9}{20}, \text{ and } 100\,\% \text{ is } \frac{9}{\cancel{20}} \times \frac{\overset{5}{\cancel{100}}}{1} = 45.$$

RULE. *Divide the percentage by the rate. The quotient will be the base.*

REMARK. When the given quantities are fractional, ***multiply the percentage by the rate with its terms inverted, and that product by*** 100. ***The result will be the base.***

1. 14 is 4 % of what number?
2. 18 is 8 % of what number?
3. 35 is 14 % of what number?
4. 81 is 45 % of what number?
5. 176 is 55 % of what number?
6. 51 is 68 % of what number?

7. 22.5 is 3.6% of what number?
8. 24.3 is 7.5% of what number?
9. 73.5 is 8.4% of what number?
10. 160 is 12.8% of what number?
11. 152.1 is 32.5% of what number?
12. 3.6 is 9.6% of what number?
13. 704 is 68.75% of what number?
14. 24 is 25.6% of what number?
15. 83.25 is 44.4% of what number?
16. 8.8 is .32% of what number?
17. 189 is 108% of what number?
18. 691.2 is 135% of what number?
19. $8\frac{4}{5}$ is $6\frac{2}{3}$% of what number?
20. $6\frac{3}{4}$ is $4\frac{1}{2}$% of what number?
21. $41\frac{2}{3}$ is $44\frac{4}{9}$% of what number?
22. 1 is $\frac{5}{8}$% of what number?
23. $\frac{3}{4}$ is $\frac{2}{9}$% of what number?
24. $\frac{8}{15}$ is $\frac{5}{6}$% of what number?
25. \$8.10 is $2\frac{1}{2}$% of what?
26. 26 bu. are $13\frac{1}{3}$% of what?
27. 2 mi. are $\frac{4}{9}$% of what?
28. 1332 men are 555% of what?
29. 315 A. are 1125% of what?
30. 11 gal. are 8% of what?

31. Mr. Brown gave $15 for a saddle, which was 12% of what he paid for a horse. How much did the horse cost?

32. I sold a carriage at a loss of 16%, thereby losing $60. How much did I pay for it?

33. A farmer planted 24 acres in corn, which was 32% of his land. How many acres did he own?

34. A dealer sold 481 bu. of coal, which was 26% of his stock. How many bushels had he?

35. A clerk pays $22.50 a month for board, which is 25% of his salary. What is his yearly salary?

36. A farmer sold 648 bu. of corn, which was 45% of his crop. How much was the remainder worth at 45 ct. a bushel?

37. 81 men out of a regiment were killed in a skirmish. If that was $7\frac{1}{2}$% of the entire number, how many remained?

38. A merchant bought lumber for $708.75, which was $31\frac{1}{2}$% of his capital. How much money had he left?

39. A owes B $68.75, which is $27\frac{1}{2}$% of what he owes C. How much does he owe to both?

40. A teamster charged $3.50 for hauling 125 bu. of wheat, which was $3\frac{1}{3}$% of its cost. What was the cost per bushel?

41. A drover bought 8 sheep, which was $6\frac{1}{4}$% of the number he already had. If he sold all at $3\frac{3}{4}$ apiece, how much did he realize?

42. A owns 49.5 A. of land, which is 88% of what B owns, or 99% of what C owns. How much more land has B than C?

43. A fruit grower sold 714 bu. of apples, which was 68% of his crop. What was the remainder worth at $0.85 a bushel?

44. A man paid $17.50 for a chain, which was 28% of the cost of his watch. How much did he pay for both?

45. 36 pupils failed at an examination. If this was 9.6% of the entire number, how many passed?

46. A speculator sold 4.5 bbl. of pork, which was 7.2% of his stock. If he paid $16 a barrel for the lot, how much did it cost him?

47. A owns 41¼% of a tract of land, B 32½% of it, and C owns the remainder. If A has 14 acres more than B, how much has C?

48. A man owning 72% of a vessel sold 45% of his share for $4050. At that rate what was the value of the vessel?

49. I sold a horse for $21 more than I gave for him, thereby gaining 23⅓%. What was the selling price?

50. I sold a watch for 22½% less than its cost, thereby losing $13.50. How much did I receive for it?

51. A manufacturer owning 75% of a factory sold 56% of his share for $7350. At that rate, what was his remaining interest worth?

52. A has 192 hogs, which is 128% of B's number. If B should buy 12½% of A's, how many would each then have?

53. A speculator sold 175 barrels of flour for $761.25, which was 116% of the cost. What was the cost per barrel?

54. C, owning $62\frac{1}{2}\%$ of a farm, sold 48% of his share to D, and had 39 acres remaining. How many acres were in the farm, and how many did C own at first?

55. I paid \$750 for a lot, which was $31\frac{1}{4}\%$ of the cost of the house built upon it. I sold the property for $116\frac{2}{3}\%$ of its value. What was the selling price?

56. $22\frac{1}{2}\%$ of $66\frac{2}{3}\%$ of 750 is 32% of $37\frac{1}{2}\%$ of what number?

57. The fence around a rectangular lot cost \$93.75 at \$.75 a rod, and the value of a crop of potatoes produced upon it was, at $62\frac{1}{2}$ ct. a bushel, 240% of the cost of the fence. If the width of the lot was $56\frac{1}{4}\%$ of its length, how many bushels per acre did it produce?

CASE IV

147. To find the base when the rate and the sum or the difference of the base and percentage are given.

(*a*) What number increased by 12% of itself will equal 196?

OPERATION

$$\begin{array}{r} 1.00 \\ .12 \\ \hline 1.12\overline{)196.00} \\ 175 \end{array}$$

EXPLANATION. $12\% = .12$. The number increased by .12 of itself = 1.12 of it. If 196 is 1.12 of a number, that number is $196 \div 1.12 = 175$.

(*b*) What number diminished by 8% of itself will equal 207?

OPERATION

$$\begin{array}{r} 1.00 \\ .08 \\ \hline .92\overline{)207.00} \\ 225 \end{array}$$

EXPLANATION. $8\% = .08$. The number minus .08 of itself = .92 of it. If 207 is .92 of a number, that number is $207 \div .92 = 225$.

RULE. *Divide the sum by* (1 + *the rate*), *or divide the difference by* (1 − *the rate*). *The quotient will be the base.*

REMARK. After adding the rate to 1, or subtracting it from 1, this case becomes identical with Case III.

1. What number increased by 4% of itself = 78?
2. What number increased by 5% of itself = 294?
3. What number diminished by 8% of itself = 299?
4. What number diminished by 15% of itself = 391?
5. 295 is 18% greater than what number?
6. 800 is 28% greater than what number?
7. 496 is 36% less than what number?
8. 559 is 35% less than what number?
9. 2214 is 116% more than what number?
10. 54 is 96% less than what number?
11. What number + 4.5% of itself will = 1713.8?
12. What number − 7.5% of itself will = 1998?
13. What number + $13\frac{1}{3}$% of itself will = 510?
14. What number − $22\frac{1}{2}$% of itself will = 1587.2?
15. Find a number which, added to 268% of itself, will equal 6789.6.
16. Find a number which, added to 2.5% of itself, will equal 4555.1.
17. Find a number which, added to $\frac{2}{5}$% of itself, will equal 1255.
18. Find a number which, diminished by .8% of itself, will equal 1860.

19. Find a number which, diminished by $99\frac{3}{5}\%$ of itself, will equal 12.5.

20. Find a number which, diminished by $62\frac{1}{2}\%$ of itself, will equal $62\frac{1}{2}$.

21. I sold a team for $350, which was 25% more than its value. What was its value?

22. A's age is 36 years, which is 28% less than B's. How old is B?

23. Smith's farm contains 87 A., which is 16% more than Brown's contains. How many acres does Brown own?

24. I paid $396 for my library, which was 12% less than its value. What is its value?

25. A flouring mill produced 1440 barrels in June, which was $12\frac{1}{2}\%$ more than it produced in May. What was its product in May?

26. The population of a town in 1898 was 1073, which was 7.5% less than it was in 1899. What was the population in 1899?

27. I sold a farm for $3870, which was 3.2% more than I paid for it. How much did it cost me?

28. I sold a stock of goods at a sacrifice of $13\frac{1}{2}\%$, receiving only $1868.40 for it. How much did I pay for it?

29. A speculator gained 36% of his capital, and then had $4590. How much did he gain?

30. A trader disposed of his stock for $3117.50, thereby losing 14%. How much did he lose?

31. A wholesale dealer sold 325 gal. of wine for $663, which was at a loss of 15%. What was the cost per gal.?

32. A merchant sold 160 bbl. of flour for $723.80, which was at a gain of $17\frac{1}{2}\%$. What was the cost per barrel?

33. The rent of my house last year was $330, which is 10% more than I paid this year. The rent this year is 9% less than the value of the property. How much is the property worth?

34. A grocer's profits in 1898 were $1617, which was 16% less than in 1899. In 1899 they were 72% less than his capital in 1898. What was his capital in 1898?

35. A has $3036, which is 15% more than B's money, or 12% less than C's. How much have B and C each?

36. A trader spent 36% of his money for horses, and 25% of the remainder for cattle. He then had $1560 remaining. How much had he at first?

37. A stock company increased its capital 32% at one time, and 28% of that amount at another. It was then $147,840. What was its original capital?

38. A real estate dealer invested 52% of his money in a farm, 45% of the remainder in city property, and the balance in 25 town lots at $165 each. How much money had he at first?

39. A drover bought 75 hogs at $4.80 each, and the money he paid for them was 76% less than what he received for a herd of cattle at $18.75 apiece. How many head of cattle did he sell?

40. An improvement company having bought a farm of 160 acres, sold $31\frac{1}{4}\%$ of it for $4000, at a gain of 28%. How much did the farm cost per acre?

41. A speculator invested 32% of his money in sugar, $37\frac{1}{2}\%$ of the remainder in tobacco, and with the balance bought 1250 bu. of wheat at \$0.85 a bushel. How much did he pay for the sugar?

42. A lady spent 20% of her money in one store, 35% of the remainder in another, and $37\frac{1}{2}\%$ of what then remained in another. She then had \$48.75. How much had she at first?

43. A trader gained 28% of his capital in one speculation, and 15% of the sum in another, but lost $37\frac{1}{2}\%$ of all his money in a third. He then had \$2070. What was his loss on the whole transaction?

44. A dealer sold 640 bu. of coal, which was 48.8% less than what he had remaining. How much did the entire stock cost at $8\frac{1}{3}$ ct. a bushel?

45. $33\frac{1}{3}\%$ of $56\frac{1}{4}\%$ of 2304 is 8% more than 40% of 80% of what?

General Formulas for the Four Cases of Percentage

148. Case I. (1) $Base \times Rate = Percentage.$

(2) $Base \times (1 + Rate) = Sum.$

(3) $Base \times (1 - Rate) = Difference.$

Case II. $Percentage \div Base = Rate.$

Case III. $Percentage \div Rate = Base.$

Case IV. (1) $Sum \div (1 + Rate) = Base.$

(2) $Sum \div (1 - Rate) = Base.$

Remark. These formulas are adapted to all the applications of percentage, and may be used instead of the rules if preferred to them.

REVIEW OF PERCENTAGE

149. 1. I sold 75 A. of land at $62.50 an A. and invested $66\frac{2}{3}\%$ of the money in village lots at $125 apiece. How many lots were bought?

2. A speculator having 120 acres of land, sold a part of it for $2025, at $45 an acre. What per cent of the land was sold?

3. A fruit grower sold 324 bu. of apples, which was 72% of his entire crop. How much was the remainder worth at $.75 a bushel?

4. A merchant sold 150 bbl. of flour for $675, which was 25% more than it cost. What was the cost per barrel?

5. A wholesale dealer sold 320 gal. of wine for $420, which was at a loss of $12\frac{1}{2}\%$. What was the cost per gallon?

6. I bought 80 acres of land for $6000, and sold 45% of it for $3240. What % was gained on the part sold?

7. I owe A $196.35, which is 32% more than I owe B, and 15% less than I owe C. How much do I owe all?

8. A owns 56.25 A. of land, which is 60% of what B owns. If each should sell $33\frac{1}{3}\%$ of his land to the other, how much more than A would B then have?

9. A trader having $2250, invested $33\frac{1}{3}\%$ of it in horses, 20% of the remainder in cattle, and the balance in mules at $37.50 a head. How many mules did he buy?

10. The rent of my house last year was $180, which is 20% more than I paid this year. The rent this year is 92% less than the value of the property. What is the property worth?

11. A poulterer had 18 dozen ducks, but $8\frac{1}{3}\%$ of them were killed. If he sold the remainder at 30 cents apiece, how much did he realize?

12. A merchant having 360 bbl. of flour sold a part of it for \$1068.75, at \$4.75 a barrel. What % of the flour was sold?

13. A grain dealer sold 1394 bu. of wheat, which was 85% of the entire quantity. How much was the remainder worth at $\$1.12\frac{1}{2}$ a bushel?

14. $22\frac{1}{2}$ acres of land yielded 64 bu. of corn per acre, and a part of the crop was sold for \$675, at $62\frac{1}{2}$ ct. a bushel. What % of it was sold?

15. I paid \$630 for grain at $\$.87\frac{1}{2}$ a bushel and sold 75% of it for \$594. At what price per bushel was it sold?

16. A dealer paid \$324 for 144 bbl. of apples, and sold 85 bbl. for \$275.40. What per cent did he gain on the part sold?

17. I sold a carriage at a profit of $18\frac{3}{4}\%$, thereby gaining \$45. What was the selling price?

18. A has \$238.05, which is 15% more than B has, or 8% less than C has. How much have all?

19. A dealer paid \$1750 for corn at 56 ct. a bushel, and sold 48% of it at a profit of $12\frac{1}{2}\%$. How much did he receive for what he sold?

20. A trader paid \$1200 for cattle at \$37.50 a head. 2 having died, he sold the remainder at \$45 each. What % did he gain on the transaction?

21. A farmer sold 1190 bu. of wheat, which was 68% of his crop. What was the remainder worth at $93\frac{3}{4}$ ct. a bushel?

22. A speculator sold 165 bbl. of flour for $910.80, which was 15% more than he paid for it. What was the cost per barrel?

23. 48 acres of land yielded 66 bu. of corn per acre, and $56\frac{1}{4}\%$ of the crop was sold for $801.90. At what price per bushel was it sold?

24. I bought 240 cords of wood for $1080, and sold $68\frac{3}{4}\%$ of it for $866.25. What % was gained on the part sold?

25. A man owning 64% of a farm, sold $87\frac{1}{2}\%$ of his share, and had 12 A. remaining. How many acres were in the entire farm, and how many did he sell?

26. A trader bought 128 hogs at $7\frac{1}{2}$ each, and the money he paid for them was 68% less than what he received for a herd of cows at $31.25 apiece. How many cows did he sell?

27. I sold 320 sheep at $3.62\frac{1}{2}$ each, and invested $56\frac{1}{4}\%$ of the money in hogs at $4.50 apiece. How many hogs did I buy?

28. I bought a farm of 48 acres for $3600, and sold 25 acres for $2125. What per cent was gained on the part sold?

29. A speculator invested 36% of his money in real estate, $37\frac{1}{2}\%$ of the remainder in bonds, and with the balance bought 160 bales of cotton, each weighing 250 lb. at $12\frac{1}{2}$ ct. a pound. How much money had he at first, and how much did he pay for the bonds?

30. I sold 75 bbl. of flour for $330, thereby losing 12%. At what price per barrel should it have been sold to gain 15%?

31. A grain dealer paid $1890 for wheat at $87\frac{1}{2}$ ct. a bushel and sold $66\frac{2}{3}\%$ of it at a loss of $16\frac{2}{3}\%$. How much did he receive for the part sold?

32. A merchant paid $300 for potatoes at $62\frac{1}{2}$ ct. a bushel and sold $83\frac{1}{3}\%$ of them for $320. What % did he gain on the quantity sold?

33. 16 of my sheep died, which was $6\frac{2}{3}\%$ of what I had remaining. If I paid $2.50 apiece for them, how much did the entire flock cost?

34. I lent A $171.50, which is $22\frac{1}{2}\%$ more than I lent B, and $37\frac{1}{2}\%$ less than I lent C. How much did I lend?

35. If a lot 56.25 rd. long and 38.4 rd. wide produces 50 bu. of barley per acre, what would be the value of $55\frac{5}{9}\%$ of the crop at 48 ct. a bushel?

36. I sold sugar at $14.50 a barrel, and gained 16%. Had I bought it for 4% less, and sold it at $15 a barrel, what per cent should I have gained?

37. A man owning 64% of a farm, sold $62\frac{1}{2}\%$ of his share, and had 38.4 acres remaining. How many acres had he at first, and how many were in the farm?

38. A lady spent 36% of her money in one store, $37\frac{1}{2}\%$ of the remainder in another, and 30% of what then remained in another; she then had $52.50. How much money did she spend?

39. A trader bought a number of sheep; 4 were killed, and he sold 75% of the remainder at a gain of 28%, and received $144, which was 10% less than all cost. How many sheep did he buy?

40. $18\frac{3}{4}\%$ of 1040 is $18\frac{3}{4}\%$ less than $18\frac{3}{4}\%$ of what number?

PROFIT AND LOSS

DEFINITIONS AND PROBLEMS

150. The principal applications of percentage are *Profit and Loss, Trade Discount, Commission, Stock Transactions and Brokerage, Insurance, Taxes, Customs, Interest, Discount,* and *Exchange.*

REMARK. In applying the principles of percentage to the solution of the problems presented in its applications, it is necessary to observe the quantities *given* and *required.* Especial attention should be devoted to a thorough consideration of the *base* upon which the percentage is estimated, before applying the rule or formula for the *case* to which the given problem belongs.

151. 1. *Profit* and *loss* are terms used to denote the gain or loss in business. The profit or the loss is the difference between the cost price and the selling price of merchandise.

2. In Profit and Loss four quantities are considered: 1st, the *cost;* 2d, the *per cent of profit* or *loss;* 3d, the *amount of profit* or *loss;* 4th, the *selling price.*

3. The cost is the *base;* the per cent of profit or loss is the *rate;* the profit or loss is the *percentage;* and the selling price is the *sum or the difference of the base and percentage.*

REMARK. The *per cent* of profit or loss is always estimated upon the *cost.*

1. I paid $250 for a team of horses, and sold them at a profit of 18%. What was the gain?

2. A carriage that cost $450 was sold at a loss of $13\frac{1}{3}\%$. What was the loss?

3. The cost of a farm was $2560, but the owner sold it at a loss of $12\frac{1}{2}\%$. How much did he receive for it?

4. A lot of cattle that cost $1760 was sold at a gain of $18\frac{3}{4}\%$. What was the selling price?

5. A speculator having a capital of $3125, lost 12% of it at one time, and 8% of the remainder at another. How much money had he left?

6. A trader having $1920, gained $16\frac{2}{3}\%$ of it in one speculation, and $12\frac{1}{2}\%$ of the amount in another. How much money had he then?

7. Goods costing $1750 were sold for $1671.25. What per cent was the loss?

8. Goods costing $1440 were sold for $1560. What per cent was the gain?

9. I sold a house for $3375, losing $225. What per cent was lost?

10. I sold a farm for $3600, gaining $225. What per cent was gained?

11. I sold a lot of lumber at a profit of $22\frac{1}{2}\%$, thereby gaining $303.75. How much did I receive for it?

12. A stock of goods was sold at a loss of $12\frac{1}{2}\%$. If the loss was $270, how much did it bring?

13. I sold my property at a gain of $840, or $18\frac{3}{4}\%$. What was the selling price?

14. Smith sold his farm for $3150, thereby losing 16%. For how much should he have sold it to gain $16\frac{2}{3}\%$?

15. A quantity of merchandise was sold for $1189.50, which was at a gain of 22%. Had it been sold at a loss of $8\frac{1}{3}\%$, how much would it have yielded?

16. A merchant increased his capital 28% the first year, and gained 25% of the sum the following year. He then had $7600. What was his original capital?

17. A trader lost 16% of his money in one speculation, and 15% of the remainder in another. He then had $1606.50. How much had he at first?

18. A real estate dealer gained $17\frac{1}{2}\%$ of his capital in 1897, and $12\frac{1}{2}\%$ of the amount in 1898. He then had $16,920. How much was his gain in the two years.

19. A speculator lost 18% of his money in one trade, and 12% of the remainder in another. He then had $4510. Find his entire loss.

20. A merchant paid $250 for coal at $0.08 a bushel, and sold 48% of it at a profit of 25%. How much did he receive for the part sold?

21. I paid $1181.25 for wheat at $87\frac{1}{2}$ ct. a bushel, and sold $55\frac{5}{9}\%$ of it at a profit of 12%. How much did I receive for the quantity sold?

22. I paid $360 for corn at $.48 a bushel, and sold 44% of it for $198. What per cent was gained on what was sold?

23. I paid $3000 for land at $62.50 an acre, and sold $31\frac{1}{4}\%$ of it for $1087.50. What per cent was gained on the part sold?

24. A sold B a farm for $3625, and gained 16%. B then sold it, and lost 16% on what it cost him. How much more did B lose than A gained?

25. C sold D a team for $225, and lost $16\frac{2}{3}\%$. D then sold it and gained $33\frac{1}{3}\%$ on what it cost him. How much more did D gain than C lost?

26. A dealer sold 48 head of cattle for $1575, thereby losing $12\frac{1}{2}\%$. At what price per head should he have sold them to gain $33\frac{1}{3}\%$?

27. A drover bought 96 cows at $31.25 apiece. 8 having died, at what price per head must he sell the remainder to realize a clear profit of 10% on the whole transaction?

28. I sold coal for $48, at 15 ct. a bushel, thereby gaining 20%. If the quantity sold was 12.8% of my entire stock, how much did the stock cost?

29. I sold two watches for $20.25 apiece, gaining $12\frac{1}{2}\%$ on one, losing 10% on the other. What was the gain or loss by the two sales?

30. I bought a lot of lumber for $212.50, which was 15% less than its value, and sold it for 12% more than its value. How much was gained?

31. I paid $2175 for a vessel, which was 16% more than its value, and sold it for 12% less than its value. What was the loss?

32. I bought a carriage for $330, which was 12% less than its value, and sold it for 10% more than its value. What was the per cent of gain on the whole transaction?

33. If 48% of the gain = .15 of the selling price, for how much should 80 bbl. of apples be sold that cost $2.75 a barrel?

34. If 24% of the loss = .048 of the selling price, for how much should 125 bbl. of flour be sold that cost $3.60 a barrel?

TRADE DISCOUNT

DEFINITIONS AND PROBLEMS

152. 1. *Trade discount* is a deduction from the regular price of merchandise, or from the face of a bill, etc. These deductions are usually made by wholesale dealers, and are termed *per cents off*. Thus, 25% *off* or 25 *off* means a discount of 25% from the regular price.

2. The *list* or *regular* price of goods is a fictitious value set upon them by the seller to serve as a basis for calculating discounts.

REMARK. *List price*, *marked price*, and *retail price* are synonymous terms.

3. The *net price* of goods is what is *paid* for them.

4. There may be *several successive discounts*. Thus, 25 and 10% off, means a discount of 25% from the list price, and another discount of 10% from the remainder.

5. The discount is sometimes expressed as a common fraction. Thus, $\frac{1}{5}$ off means 20% off; $\frac{1}{6}$ and 10 off means $16\frac{2}{3}$% and 10% off.

6. The list price is the *base*. The amount of the discounts is the *percentage*, and the sum paid for the goods is the *base minus the percentage*.

1. The list price of a piano was $500, on which discounts of 20 and 5% were given. How much was paid for it?

SOLUTION. 100% − 20% = 80%. $500 × .80 = $400. 100% − 5% = 95%. $400 × .95 = $380.

2. I paid $270 for a bill of goods, which was 25 and 10% off. What was the retail price?

SOLUTION. 100% − 10% = 90%. $270 ÷ .90 = $300. 100% − 25% = 75%. $300 ÷ .75 = $400.

RULE. 1. *Subtract the given discounts or per cents from* 1.

2. *To find the net price, multiply the list price by the first remainder, and that product by the second remainder, and so on. The last product will be the net price.*

3. *To find the list price, divide the net price by the last remainder, and that quotient by the preceding remainder, and so on. The last quotient will be the list price.*

REMARK. The list price may also be found by dividing the net price by the product of all the remainders.

1. A bill of goods amounted to $480, but the purchaser was allowed $\frac{1}{3}$ off. How much did he pay?

2. The retail price of an invoice of boots and shoes was $875, but the buyer received discounts of $\frac{1}{5}$ and 5%. What was the cost?

3. The list price of a lot of groceries was $960, but the dealer allowed $\frac{1}{6}$ and $12\frac{1}{2}$% off. How much was paid for the lot?

4. The cost of a bill of dry goods was $285, which was a reduction of 25% from the retail price. What was the retail price?

5. The net price of an invoice of toys was $315, after discounts of 20 and 12½% had been given. What was the list price?

6. A consignment of carpets cost the dealer $1652.40, after deducting 15 and 10%. What was the marked price?

7. I bought 48 kegs of nails, each containing 100 lb. at 5 ct. a pound, but received discounts of ¼ and 12½%. What was the net price?

8. The marked price of a lot of furniture was $1250, but the manufacturer allowed 16, 12, and 5% off. What was the cost to the retailer?

9. The retail price of an invoice of marble was $2250, but the wholesale dealer allowed 20, 15, and 10% off. What was paid for the marble?

10. The wholesale price of a bill of hardware was $950.40, which was a reduction of ⅕ and 12% from the retail price. What was the retail price?

11. I paid $1567.50 for the stock in my store, which was a reduction of ⅛, 12, and 5% from the marked price. What was the amount of the discounts?

12. The list price of a shipment of canned fruits was $480, but the purchaser was allowed 25, 15, and 10% off. How much did they cost him?

13. The cost of an invoice of toys was $1224, the buyer having received ⅕, 15, and 4% off. If he sold them at the retail price, how much did he gain?

14. The marked price of a lot of books was $1500, but the purchaser was allowed ⅛, 10, and 8% off. How much did he pay?

15. A bill of goods cost $724.50, the buyer having received discounts of $16\frac{2}{3}$, $12\frac{1}{2}$, and 8% off. What was the marked price?

16. The retail price of a stock of merchandise was $875, but the dealer gave discounts of 20, 10, and 5% off. What was the cost of the goods?

17. The net price of a bill of hardware was $808.50, which was a reduction of 20, 16, and $12\frac{1}{2}$% from the regular price. What was the amount of the discounts?

18. I sold goods at 20, $16\frac{2}{3}$, and $12\frac{1}{2}$% off, and the discounts amounted to $260.75. How much did the purchaser pay?

19. I bought merchandise at 25, $16\frac{2}{3}$, and 10% off, and the net price exceeded the discounts by $157.50. What was the net price?

20. I sold goods at $37\frac{1}{2}$, 20, and $12\frac{1}{2}$% off, and the discounts exceeded the net price by $125. What was paid for the goods?

21. I bought goods at 20, $16\frac{2}{3}$, and $6\frac{1}{4}$% off, and the discounts amounted to $506.25. If the goods were sold at the list price, what per cent was gained?

22. A dealer bought goods at $16\frac{2}{3}$, 10, and 4% off, and the net price exceeded the discounts by $660. If he sold them at 90% of the list price, what was his profit?

23. A man is offered goods for $1000 at one house with discounts of 20, 10, and 5%; at another house with discounts of 10, 5, and 20%, and at a third house with discounts of 10, 20, and 5%. Which is the best offer?

COMMISSION

DEFINITIONS AND PROBLEMS

153. 1. A *commission merchant* is one who buys or sells merchandise for another.

2. An *agent* is one who transacts business for another. He is sometimes called a *factor* or a *correspondent.*

3. *Commission* is a percentage paid to an agent for transacting business.

4. *Charges* are expenses incurred in the transaction of business.

5. The *net proceeds* of a sale is the sum remaining after the commission and the charges have been deducted.

6. The amount of the sales, or the sum invested, is the *base;* the commission is the *percentage;* the net proceeds is the *difference* between the base and the percentage.

REMARK. The commission is estimated at a certain per cent of the *base.*

154. (*a*) An agent sold $560 worth of goods on a commission of 4½%. What was his commission? The net proceeds?

OPERATION

$560	$560.
.04½	25.20
22 40	$534.80, net proceeds.
2 80	
$25.20, com.	

EXPLANATION. His commission was 4½%, or .04½ of $560 = $25.20. The net proceeds was $560 − $25.20 = $534.80.

(*b*) An attorney collected a claim at $4\frac{1}{2}\%$ and sent the owner $2721.75. What was the amount of the claim, and what was his commission?

OPERATION.
$$\begin{array}{r} 1.000 \\ .045 \\ \hline .955 \overline{)\$2721.75} \\ \$2850 = \text{claim} \end{array}$$
$\$2850 \times 4\frac{1}{2}\% = \$128.25$, commission.

EXPLANATION. Since $4\frac{1}{2}\%$ has been deducted from 100% of the claim, the remainder $\$2721.75 = 95\frac{1}{2}\%$ of the claim. $\$2721.75 \div .955 = \$2850$, the claim, and $4\frac{1}{2}\%$ of $\$2850 = \$128.25$, the commission.

RULE I. 1. *Multiply the amount of the sales by the rate. The product will be the commission.*

2. *Subtract the commission from the amount of the sales. The result will be the net proceeds.*

RULE II. *Divide the sum returned by* (1 — *the rate*). *The quotient will be the sum collected, which multiplied by the rate will give the commission.*

1. An agent sold a farm for $4875, and charged 2% commission. What was his commission?

2. A factor sold 125 bbl. of sugar, each weighing 192 lb. at $6\frac{1}{4}$ ct. a pound. Find his commission at $3\frac{1}{3}\%$.

3. A commission merchant sold 160 bbl. of pork, each weighing 200 lb., at 8 ct. a pound, and 45 hhd. of tobacco, of 320 lb. each, at $22\frac{1}{2}$ ct. a pound. Find his entire commission at $2\frac{1}{4}\%$.

4. A commission house sold 316 bales of cotton, each weighing 240 lb., at $18\frac{3}{4}$ ct. a pound, and 250 bbl. of rice, of 150 lb. each, at $6\frac{2}{3}$ ct. a pound. What was the entire commission at $1\frac{1}{2}\%$?

5. A real estate agent sold property for $108,000 at $1\frac{3}{4}\%$ commission. What sum did he return to the owner?

6. An agent sold a vessel for $4800, and sent the owner $4680. What per cent was his commission?

7. My commission for buying $6840 worth of Texas cattle was $256.50. What was the rate of commission?

8. A factor sold a cargo of rice on a commission of $2\frac{3}{4}\%$, and sent the owner $5601.60. What was the amount of the sale?

9. An agent sold lumber for $4320, and sent the owner $4190.40. What per cent did he receive?

10. A commission merchant sold a consignment of grain on a commission of $2\frac{1}{4}\%$, and sent the owner $3870.90. What was his commission?

11. The commission for selling town lots at $1\frac{1}{2}\%$ was $100.50. What sum did the owner realize?

12. An attorney collected a claim at $3\frac{1}{2}\%$, and sent the owner $2895. What was the amount of the claim, and what was his commission?

13. An auctioneer sold goods on a commission of 3%, and sent the owner $4777.25. What was his commission?

14. As net proceeds of a sale of city property an agent returned $10,150.50, after deducting $113.25 charges, and a commission of $2\frac{1}{4}\%$. What was the amount of the sale?

15. A sale of 45 acres of land realized $3425 to the owner, after paying $40 charges and a commission of $3\frac{3}{4}\%$. At what price per acre was the land sold?

16. A sale of farm land returned as net proceeds $3125, after deducting the agent's commission of 3% and $27.50 expenses. What was the commission?

17. An auctioneer sold 50 horses and remitted $3600 to the owner, after paying $75 charges and retaining his commission of 2%. At what price per head were the horses sold?

18. An agent sold wood at $4.75 a cord, and remitted $687.50 to the owner, after deducting 2% commission and $10.75 charges. If the wood cost $4.25 a cord, what was the owner's profit?

19. A speculator bought corn at 45 ct. a bushel, and gained 25% by selling it through an agent who charged him 4% of the sale. If the agent's commission was $30, how many bushels were sold?

STOCK TRANSACTIONS

DEFINITIONS

155. 1. *Stock* is capital in the form of transferable shares, each for a certain amount.

REMARK. The term *stock* is applied to the capital of various corporations, as railroad companies, banks, insurance companies, etc., whose organizations are effected in conformity with certain legal requirements.

2. The owners of stock are *stockholders*.

3. A *share* is one of the equal parts into which stock is divided. It is usually $100, but sometimes it is $50 or some other number.

4. *Stock transactions* are those relating to the purchase and sale of stocks, bonds, and gold.

5. *Stocks* is a general name applied to the shares or bonds of a corporation, and to government bonds and public securities.

6. A *bond* is a written or printed obligation to pay a certain sum of money at a specified time.

7. *United States bonds* are bonds of the United States, payable at or within a specified time, with interest at a fixed rate.

8. *Currency* is the money of the country in circulation. It now consists of government notes and national bank notes.

9. The *par value* of stocks and bonds is the face or nominal value written or printed upon them.

10. The principal stock transactions which involve an application of percentage are (1) *Brokerage*, (2) *Assessments and Dividends*, (3) *Stock Values*, and (4) *Stock Investments*.

REMARK. In solving the problems involving these transactions, consider a share as $100 unless some other value is given.

BROKERAGE

156. 1. A *broker* is an individual who buys and sells stocks, bonds, gold, etc., for others.

2. *Brokerage* is a percentage paid to the broker for transacting the business. It is always calculated upon the *par value*.

3. The par value is the *base;* the brokerage is the *percentage*.

What would be the brokerage on 150 shares of stock at $\frac{1}{4}\%$?

OPERATION

$$\begin{array}{r} \$100 \\ 150 \\ \hline \$15000 \\ .01 \\ \hline 4)\$150.00 \\ \$37.50 \end{array}$$

EXPLANATION. The par value of 150 shares is $\$100 \times 150 = \$15000$. $\frac{1}{4}\%$ of $\$15000 = \$37.50$.

1. A broker bought 85 shares of railroad stock for an operator who paid him $\frac{1}{2}\%$. What was the brokerage?

2. A broker sold 96 shares Erie stock, and charged $57.60. What per cent was the brokerage?

3. I paid a broker $37.50, or $\frac{3}{4}\%$, for buying insurance stock. How many shares did he buy?

4. Find the brokerage on 120 shares B. and O. stock at $\frac{3}{8}\%$.

5. I paid a broker $48.75 for selling 78 shares of bank stock. What per cent did he charge?

6. The brokerage at $\frac{3}{5}\%$ for buying Pennsylvania R.R. stock was $75. What was the number of shares?

7. A broker charged $\frac{7}{10}\%$ for selling 135 shares of telegraph stock. What was the brokerage?

8. The brokerage for selling 175 shares Northern Pacific stock was $131.25. What was the per cent?

9. A broker received $56.25, or a brokerage of $\frac{3}{8}\%$ for buying O. and M. R.R. stock, and charged $\frac{1}{4}\%$ for selling it. How many shares were there, and what was his brokerage for selling the stock?

10. I paid a broker $\frac{3}{8}\%$ for buying telephone stock, and $\frac{3}{10}\%$ for selling it. If he received $50.40 for selling, how much did he charge for buying?

ASSESSMENTS AND DIVIDENDS

157. 1. An *assessment* is a percentage paid by stockholders to meet the losses or expenses of the company.

2. A *dividend* is a percentage paid to stockholders out of the earnings of the company.

3. The *gross earnings* of a company are the entire receipts from its business. The *net earnings* are what remains after deducting all losses and expenses from the gross earnings. The dividends are paid, out of the net earnings.

1. The capital stock of a bank is $250,000. If a dividend of $7\frac{1}{2}\%$ is declared, how much will it be?

2. I own 48 shares of steamboat stock. If the company declare a dividend of $8\frac{1}{3}\%$, payable in stock, how many shares shall I then own?

3. Mr. B paid an assessment of $18\frac{3}{4}\%$, or $843.75, on his mining stock. How many shares did he own?

4. A telephone company declared a dividend of $10\frac{1}{2}\%$, or $19,687.50. What was the capital stock?

5. A gas company's capital stock is $375,000, and its net earnings for the year are $31,250. What will be the rate of dividend?

6. An insurance company has a capital stock of $225,000. Its gross earnings for the year are $35,437.50, and its expenses $7312.50. What per cent will the dividend be?

7. What assessment is made when the owner of 85 shares pays $450 more than the owner of 49 shares?

8. A $6\frac{1}{4}\%$ dividend having been declared, $10,000 was paid out, and $1718.75 was unclaimed. What was the number of shares?

9. A street car company declared a dividend of $8\frac{3}{4}\%$, or $13,125. What was the capital stock?

10. A steamboat company has a capital stock of $125,000. Its gross earnings for a certain year were $29,687.50, and its expenses $12,812.50. What was the rate of dividend, and how much did a stockholder receive who owned 25 shares?

11. The capital stock of a mining company is $112,500 in $50 shares. The gross earnings for the last year were $21,093.75, and the expenses $7031.25. What dividend will a stockholder receive who owns 50 shares?

12. A national bank declared a dividend of $28,125. If a stockholder who owned 40 shares received $300, what was the total number of shares?

13. What assessment was made when the owner of 38 shares paid $144.50 less than the owner of 55 shares?

14. A manufacturing company has a capital stock of $275,250 in $50 shares. The gross earnings for the past year were $44,040, and the expenses $19,267.50. If I own 125 shares, what will my dividend be?

15. The capital stock of a turnpike company is $165,000. Mr. B, who owns 45 shares, receives a dividend of $375. What is the entire amount of dividend?

STOCK VALUES

158. 1. The *market value* of stocks, bonds, and gold is the price at which they sell. The market value of stocks and bonds may be more or less than the par value.

Thus, "Illinois Central $97\frac{1}{2}$," indicates that the stock of that railroad is selling for $97\frac{1}{2}\%$ of the par value. "Gold $106\frac{1}{4}$," means that the gold dollar is worth $106\frac{1}{4}\%$ of the currency dollar, or $1.06\frac{1}{4}$.

2. When stock sells for its *nominal value*, it is at *par;* when it sells for more than its nominal value, it is *above par*, or *at a premium;* when it sells for less than its nominal value, it is *below par*, or *at a discount.*

3. The par value is the *base*, the premium or discount is the *percentage*, and the market value is the *sum* or the *difference*.

Find the cost of 20 U. S. 100-dollar bonds at $107\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

OPERATION

$100	$1.07\frac{1}{2}$
20	$.00\frac{1}{8}$
$2000	$1.07\frac{5}{8}$
$1.07\frac{5}{8}$	
$2152.50	

EXPLANATION. The par value is $100 \times 20 = \$2000$. The market rate + the brokerage is $107\frac{5}{8}\%$. The cost is $\$2000 \times 1.07\frac{5}{8} = \$2152.50$.

1. Find the cost of 25 Cincinnati Southern R.R. bonds at par, brokerage $\frac{1}{2}\%$.

2. Find the cost of 32 city bonds at 4% premium, brokerage $\frac{3}{8}\%$.

3. Find the cost of 45 corporation bonds at 4% discount, brokerage $\frac{3}{8}\%$.

4. Find the cost of 50 shares C. and O. R.R. stock at 105, brokerage $\frac{1}{4}\%$.

5. Find the cost of 68 shares P. C. R.R. stock at 94, brokerage $\frac{3}{10}\%$.

6. I paid $7380 for B. and O. stock at 102, brokerage $\frac{1}{2}\%$. How many shares did I buy?

7. A speculator paid $9528.75 for canal stock at $4\frac{3}{8}\%$ discount, brokerage $\frac{5}{8}\%$. How many shares did he buy?

8. How many shares Erie R.R. stock at 107, brokerage $\frac{1}{2}\%$, will cost $11,610?

9. How many shares of bridge stock at $93\frac{1}{2}$, brokerage $\frac{1}{4}\%$, will cost $15,000?

10. How many shares N. Y. Central stock at 6% premium, brokerage $\frac{1}{4}$%, could be bought for $8925?

11. A broker sold 75 shares Union Pacific, brokerage $\frac{4}{5}$%, and remitted the owner $6540. At what rate did the stock sell?

12. My broker sold 128 shares Pennsylvania stock, brokerage $\frac{3}{8}$%, and sent me $13,520. What was the market rate?

13. A broker sold 148 shares of bank stock at $105\frac{3}{4}$, brokerage $\frac{3}{4}$%, and with the proceeds bought telegraph stock at 8% discount, brokerage $\frac{1}{2}$%. How many shares did he buy?

14. A broker sold 1750 shares of mining stock at $69\frac{1}{2}$, brokerage $\frac{3}{4}$%, and invested the money in Southern Pacific stock at 13% discount, brokerage $\frac{1}{2}$%. How many shares did he buy?

15. Find the cost in currency of $10,000 in gold at 4% premium, brokerage $\frac{3}{8}$%.

16. When gold was at 160, what was the value in gold of a currency dollar?

17. When currency was at a discount of 20%, what was the value in currency of a gold dollar?

18. I bought stock at $3\frac{1}{2}$% discount, and sold it at $2\frac{3}{4}$% premium, thereby gaining $693.75. How many shares were sold?

19. I bought stock at 101, and sold it at $1\frac{1}{2}$% discount, thereby losing $375. How much did I get for it?

20. A broker sold gold, and remitted the owner $10,020. His brokerage at $\frac{5}{8}$% was $60. What was the market value of gold?

21. A broker sold gold, and remitted the owner $96.50. His brokerage at $\frac{2}{5}\%$ was $37.50. What was the price of gold?

STOCK INVESTMENTS

159. 1. *United States* or *government bonds*, and those issued by states, corporations, etc., are named from the rate of interest they bear. Thus, government bonds that bear 4% interest are called United States 4's; New York bonds bearing 6% interest are called New York 6's, etc.

2. The *income* from stocks or bonds is the annual profit which they yield.

(*a*) What income will be derived by investing $5040 in U. S. 5% bonds at 105?

SOLUTION. $5040 ÷ 1.05 = $4800. $4800 × .05 = $240.

EXPLANATION. $1 worth of the bonds cost $1.05; then the *par value* is $5040 ÷ 1.05 = $4800. The *income* is 5% of $4800 = $240.

(*b*) If 6% stock is bought at 80, what per cent will it yield on the investment?

SOLUTION. $1 × .06 = $.06. $.06 ÷ $.80 = .07½ = 7½%.

EXPLANATION. $1 worth of stock costs $.80, and the income from it is $.06; then the *rate* is $.06 ÷ $.80 = 7½%.

(*c*) What sum must be invested in 8% stock at 12% premium, to secure an annual income of $1000?

SOLUTION. $1000 ÷ .08 = $12500. $12500 × 1.12 = $14000.

EXPLANATION. $1 worth of stock yields $.08 income. Then the par value is $1000 ÷ .08 = $12500. $1 worth of stock costs $1.12. Then the sum invested will be $12500 × 1.12 = $14000.

1. What will be the income from an investment of $7280 in U. S. 4½% bonds at 104?

2. If a man pays $7280 for 4½% stock at 91, what income will he secure?

3. How much more income will be derived by investing $4320 in 6% bonds at 108, than by investing it in 5% bonds at 96?

4. What will be the income from an investment of $6880 in Erie R.R. stock at 7% premium, brokerage ½%, if it pays 6¼%?

5. If a capitalist invests $11,550 in mining stock at 32% discount, brokerage ¾%, what will be his dividend at 7½%?

6. What must be paid for 7½% bonds to make 6% on the investment?

7. What must be paid for 6% bonds to realize 7½% on the investment?

8. At what discount must 7½% bonds be bought to pay 8% on the investment?

9. What sum should be invested in 5% stock at 12½% discount to secure an annual income of $540?

10. What amount must be paid for 8% stock at 10% premium to realize an annual income of $840?

11. A broker bought 4½% stock at 6¼% below par, charging ½% brokerage. If the owner realized an income of $225, how much did the broker pay for the stock?

12. What income in currency would be derived from an investment of $13,760 in U. S. 6% bonds at 107½, when gold is at 105?

13. If I invest $9180 in U. S. 5% bonds at 8% premium, what will be my income in currency if gold is at 106?

14. I sold 150 shares of stock at 108, and invested the money in 6% stock at 90. What income will I realize?

15. Brown sold 160 5% bonds at $93\frac{3}{4}$, and invested the proceeds in 7% bonds at 120. How much was his income increased?

16. A capitalist sold 108 $7\frac{1}{2}$% bonds at 5% premium, and invested the money in 6% stock at 16% discount. How much was his income increased or decreased?

17. Which is better, and by how many per cent, to buy $4\frac{1}{2}$% stock at 90, or $7\frac{1}{2}$% stock at 125?

18. Which is better, and how much per annum, to invest $10,080 in 5% bonds at 12% premium, or in 4% stock at 4% discount?

19. What sum invested in U. S. 5% bonds at 106 will yield an annual income of $1170 in currency, if gold is at 4% premium?

20. What sum invested in 6% Government bonds at 110 will yield an annual income of $1102.50 in currency when gold is at 105?

REVIEW OF STOCKS

160. **1.** A capitalist paid $15,480 for N. Y. P. and O. stock at 107, brokerage $\frac{1}{2}$%. How many shares did he buy?

2. How many shares at 2% discount must be given for 140 shares at 5% premium to sustain no loss?

3. The brokerage for buying insurance stock was $40.50. If the rate of brokerage was $\frac{3}{8}\%$, how many shares were bought?

4. I bought stock at $97\frac{1}{2}$, and sold it at $101\frac{1}{4}$, gaining $281.25. What was the par value?

5. I bought bonds at 103, and sold them at $98\frac{1}{2}$, thereby losing $225. How much did I receive for them?

6. An operator bought stock at 2% discount, and sold it at $4\frac{1}{2}\%$ premium, brokerage $\frac{3}{4}\%$ on each transaction. If he gained $320 by the operation, for how much did the stock sell?

7. When gold is at a premium of $2\frac{1}{2}\%$, what is the value in gold of $2818.75 in currency?

8. When currency is at a discount of 4%, what is the value in currency of $2640 in gold?

9. When a "greenback" dollar was worth only 60 cents in gold, what was the price of gold?

10. When 75 cents in gold would buy a paper dollar, what was the premium on gold?

11. I exchanged 159 shares of stock at 88, for stock at 106. How many shares of the latter did I receive?

12. How much will a man increase his income by selling 135 shares 5% Electric Light stock at 96, and investing the money in 6% bank stock at 108?

13. I bought 104 shares at $98\frac{3}{4}$, and sold them at $101\frac{1}{2}$, brokerage $\frac{1}{4}\%$ on each transaction. What was the gain?

14. I bought 148 shares Street Car stock at $93\frac{1}{4}$, and sold it at $89\frac{3}{4}$, brokerage $\frac{1}{2}\%$ on each transaction. Find the loss.

15. How many shares bought at $3\frac{3}{4}$ % premium, and sold at $2\frac{1}{2}$ % discount, brokerage $\frac{3}{8}$ % on each transaction, will cause a loss of $1050?

16. A broker sold 160 shares Illinois Central at 104, and sent the owner $16,600. What per cent was the brokerage?

17. Which is more profitable, and how much more; to invest $17,325 in U. S. 5% bonds at 105, or in L. and N. $4\frac{1}{2}$'s at 99?

18. I bought stock at 10% discount, and sold it at a profit of $16\frac{2}{3}$%. At what premium was it sold?

19. I bought canal stock at 4% premium, and sold it at a loss of $12\frac{1}{2}$ %. At what discount was it sold?

20. A broker sold a certain amount of gold, and sent the owner $10,762.50; his brokerage at $\frac{1}{2}$% was $52.50. What was the price of the gold?

21. A broker sold gold at 105. After retaining his brokerage of $\frac{3}{8}$%, or 46.87\frac{1}{2}$, he sent the owner the balance. How much was it?

22. I bought bonds at $96\frac{1}{4}$ and sold them at $2\frac{1}{2}$% premium, thereby gaining $187.50. How much did I receive for them?

23. Which will yield the greater per cent of income, 6% stock at 90, or 5% stock at 80?

24. I bought stock at $3\frac{3}{4}$% premium, and sold it at $1\frac{1}{2}$% discount, brokerage $\frac{1}{2}$% on each transaction. If I lost $425 by the operation, how much did I realize?

25. A broker sold bonds, and remitted $6462.50 to the owner. His brokerage at $\frac{3}{5}$% was $37.50. At what premium did the bonds sell?

INSURANCE

DEFINITIONS AND PROBLEMS

161. *Insurance* is security against loss or damage. It is of two kinds, *property insurance* and *life insurance.*

162. 1. *Property insurance* includes two kinds, *fire insurance* and *marine insurance.*

2. *Fire insurance* is security against loss or damage by fire.

3. *Marine insurance* is security against loss or damage by navigation.

4. The *policy* is the written contract or agreement between the company and the person insured.

5. The *premium* is the sum paid for insurance. It is a certain percentage of the amount insured.

REMARK. In order to prevent fraud insurance companies seldom insure property for its full value. When losses occur, the companies have the choice of replacing the property or of paying the insurance.

6. The amount insured is the *base*, the rate of insurance is the *rate*, and the premium is the *percentage.*

A house worth $3750 was insured for $\frac{4}{5}$ of its value, at $1\frac{3}{4}\%$. What was the premium?

OPERATION

$5)\$3750$
750
4
$\$3000$
$.01\frac{3}{4}$
3000
2250
$\$52.50$

EXPLANATION. $\frac{4}{5}$ of the value is \$3000. As the rate is $1\frac{3}{4}\%$ of the sum insured, the premium is $1\frac{3}{4}\%$ of \$3000 = \$52.50.

1. I insured a house worth \$2160 for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$. What was the premium?

2. A vessel worth \$13,500 was insured for $\frac{7}{10}$ of its value at $1\frac{2}{3}\%$. What was the cost of insurance?

3. A store is worth \$6750 and the stock \$5000. I insured the building for $\frac{5}{6}$ of its value at 2%, and the stock for $\frac{3}{4}$ of its value at $1\frac{4}{5}\%$. Find the total cost of insurance.

4. A vessel is appraised at \$10,800, and its cargo at \$10,500. I insured $\frac{4}{5}$ of the value of the vessel at $2\frac{1}{4}\%$, and $\frac{3}{4}$ of the value of the cargo at $1\frac{2}{3}\%$. What sum was paid for insurance?

5. A dwelling worth \$3840 was insured for $\frac{7}{8}\%$ of its value, and the premium was \$50.40. What was the rate of premium?

6. I insured a steamer worth \$16,500 for $\frac{5}{6}$ of its value, paying \$220 premium. What was the rate of premium?

7. The cost of insuring property worth \$3600, for $\frac{4}{5}$ of its value was \$64.80. What per cent was charged?

8. \$294 was paid for insuring $\frac{7}{10}$ of a factory valued at \$12,600. What per cent was the premium?

9. I insured a residence for $\frac{4}{5}$ of its value at $1\frac{3}{4}\%$, paying $44.10 premium. Find the value of the residence.

10. The cost of insuring $\frac{9}{10}$ of the value of a cargo of grain, at $\frac{3}{8}\%$, was $17.82. What was the value of the cargo?

11. A manufacturer insured $\frac{5}{6}$ of the value of his mill, at $2\frac{1}{2}\%$, and $\frac{5}{8}$ of the value of the machinery at $1\frac{3}{4}\%$, paying $156.25 premium on the mill, and $63 on the machinery. What was the entire value of the property insured?

12. The cost of insuring a store for $\frac{4}{5}$ of its value, at $1\frac{3}{4}\%$, was $96.25, and the stock for $\frac{3}{5}$ of its value, at $1\frac{2}{3}\%$, was $31.25. What was the combined value of the store and stock?

13. A paid $51.75 for insuring $\frac{5}{6}$ of the value of his house at $2\frac{1}{4}\%$. B paid $70.50 for insuring $\frac{3}{4}$ of the value of his house at $2\frac{1}{2}\%$. What was the difference in the value of the houses?

14. The premium for insuring a vessel worth $13,500, at $\frac{3}{4}\%$, was $81. What part of the value was insured?

15. The cost of insuring property worth $7680, at $3\frac{1}{3}\%$, was $224. What fraction of the value was insured?

16. An ocean steamer valued at $75,000, of which $\frac{1}{4}$ belonged to A, $\frac{1}{3}$ to B, and the remainder to C, was wrecked. What loss did each sustain, if the vessel was insured for $\frac{3}{4}$ of its value?

17. A block of buildings worth $150,000 is insured for $\frac{5}{6}$ of its value, at $2\frac{1}{2}\%$. If it should be destroyed by fire, what would be the actual loss to the company?

18. A vessel is valued at $62,500, and its cargo at $48,000. The vessel is insured for $\frac{4}{5}$ of its value, at $1\frac{3}{4}\%$, and the cargo for $\frac{3}{4}$ of its value, at $1\frac{2}{3}\%$. If the vessel should be lost at sea, what would be the net loss to the insurance company?

19. What amount must be insured on property worth $5775, for $\frac{4}{5}$ of its value, at $3\frac{3}{4}\%$, to include the premium in case of loss?

20. What sum must be insured on property worth $4740, for $\frac{2}{3}$ of its value, at $1\frac{1}{4}\%$, to cover both property and premium?

LIFE INSURANCE

163. 1. *Life insurance* is a contract in which a company agrees to pay a specified sum of money at the death, or at a certain time in the life, of the person insured.

2. The principal kinds of policies issued by life insurance companies are (1) *life policies*, and (2) *endowment policies*.

3. A *life policy* is payable at the death of the person insured.

4. An *endowment policy* is payable at a specified time in the life of the person insured, or at his death, if it occurs before that time.

REMARK. The premium is usually a regular annual or semi-annual payment based upon the *expectation of life* of the person insured at the time the policy is issued.

Mr. Roberts, aged 25, secures a life policy for $3000, at an annual premium of $20.40 per $1000. If he dies at the age of 50, what amount will his insurance have cost?

OPERATION

$20.40
3
$61.20
25
306 00
1224 0
$1530.00

EXPLANATION. As the annual premium on $1000 is $20.40, on $3000 it is $20.40 × 3 = $61.20. In 25 years it will have cost $61.20 × 25 = $1530.

1. James Gordon at the age of 35 takes an endowment policy for $5000, payable in 10 years, at an annual premium of $115.87 per $1000. Should he live to receive the endowment, how much more will he have paid than he will secure?

2. Mr. Smith paid $1650 for 8 years' insurance, on a life policy for $7500. What was the annual premium per $1000?

3. At the age of 33 a gentleman secured a life policy for $10,000, at the rate of $25.70 per $1000 annually. He died at the age of 48. How much more did his heirs receive than he paid out?

TAXES

DEFINITIONS AND PROBLEMS

164. 1. A *tax* is a sum of money levied upon the citizens of a country for the support of government, or for other public purposes.

2. A *property tax* is assessed upon the property of citizens.

3. *Real estate* is fixed property, as lands, houses, etc.

4. *Personal property* is movable property, as money, merchandise, etc.

5. A *poll tax* is assessed upon male citizens above twenty-one years of age, without regard to property.

6. An *assessor* is an officer appointed by the government to make a list of taxable property, and to appraise it for taxation.

7. The *valuation* is the assessor's appraisement. It includes the names of the persons subject to taxation, together with the estimated value of their property.

8. The *rate* of taxation is a certain per cent of the valuation. It is usually expressed as a given number of mills on each dollar of valuation.

165. To find the rate of taxation.

(*a*) A tax of $6562.50 is to be assessed in a village whose taxable valuation is $437,500. What will be the rate?

	Valuation	Tax	Rate
OPERATION	$437500)	$6562.500	(.015
		4375 00	
		2187 500	
		2187 500	

EXPLANATION. As the valuation was *multiplied* by a certain rate to produce the *tax*, the tax must be *divided* to find the *rate*. Dividing $6562.50 by $437,500, the rate is .015, or 15 mills on $1.

RULE. *Divide the property tax by the valuation; the quotient will be the rate of taxation.*

REMARK. When there is a poll tax, find its amount by multiplying the tax on one poll by the number of polls, and subtract the product from the total amount of tax. The remainder will be the property tax.

(*b*) A tax of $10,000 is assessed upon a district whose valuation is $562,000. There are 350 persons who pay a poll tax of $1.25 each. Find the rate of taxation, and A's tax, who owns property valued at $1850, and pays 1 poll.

OPERATION

Valuation	Tax	Polls
$562500	$10000	350
		1.25
		1750
		4200
		$437.50 = poll tax

$10000.00
437.50
$562500)$9562.500(.017 = rate
5625 00
3937 500
3937 500

$1850
.017
12 950
18 50
$31.450 = A's property tax
1.25 = A's poll tax
$32.70 = A's entire tax

EXPLANATION The amount of poll tax is

$1.25 × 350 = $437.50;

$10000 − $437.50 = $9562.50,

which is the property tax. Dividing the property tax by the valuation gives the *rate*, .017, or 17

mills. Multiplying $1850 by .017 = $31.45, which is A's property tax. Adding $1.25 poll tax gives $32.70 = A's entire tax.

1. The valuation is $321,400, and the rate 16 mills. Find the amount of tax.

2. The tax is $2761.50, and the rate 14 mills. What is the valuation?

3. The valuation is $548,950, and the tax $9881.10. What is the rate?

4. The taxable property of a town is assessed at $473,280, upon which a tax of $7099.20 is levied. If Mr. Brown's property is valued at $3960, how much tax must he pay?

5. If the valuation is $604,720, and the tax $11,489.68, what will be the tax on property assessed at $5780?

6. At $22\frac{1}{2}$ mills, the tax levy was $30,955.50. What was the amount of taxable property?

7. Valuation, $722,500; A's property is assessed at $2750, and his tax is $60.50. Find amount of tax.

8. Tax, $5860.80; B's real estate is valued at $2450, and his tax is $44.10. What is the entire valuation?

9. Mr. Smith's property is valued at $3750, and he pays $63.75 tax. If the valuation is $439,750, what is the amount of tax?

10. Mr. Wright's property is assessed at $4320, upon which he pays $75.60 tax. What is the valuation, if the amount of tax is $11,314.80?

11. A tax of $20,501.25 is levied upon a town whose valuation is $826,250. There are 537 persons subject to a poll tax of $1.25 each. Find the rate, and A's tax, who owns property valued at $5250, and pays 2 polls.

12. The taxable value of a district is $537,520, and a tax of $12,027.42 is to be levied. 493 persons pay a poll tax of $1.50 each. What will be the tax bill of a man owning real estate assessed at $6480, and paying 1 poll?

13. Valuation, 468,750; tax, $8000; number of polls 400, taxed at $1.25 each. Find A's tax on property valued at $6250, including 1 poll.

14. Valuation, $655,360; tax, $11,937.55; number of polls 625, at 75 ct. each. Mr. Thompson owns real estate assessed at $7500, personal property valued at $1980, and pays 2 polls. What is the amount of his tax?

15. Tax, $5474.64; number of polls 250, taxed at $1.20 each. C's property is valued at $2250, and he pays $42.90 tax, including 2 polls. Find the property valuation.

16. Valuation, $756,250; number of polls 575, each taxed at 75 cents. B owns property listed at $6750, and he pays 1 poll. If his tax amounts to $133.05, what is the entire tax?

17. Tax, $15,294.80; there are 746 voters, each of whom pays a poll tax of $1.25. A's property is assessed at $3250, and he pays $55.80 tax, including 2 polls. What is the valuation?

18. In a certain district the realty was listed at $583,680, and the personalty at $175,160. There were 631 voters, each taxed at $1.50. Mr. Roberts owned real estate valued at $3760, personal property at $1520, and paid 1 poll. If his tax was $80.70, what was the total tax?

19. The tax levied in a certain district was $11,000. There were 608 polls, each taxed at $1.25. A citizen who owned property valued at $4820 paid $62.75 tax, including 2 polls. What was the valuation?

20. The real estate of a village is assessed at $493,750, and the personal property at $106,250. 400 citizens pay a poll tax of $1 each. Mr. Johnson's real estate is valued at $3750, his personal property at $1000, and he pays 2 polls. His tax bill is $93.60, including a penalty of 20% for delinquency. What is the whole amount of tax?

166. To apportion a tax.

(*a*) A tax of $13,102.50 is levied upon a district whose taxable property is valued at $837,250. 725 persons pay a poll tax of $0.75 each. Find the rate of taxation, and construct a tax table to $9000.

TAX TABLE

Rate 15 mills on a $1

Value	Tax	Value	Tax	Value	Tax	Value	Tax
$1	$0.015	$10	$0.15	$100	$1.50	$1000	$15.00
2	.03	20	.30	200	3.00	2000	30.00
3	.045	30	.45	300	4.50	3000	45.00
4	.06	40	.60	400	6.00	4000	60.00
5	.075	50	.75	500	7.50	5000	75.00
6	.09	60	.90	600	9.00	6000	90.00
7	.105	70	1.05	700	10.50	7000	105.00
8	.12	80	1.20	800	12.00	8000	120.00
9	.135	90	1.35	900	13.50	9000	135.00

REMARK. Tables like the above are commonly used in county auditors' offices. It is more convenient to use such a table than to calculate each person's tax.

(*a*) Samuel Mason's property is valued at $3548, and he pays poll tax for 1 person. Find his tax.

OPERATION

$3548.

$45.00
7.50
.60
.12
$53.22
.75
$53.97

EXPLANATION. $3548 = $3000 + $500 + $40 + $8. Referring to the table, the tax on $3000 is $45; on $500, $7.50; on $40, $0.60; on $8, $0.12. The tax on $3548 is $45 + $7.50 + $0.60 + $0.12 = $53.22 = his property tax. Adding $0.75 poll tax, his entire tax is $53.97.

REMARK. It will be observed that the same result would have been attained by multiplying $3548 by the rate, .015, and adding the poll tax.

By the preceding table, find the taxes to be paid by the following:

1. Charles Friend, property $2160, 2 polls.
2. William Ellis, property $3750, 1 poll.
3. Eliza Carson, property $1296.
4. Jesse Dunn, property $4132, 3 polls.
5. Henry White, property $1098, 1 poll.
6. Edwin Phillips, property $3974, 2 polls.
7. George Harding, property $9786, 3 polls.
8. The taxable property of a city is $12,532,620; the tax to be raised, $300,782.88. Find the rate, and construct a tax table to $10,000.

By the table constructed in Ex. 8, *find the tax to be paid by the following persons:*

9. Robert Kennedy, property valued at $926.
10. Francis Wilson, property valued at $19,580.
11. Andrew Addison, property valued at $257,320.
12. Simon Cameron, property valued at $1068.
13. Franklin Taylor, property valued at $64,375.
14. Clifford McGrew, property valued at $10,860.
15. Herman Dumont, property valued at $316,794.

INTERNAL REVENUE

167. *United States revenue* is money collected for the support of the government. It includes *internal revenue*, and the revenue from *duties* or *customs*.

168. *Internal revenue* is derived from taxes upon the manufacture of liquors and tobacco, from licenses for selling them, from the sale of public lands, postage stamps, etc.

1. Letter postage is 2 cents for each ounce or fraction of an ounce. What would be the postage on a package of manuscript weighing 1 lb. $8\frac{1}{2}$ oz.?

2. The public lands are sold at $1.25 an acre. What would be the cost of five sections of these lands?

3. The tax on proof spirits is 70 cents a gallon. What would be the amount of tax on 25 barrels, each containing 50 gallons?

4. The tax on cigars is $3 per M. How much tax would be collected on 256 boxes, each containing 50 cigars?

DUTIES OR CUSTOMS

169. 1. *Duties* or *customs* are taxes levied by the government upon goods imported from foreign countries. They are of two kinds, *ad valorem* and *specific*.

2. An *ad valorem* duty is a certain per cent of the cost of the goods in the country from which they were imported.

3. A *specific duty* is a fixed tax levied upon certain articles without reference to their value.

REMARKS. 1. *Tare* is an allowance made for the weight of the box, package, cask, etc., containing the goods. *Draft* is an allowance made for waste. *Leakage* is allowed for the waste of liquors imported in barrels or casks, and *breakage* for that of liquors imported in bottles. *Gross weight* is the weight of merchandise before any allowances are made. *Net weight* is the weight after all the allowances are made.

2. Duties are collected at the customhouses by government officers. The cost of the goods is shown by an invoice, or their value is appraised at the customhouse. This value, upon which the duty is estimated, is always the nearest integer in dollars, pounds, marks, francs, etc. *The duty must be paid in coin.*

Find the duty on 12 hhd. of wine, each containing 63 gal. at 75 ct. a gallon, allowing 4 % for leakage.

OPERATION.

63 gal.
12
756 gal. gross.
.04
30.24 gal. leakage.

756
30.24
725.76 gal. net.
$0.75
362880
508032
$544.3200 = duty.

EXPLANATION. The gross quantity is 63 × 12 = 756 gal. Allowing 4 % or 30.24 gal. for leakage, the net quantity is 725.76 gal., and the duty at 75 ct. per gallon is $544.32.

1. What is the duty at 4 ct. a pound on 50 boxes of raisins, gross weight 30 lb. each, allowing $16\frac{2}{3}\%$ tare?

2. What is the duty at $33\frac{1}{3}\%$ on 160 chests of tea, each weighing 45 lb. and invoiced at 45 ct. a pound, the tare being 5 lb. per chest?

3. Find the duty on 32 tons of steel rails at $37\frac{1}{2}\%$, their cost being £6 5*s.* per ton, and reckoning the pound at $4.88.

4. I imported 50 baskets of champagne costing 125 fr. per basket in Bordeaux. Allowing 4% for breakage, and estimating the franc at $0.193, what was the duty at 45%?

5. A manufacturer imported 24 bales of wool, each weighing 375 lb. and costing $43\frac{3}{4}$ ct. a pound. Allowing 4% draft, find the duty at 5 ct. a pound and 15% ad valorem.

6. An importer received a consignment of 64 rolls of Brussels carpet, each measuring 75 meters, and costing 4 fr. per meter in Tournay. The charges were 10 fr. per roll, and the duty 25% ad valorem. Reckoning the franc at $0.193, what was the total cost?

7. I imported 10 cases Irish linens, each weighing 160 lb., tare $6\frac{1}{4}\%$. The cost in Ireland was £562 10*s.*, the charges £10, the duty 40 ct. per pound, and 20% ad valorem. What was the entire cost, estimating the pound at $4.86?

8. A wholesale dealer imported from Germany 120 cases of cheese, for which he paid 25 marks per case. The charges were 4 marks per case, and the duty $33\frac{1}{3}\%$ ad valorem. At what price per case must he sell it to make a clear profit of 25%, estimating the mark at 24 ct.?

INTEREST

DEFINITIONS

170. 1. *Interest* is money paid by a borrower to the lender for the use of money.

2. The *principal* is the sum of money borrowed.

3. The *amount* is the sum of the principal and interest.

4. The *rate* is a certain per cent of the principal, paid for its use for *one year.*

5. The *time* is the period during which the principal bears interest.

6. A *promissory note* is a written promise to pay a certain sum of money at a specified time.

7. The sum specified in the note is its *face* or the *principal.*

REMARK. It is customary for the borrower to give the lender a note for the money. The following is the usual form:

$1000.00. CINCINNATI, OHIO, May 4, 1901.

One year after date, I promise to pay to Henry Cooper or order, One Thousand Dollars, with interest at 6%, for value received.

JESSE WILSON.

8. *Legal interest* is interest at a rate established or allowed by law.

9. *Usury* is interest charged at a higher rate than that allowed by law.

REMARK. The legal rate of interest in the different states and territories in 1901 was as follows:

STATES AND TERRITORIES	LEGAL	SPECIAL	STATES AND TERRITORIES	LEGAL	SPECIAL
Alabama	8	Limit 8	Montana	8	No limit
Arizona	7	No limit	Nebraska	7	Limit 10
Arkansas	6	Limit 10	Nevada	7	No limit
California	7	No limit	New Hampshire	6	Limit 6
Colorado	8	No limit	New Jersey	6	Limit 6
Connecticut	6	Limit 6	New Mexico	6	Limit 12
Delaware	6	Limit 6	New York	6	Limit 6
Dist. of Columbia	6	Limit 10	North Carolina	6	Limit 6
Florida	8	Limit 10	North Dakota	7	Limit 12
Georgia	7	Limit 8	Ohio	6	Limit 8
Idaho	7	Limit 12	Oklahoma	7	Limit 12
Illinois	5	Limit 7	Oregon	6	Limit 10
Indiana	6	Limit 8	Pennsylvania	6	Limit 6
Indian Territory	6	Limit 10	Rhode Island	6	No limit
Iowa	6	Limit 8	South Carolina	7	Limit 8
Kansas	6	Limit 10	South Dakota	7	Limit 12
Kentucky	6	Limit 6	Tennessee	6	Limit 6
Louisiana	5	Limit 8	Texas	6	Limit 10
Maine	6	No limit	Utah	8	No limit
Maryland	6	Limit 6	Vermont	6	Limit 6
Massachusetts	6	No limit	Virginia	6	Limit 6
Michigan	5	Limit 7	Washington	6	Limit 12
Minnesota	6	Limit 10	West Virginia	6	Limit 6
Mississippi	6	Limit 10	Wisconsin	6	Limit 10
Missouri	6	Limit 8	Wyoming	8	Limit 12

NOTE. The first column gives the legal rate. The second column the rate that may be collected if so stipulated in the note.

SIMPLE INTEREST

171. 1. *Simple interest* is interest upon the principal only.

2. Simple interest includes five quantities: (1) the *principal*, (2) the *time*, (3) the *rate*, (4) the *interest*, (5) the *amount*. *Three of the first four* must be given to find the fourth. There are *five cases*.

CASE I

172. 1st. To find the interest or the amount for one or more years when the principal, time, and rate are given.

(*a*) Find the interest of \$125 for 2 years at 4%.

	Prin.	Time	Rate
OPERATION.	\$125	2 yr.	4%
	.04		
	\$5.00 = int. for 1 yr.		
	2		
	\$10.00 = int. for 2 yr.		

EXPLANATION. As the rate is 4%, the interest for 1 year is .04 of \$125 = \$5. For 2 years it is \$5 × 2 = \$10.

(*b*) Find the amount of \$204.80 for 4 years at 5%.

	Prin.	Time	Rate
OPERATION.	\$204.80	4 yr.	5%
	.05		
	\$10.2400 = int. for 1 yr.		
	4		
	\$40.96 = int. for 4 yr.		
	204.80 = principal.		
	\$245.76 = amount.		

EXPLANATION. The interest of \$204.80 for 1 year at 5% is \$10.24; for 4 years it is \$40.96. The amount is the sum of \$40.96 and \$204.80 = \$245.76.

RULE. 1. *Multiply the principal by the rate; the product will be the interest for one year.*

2. *Multiply the interest for one year by the given number of years; the product will be the interest required.*

3. *To find the amount, add the principal to the interest.*

Find the interest of:	*Find the amount of:*
1. $180 for 1 yr. at 3%.	7. $777 for 2 yr. at $6\frac{2}{3}$%.
2. $240 for 2 yr. at $3\frac{1}{3}$%.	8. $830 for 3 yr. at 7%.
3. $315 for 3 yr. at 4%.	9. $999 for 4 yr. at $7\frac{1}{2}$%.
4. $420 for 4 yr. at $4\frac{1}{2}$%.	10. $1025 for 5 yr. at 8%.
5. $556 for 5 yr. at 5%.	11. $1182 for 3 yr. at $8\frac{1}{3}$%.
6. $643 for 3 yr. at 6%.	12. $1245 for 5 yr. at 9%.

2d. When the time is any number of months.

Find the interest of $180 for 10 months at 7%.

	Prin.	Time	Rate
OPERATION.	$180	10 mo.	7%
	.07		
6)	$12.60		
	2.10		
	5		
	$10.50		

EXPLANATION. The interest of $180 for one year at 7% is $12.60. As 10 mo. are $\frac{5}{6}$ of a year, the interest for 10 mo. is $\frac{5}{6}$ of $12.60 = $10.50.

RULE. *Find the interest for one year; then take such a part of this interest as the number of months is part of a year.*

Find the interest of:

13. $280 for 3 mo. at 3%.
14. $396 for 4 mo. at $3\frac{1}{3}$%.
15. $468 for 5 mo. at 4%.
16. $576 for 6 mo. at $4\frac{1}{2}$%.
17. $660 for 7 mo. at 5%.
18. $750 for 8 mo. at 8%.

Find the amount of:

19. $825 for 9 mo. at $6\frac{2}{3}$%.
20. $936 for 10 mo. at 7%.
21. $1080 for 11 mo. at $7\frac{1}{2}$%.
22. $1140 for 8 mo. at 8%.
23. $1260 for 7 mo. at $8\frac{1}{3}$%.
24. $1350 for 10 mo. at 9%.

3d. When the time is any number of days.

Find the interest of $945 for 20 days at 8%.

	Prin.	Time	Rate
OPERATION.	$945	20 da.	8%
	.08		
12)	$75.60		
3)	6.30		
	2.10		
	2		
	$4.20		

EXPLANATION. The interest of $945 for one year at 8% is $75.60; for one month it is $\frac{1}{12}$ of $75.60 = $6.30. As 20 days are $\frac{2}{3}$ of a month, the interest for 20 days is $\frac{2}{3}$ of $6.30 = $4.20.

RULE. *Find the interest for one month; then take such a part of this interest as the number of days is part of a month.*

Find the interest of:

25. $840 for 15 da. at 3%.
26. $990 for 18 da. at $3\frac{1}{3}$%.
27. $750 for 21 da. at 4%.
28. $1280 for 9 da. at $4\frac{1}{2}$%.
29. $720 for 27 da. at 5%.
30. $540 for 20 da. at 6%.

Find the amount of:

31. $675 for 12 da. at $6\frac{2}{3}$%.
32. $480 for 24 da. at 7%.
33. $1024 for 15 da. at $7\frac{1}{2}$%.
34. $1170 for 10 da. at 8%.
35. $1350 for 24 da. at $8\frac{1}{3}$%.
36. $1440 for 21 da. at 10%.

4th. When the time is years, months, and days, or any two of these periods.

Find the interest of $560 for 3 yr. 4 mo. 15 da. at 5%.

	Prin.	Time			Rate
		yr.	mo.	da.	%
OPERATION.	$560	3	4	15	5%
	.05				
	$28.00				
	$3\frac{3}{5}$				
	84 00				
	10 50				
	$94.50				

EXPLANATION. 15 da. are $\frac{1}{2}$ of a month; $4\frac{1}{2}$ mo. are $\frac{3}{8}$ of a year. The interest for one year is \$28. For $3\frac{3}{8}$ years it is $\$28 \times 3\frac{3}{8} = \$94.50$.

RULE. 1. *Reduce the months and days to the fraction of a year.*

2. *Find the interest for one year at the given rate and multiply it by the time expressed in years.*

Find the interest of:

37. \$175 for 1 yr. 2 mo. 12 da. at 3%.

38. \$225 for 1 yr. 10 mo. at $3\frac{1}{3}$%.

39. \$320 for 1 yr. 7 mo. 15 da. at 4%.

40. \$450 for 1 yr. 8 mo. at $4\frac{1}{2}$%.

41. \$575 for 1 yr. 9 mo. 18 da. at 5%.

42. \$660 for 2 yr. 1 mo. at 6%.

43. \$729 for 2 yr. 2 mo. 20 da. at $6\frac{2}{3}$%.

44. \$880 for 2 yr. 3 mo. 18 da. at 7%.

45. \$920 for 2 yr. 10 mo. 15 da. at 8%.

46. \$1080 for 2 yr. 8 mo. 12 da. at $7\frac{1}{2}$%.

Find the amount of:

47. \$115.20 for 3 yr. 1 mo. 15 da. at $8\frac{1}{3}$%.

48. \$222.50 for 3 yr. 6 mo. 20 da. at 9%.

49. \$325.60 for 3 yr. 7 mo. 15 da. at 10%.

50. \$456.25 for 3 yr. 7 mo. 6 da. at 12%.

51. \$517.50 for 3 yr. 5 mo. 10 da. at 6%.

52. \$618.75 for 4 yr. 1 mo. 6 da. at 8%.

53. $731.25 for 4 yr. 6 mo. 12 da. at $6\frac{2}{3}\%$.

54. $819.20 for 4 yr. 4 mo. 15 da. at $6\frac{1}{4}\%$.

55. $937.50 for 4 yr. 9 mo. 18 da. at 5%.

56. $444.44 for 5 yr. 6 mo. 20 da. at $4\frac{1}{2}\%$.

173. The "Six per cent Method" of computing interest.

1st. To find the interest of $1 for any time at 6%.

REMARK. The interest of $1 for one year at 6% is 6 cents; for 1 month it is $\frac{1}{12}$ of 6 cents, or 5 mills; for 2 months it is 2 times 5 mills, or 1 cent. Therefore, *the interest of* $1 *at* 6% *for any number of months is half as many cents as there are months.*

The interest of $1 for one day at 6% is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill; for 6 days it is 6 times $\frac{1}{6}$ mill, or 1 mill. Hence *the interest of* $1 *at* 6% *for any number of days is* $\frac{1}{6}$ *as many mills as there are days.*

2d. To find the interest of any sum of money for any time at 6%.

Find the interest of $480 for 2 yr. 7 mo. 13 da. at 6%.

	Prin.	Time (yr. mo. da.)	Rate
OPERATION.	$480	2 7 13	6%
	$.157\frac{1}{2}$	12	6) .013
	3360	2) .310	$.002\frac{1}{6}$
	2400	.155	
	480	$.002\frac{1}{6}$	
	80	$.157\frac{1}{2}$	
	75.44\frac{0}{0}$		

EXPLANATION. 2 yr. 7 mo. 13 da. = 31 months 13 da. Since the interest of $1 at 6% is $\frac{1}{2}$ as many cents as there are months and $\frac{1}{6}$ as many mills as there are days, it is here $\frac{1}{2}$ of $.31, or $.155, plus $\frac{1}{6}$ of $.013, or $.002$\frac{1}{2}$, which added equal $.157$\frac{1}{2}$. Then the interest of $480 is $.157$\frac{1}{2}$ × 480 = $75.44.

RULE. 1. *Reduce the years and months to months, and express* $\frac{1}{2}$ *as many cents as there are months.*

2. *Express* $\frac{1}{6}$ *as many mills as there are days.*

3. *Add these results. The sum will be the interest of* \$1 *for the given time at* 6%.

4. *Multiply the principal by this sum. The product will be the interest required.*

REMARK. As 3 days = $\frac{1}{10}$ of a month, any multiple of 3 days, from 3 to 27 inclusive, can be written as a decimal fraction of a month. Thus, 9 mo. 18 da. is written .096; 3 yr. 7 mo. 27 da. is written .439, etc.

Find the interest of \$1, *at* 6%, *for:*

1. 8 mo. 12 da.
2. 11 mo. 15 da.
3. 5 mo. 24 da.
4. 10 mo. 21 da.
5. 6 mo. 27 da.
6. 7 mo. 18 da.
7. 4 mo. 9 da.
8. 9 mo. 6 da.
9. 1 yr. 4 mo. 10 da.
10. 1 yr. 6 mo. 3 da.
11. 2 yr. 5 mo. 12 da.
12. 2 yr. 8 mo. 17 da.
13. 3 yr. 7 mo. 25 da.
14. 3 yr. 9 mo. 3 da.
15. 4 yr. 1 mo. 11 da.
16. 4 yr. 11 mo. 19 da.

Find the interest, at 6%, *of:*

17. \$120 for 8 mo. 10 da.
18. \$175 for 10 mo. 24 da.
19. \$225 for 6 mo. 12 da.
20. \$360 for 7 mo. 15 da.
21. \$440 for 9 mo. 21 da.
22. \$550 for 11 mo. 18 da.
23. \$675 for 5 mo. 6 da.
24. \$720 for 10 mo. 20 da.
25. \$825 for 9 mo. 18 da.
26. \$1080 for 8 mo. 3 da.

27. $250 for 30 da.
28. $375 for 36 da.
29. $480 for 45 da.
30. $525 for 48 da.
31. $640 for 63 da.
32. $750 for 66 da.
33. $875 for 72 da.
34. $880 for 75 da.
35. $920 for 81 da.
36. $925 for 96 da.
37. $900 for 93 da.
38. $960 for 99 da.
39. $1040 for 105 da.
40. $1250 for 108 da.

41. $125 for 1 yr. 2 mo. 12 da.
42. $150 for 1 yr. 5 mo. 6 da.
43. $175 for 1 yr. 7 mo. 18 da.
44. $216 for 1 yr. 10 mo. 15 da.
45. $350 for 2 yr. 8 mo. 24 da.
46. $480 for 2 yr. 4 mo. 15 da.
47. $575 for 2 yr. 1 mo. 18 da.
48. $625 for 2 yr. 7 mo. 6 da.
49. $750 for 3 yr. 2 mo. 12 da.
50. $888 for 3 yr. 1 mo. 15 da.
51. $960 for 3 yr. 7 mo. 27 da.
52. $1000 for 3 yr. 9 mo. 21 da.

Find the amount, at 6%, of:

53. $325 for 1 yr. 0 mo. 12 da.
54. $480 for 1 yr. 1 mo. 10 da.
55. $510 for 1 yr. 4 mo. 20 da.
56. $640 for 2 yr. 3 mo. 15 da.
57. $720 for 2 yr. 5 mo. 21 da.
58. $880 for 2 yr. 9 mo. 9 da.
59. $950 for 3 yr. 7 mo. 24 da.

60. $1080 for 3 yr. 11 mo. 3 da.
61. $1175 for 3 yr. 3 mo. 6 da.
62. $1250 for 4 yr. 3 mo. 6 da.
63. $1325 for 4 yr. 9 mo. 18 da.
64. $1440 for 4 yr. 5 mo. 15 da.
65. $1560 for 4 yr. 7 mo. 9 da.

174. The "Twelve per cent Method" of computing interest.

1st. To find the interest of $1 for any time at 12%.

REMARK. The interest of $1 for one year at 12% is 12 cents; for 1 month it is $\frac{1}{12}$ of 12 cents, or 1 cent. Therefore, *the interest of* $1 *at* 12% *for any number of months is as many cents as there are months.*

Again, the interest of $1 for one day at 12% is $\frac{1}{30}$ of 1 cent, or $\frac{1}{3}$ of a mill; then for 3 days it is 3 times $\frac{1}{3}$ mill, or 1 mill. Hence, *the interest of* $1 *at* 12% *for any number of days is* $\frac{1}{3}$ *as many mills as there are days.*

2d. To find the interest of any sum of money for any time at 12%.

Find the interest of $348.60 for 3 yr. 5 mo. 20 da. at 12%.

OPERATION.

Prin.	Time (yr. mo. da.)	Rate
$348.60	3 5 20	12%
.416⅔	12	
209160	.416⅔	
34860		
139440		
23240		
$145.25000		

EXPLANATION. The interest of $1 for 3 yr. 5 mo. 20 da. at 12% is $0.416⅔. Then the interest of $348.60 is $0.416⅔ × 348.60 = $145.25.

RULE. 1. *Reduce the years and months to months and express as many cents as there are months.*

2. *Express $\frac{1}{3}$ as many mills as there are days.*

3. *Add these results; the sum will be the interest of* $1 *for the given time at* 12%.

4. *Multiply the principal by this sum. The product will be the interest required.*

Find the interest at 12% *of:*

1. $62.50 for 7 mo. 6 da.
2. $187.50 for 6 mo. 20 da.
3. $240 for 5 mo. 17 da.
4. $450 for 11 mo. 19 da.
5. $333.30 for 3 mo. 10 da.

Find the amount at 12% *of:*

6. $475 for 1 yr. 9 mo. 18 da.
7. $468.75 for 1 yr. 0 mo. 24 da.
8. $660 for 1 yr. 1 mo. 10 day.
9. $787.50 for 4 yr. 3 mo. 6 da.
10. $913.20 for 9 yr. 0 mo. 10 da.

175. To find the interest of any sum for any time at any rate per cent.

First. By the "Six per cent Method."

Find the interest of $420 for 3 yr. 7 mo. 6 da. at 5%.

	Prin.	Time yr. mo. da.	Rate
OPERATION.	$420	3 7 6	5%
	.18	12	
	3360	2).432	
	420	.216	
	$75.60	5	
		6)1.080	
		.18ø	

EXPLANATION. The interest of $1 for 3 yr. 7 mo. 6 da. at 6%, is $0.216; at 5% it is $\frac{5}{6}$ of $0.216 = $0.18. Then the interest of $420 is $0.18 × 420 = $75.60.

RULE. 1. *Find the interest of* $1 *for the given time at* 6%.

2. *If the given rate is other than* 6%, *multiply this interest by it, and divide the product by* 6.

3. *Multiply the principal by this decimal. The product will be the interest required.*

Second. By the "Twelve per cent Method."

Find the interest of $562.50 for 3 yr. 1 mo. 15 da. at 8%.

	Prin.	Time yr. mo. da.	Rate
OPERATION.	$562.50	3 1 15	8%
	.25	12	
	281250	3)\.375	
	112500	.125	
	$140.6250	2	
		.25¢	

EXPLANATION. The interest of $1 for 3 yr. 1 mo. 15 da. at 12%, is $0.375; at 8% it is $\frac{8}{12}$ or $\frac{2}{3}$ of $0.375 = $0.25. The interest of $562.50 is $0.25 × 562.50 = $140.63.

RULE. 1. *Find the interest of* $1 *for the given time at* 12%.

2. *If the given rate is other than* 12%, *take such a part of the interest of* $1 *as the given rate is part of* 12.

3. *Multiply the principal by this decimal. The product will be the interest required.*

REMARKS. 1. Either factor may be used as the multiplier.

2. If the mills in the result are 5 or more, add 1 to the cents; if less than 5, disregard them.

Find the interest of:

	Prin.	Time (yr. mo. da.)	Rate
1.	$285	1 1 10	3%
2.	$312	3 1 15	3⅓%
3.	$440	2 5 21	4%
4.	$525	10 20	4½%
5.	$668.75	2 4 24	5%
6.	$756.25	1 7 6	5½%
7.	$837.50	3 6 12	6%
8.	$951.20	9 18	6¼%
9.	$1024	3 4 15	6⅔%
10.	$1150	2 1 6	7%
11.	$1225.50	4 5 10	7½%
12.	$1375	1 6 9	8%
13.	$1415.50	3 7 6	8⅓%
14.	$1666.65	2 2 20	9%
15.	$1512.80	4 4 15	10%
16.	$175	5 4 24	3%
17.	$243.75	4 3 6	3¾%
18.	$320	4 0 18	3⅓%
19.	$437.50	3 2 12	4%
20.	$540	2 4 24	4½%
21.	$675	1 9 18	5%
22.	$728	5 2 15	6%
23.	$864	4 7 6	6¼%
24.	$937.50	4 9 18	6⅔%
25.	$1025	1 2 12	7%
26.	$1196	1 10 15	8%
27.	$1280	3 0 12	7½%
28.	$1320	11 21	8⅓%
29.	$1451.20	4 10 10	9%
30.	$1625.60	5 7 15	10%

Find the amount of:

	Prin.	Time (yr. mo. da.)	Rate
31.	$180	1 11 0	3%
32.	$250	1 0 18	3⅓%
33.	$333.45	2 9 10	4%
34.	$475	11 6	4½%
35.	$550	2 10 24	5%
36.	$640	2 0 27	6%
37.	$718.75	3 7 6	6⅔%
38.	$819.20	3 4 24	6¼%
39.	$960	3 0 27	7%
40.	$1062.50	4 3 6	7½%

Find the amount of:

	Prin.	Time (yr. mo. da.)	Rate		Prin.	Time (yr. mo. da.)	Rate
41.	$1187.50	4 4 24	8%	46.	$1687.50	4 9 18	$3\frac{1}{3}$%
42.	$1296	4 11 21	$8\frac{1}{3}$%	47.	$1732	2 6 20	$4\frac{1}{2}$%
43.	$1320	10 15	9%	48.	$1867.50	3 7 6	$6\frac{2}{3}$%
44.	$1440	5 9 12	10%	49.	$1989.90	1 9 10	$7\frac{1}{2}$%
45.	$1562.50	3 2 12	3%	50.	$2001.60	4 3 6	$8\frac{1}{3}$%

Case II

176. To find the time when the principal, interest, and rate are given.

(*a*) The interest of $250 for a certain time at 5% was $30. What was the time?

OPERATION

Prin.	Int.	Rate
$250	$30	5%

.05

$12.50)$30.00(2.4 yr.

250 12

500 4.8 mo.

500 30

24.0 da.

EXPLANATION. The interest of $250 for 1 year at 5% is $12.50. The time was as many years as $12.50 is contained times in $30 = 2.4 years. .4 yr. reduced to integers = 4 mo. 24 da.

	yr.	mo.	da.
Ans.	2	4	24

(*b*) I loaned $450 for a certain time at 4%, and received $483.75 on settlement. How long was the money loaned?

OPERATION. $483.75 − $450 = $33.75.

Prin.	Int.	Rate
$450	$33.75	4%
.04		

$18.00)$33.750(1.875 yr.
18 — 12
157 — 10.500 mo.
144 — 30
135 — 15.0 da.
126
90
90

EXPLANATION. The interest was $483.75 − $450 = $33.75. The interest of $450 for 1 year at 4% is $18. The time was as many years as $18 is contained times in $33.75 = 1.875 yr. = 1 yr. 10 mo. 15 da.

RULE. *Divide the given interest by the interest of the principal at the given rate for one year.*

REMARKS. 1. When the principal and the amount are given, subtract the principal from the amount to find the interest.

2. Should there be a fractional part of a year in the result, reduce the fraction to integers (Art. 111).

Find the time in the following:

	Principal	Interest	Rate		Principal	Interest	Rate
1.	$150	$10.50	4%	7.	$720	$208.80	10%
2.	$250	$31.25	5%	8.	$875	$84	3%
3.	$360	$56.70	6%	9.	$960	$28.80	$3\frac{1}{3}$%
4.	$450	$75.60	7%	10.	$1050	$120.75	4%
5.	$575	$82.80	8%	11.	$1125	$243	$4\frac{1}{2}$%
6.	$625	$202.50	9%	12.	$1280	$88	5%

	Principal	Interest	Rate		Principal	Interest	Rate
13.	$1375	$121	$5\frac{1}{2}\%$	17.	$1620	$400.95	$7\frac{1}{2}\%$
14.	$1425	$176.70	6%	18.	$1875	$468.75	8%
15.	$1567.50	$282.15	$6\frac{2}{3}\%$	19.	$1980	$346.50	$8\frac{1}{3}\%$
16.	$1750	$539	7%	20.	$2025	$992.25	10%

	Amount	Principal	Rate		Amount	Principal	Rate
21.	$194.25	$187.50	3%	25.	$561.92	$512	$7\frac{1}{2}\%$
22.	$267	$240	$3\frac{1}{3}\%$	26.	$819	$675	5%
23.	$366.40	$320	4%	27.	$913.50	$787.50	$6\frac{2}{3}\%$
24.	$540.48	$480	$4\frac{1}{2}\%$	28.	$1137.87	$968.40	$8\frac{1}{3}\%$

29. In what time will any principal double itself at $7\frac{1}{2}\%$?

30. In what time will any principal double itself at $4\frac{1}{2}\%$?

Case III

177. To find the rate when the principal, interest, and time are given.

(a) The interest of $175 for 2 yr. 4 mo. 24 da. was $21. What was the rate per cent?

OPERATION.

Prin.	Int.	Time
		yr. mo. da.
$175	$21	2 4 24

$$\begin{array}{r} .024 \\ \hline 700 \\ 350 \\ \hline \$4.200 \end{array} \qquad \begin{array}{r} 12 \\ \hline 12\overline{).288} \\ .024 \end{array}$$

$$\$4.200 \overline{)\$21.0} \quad 5\%$$

EXPLANATION. The interest of $1 for 2 yr. 4 mo. 24 da. at 1% is $0.024, and of $175 it is $0.24 × 175 = $4.20. The rate is as many per cent as $4.20 is contained times in $21, = 5%.

(*b*) The amount of $350 for 1 yr. 9 mo. 18 da. was $378.35. What was the rate per cent?

OPERATION. $378.35 − $350 = $28.35.

Prin.	Int.	Time
		yr. mo. da.
$350	$28.35	1 9 18
.018		12
2800		12)\.216
350		.018
$6.300)$28.35(4.5%		
252		
315		
315		

EXPLANATION. The interest was $378.35 − $350 = $28.35. The interest of $350 for 1 yr. 9 mo. 18 da. at 1% is $6.30. The rate was $28.35 ÷ $6.30 = 4½%.

RULE. *Divide the given interest by the interest of the principal at* 1% *for the given time.*

REMARK. When the amount and the interest are given subtract the interest from the amount to find the principal.

Find the rate in the following:

	Principal	Interest	Time yr.	mo.	da.
1.	$75	$3.75	1	8	0
2.	$120	$18	3	9	0
3.	$275	$22	1	7	6
4.	$360	$40.50	1	10	15
5.	$450	$72.45	2	3	18
6.	$555	$106.56	2	4	24
7.	$666	$133.20	2	2	20

	Principal	Interest	Time yr.	mo.	da.
8.	$725	$69.60	3	2	12
9.	$800	$88	3	3	18
10.	$960	$120	3	1	15
11.	$1080	$216	4	5	10
12.	$1110	$85.10	1	6	12
13.	$1225	$102.90	1	4	24
14.	$1320	$148.50	1	9	18
15.	$1440	$252	2	7	15
16.	$1562.50	$306.25	2	9	18
17.	$1631.25	$261	2	1	18
18.	$1787.50	$514.80	3	7	6
19.	$1856.25	$742.50	4	9	18
20.	$1968.75	$630	3	6	20

	Amount	Interest	Time yr.	mo.	da.
21.	$2208.60	$183.60	2	3	6
22.	$2295	$135	1	4	20
23.	$2492	$252	3	4	15
24.	$2889.20	$409.20	3	3	18
25.	$3269.75	$519.75	2	8	12
26.	$3138.75	$607.50	3	2	12
27.	$3818.75	$881.25	4	9	18
28.	$3700	$740	3	1	15

29. At what rate will any principal double itself in 26 yr. 8 mo.?

30. At what rate will any principal double itself in 22 yr. 2 mo. 20 da.?

CASE IV

178. To find the principal when the interest, time, and rate are given.

What principal will in 1 yr. 7 mo. 6 da. at 6% yield $36 interest?

	Interest	Time (yr. mo. da.)	Rate
OPERATION.	$36	1 7 6	6%

12
2)$.192$
$.096)\$36.000(\$375$
288
720
672
480
480

EXPLANATION. The interest of $1 for 1 yr. 7 mo. 6 da. at 6% is $0.096. It would require as many dollars to produce $36 interest as $0.096 is contained times in $36, = $375.

RULE. *Divide the given interest by the interest of* $1 *for the given time at the given rate.*

Find the principal in the following:

	Interest	Time (yr. mo. da.)	Rate		Interest	Time (yr. mo. da.)	Rate
1.	$7.40	1 4 0	3%	9.	$117	3 1 15	4%
2.	$20.25	2 3 0	4%	10.	$170	3 6 20	4½%
3.	$23.70	1 2 12	5%	11.	$249.40	4 3 18	5%
4.	$39.16	1 5 18	6%	12.	$168.30	2 4 24	5½%
5.	$88.20	2 4 24	7%	13.	$225	2 9 10	6%
6.	$134.40	2 7 15	8%	14.	$159.60	1 7 6	7%
7.	$165	2 5 10	9%	15.	$248.80	2 1 18	7½%
8.	$275.28	3 1 6	10%	16.	$374.85	3 4 15	6⅔%

	Interest	Time yr.	mo.	da.	Rate		Interest	Time yr.	mo.	da.	Rate
17.	$302.40	2	9	18	$6\frac{1}{4}\%$	24.	$290.25	2	8	12	4%
18.	$656.25	4	4	15	8%	25.	$450	4	9	18	$3\frac{1}{3}\%$
19.	$528	3	3	18	$8\frac{1}{3}\%$	26.	$1410	6	4	24	$7\frac{1}{2}\%$
20.	$729	5	7	15	6%	27.	$179.20	2	1	6	$8\frac{1}{3}\%$
21.	$560.70	3	7	6	7%	28.	$304	3	11	15	6%
22.	$607.60	4	10	24	5%	29.	$153.90	1	10	24	5%
23.	$184.50	1	7	6	$4\frac{1}{2}\%$	30.	$500	4	3	6	$7\frac{1}{2}\%$

CASE V

179. To find the principal or the interest when the amount, time, and rate are given.

The amount of a certain sum of money for 1 yr. 7 mo. 6 da. at 5% was $675. What were the principal and the interest?

	Amount	Time yr.	mo.	da.	Rate
OPERATION.	$675	1	7	6	5%

12
12)1.92
.016
5
.08ø
1.00
1.08)$675.00($625 = principal.
648
270
216
540
540

$675
625
$50 = interest.

EXPLANATION. The amount of $1 for 1 yr. 7 mo. 6 da. at 5% is $1.08. It would require as many dollars to amount to $675 as $1.08 are contained times in $675, = $625. The *interest* is $675 − $625 = $50.

RULE. 1. *Divide the given amount by the amount of* $1 *for the given time at the given rate. The quotient will be the principal.*

2. *Subtract the principal from the amount. The remainder will be the interest.*

Find the principal in the following:

	Amount	Time (yr. mo. da.)	Rate		Amount	Time (yr. mo. da.)	Rate
1.	$79.50	2 0 0	3%	11.	$1331.52	3 1 10	4½%
2.	$192.60	1 9 0	4%	12.	$1462.50	3 4 24	5%
3.	$281.25	2 6 0	5%	13.	$1689.32	4 3 6	6%
4.	$348.40	1 2 12	6%	14.	$1908.65	4 2 12	7%
5.	$510.60	1 10 15	8%	15.	$2069.10	4 9 18	6⅔%
6.	$725	1 9 10	9%	16.	$1890	1 7 6	7½%
7.	$930	2 4 24	10%	17.	$2118.20	2 4 15	8%
8.	$931	2 1 18	3%	18.	$2190.20	2 1 6	8⅓%
9.	$1102.79	2 7 15	4%	19.	$2694.60	3 10 20	9%
10.	$1148	3 7 6	3⅓%	20.	$2722.50	3 2 12	10%

Find the interest:

	Amount	Time (yr. mo. da.)	Rate		Amount	Time (yr. mo. da.)	Rate
21.	$465.75	0 10 15	4%	26.	$1128	2 7 15	6⅔%
22.	$547.05	0 11 6	4½%	27.	$1460	2 4 24	7%
23.	$688.50	1 7 6	5%	28.	$1732.50	4 3 6	7½%
24.	$780	1 4 20	6%	29.	$1805.55	3 1 15	8%
25.	$1030.40	2 4 24	6¼%	30.	$1940.04	3 3 18	8⅓%

REVIEW OF SIMPLE INTEREST

180. **1.** Find the amount of $720, from Oct. 27, 1895, to Jan. 30, 1899, at $3\frac{1}{3}\%$.

2. The amount of $444.40 for a certain time at 4% was $499.95. How long was it loaned?

3. I borrowed $450 Sept. 15, 1896, and on July 3, 1898, I paid the amount due, which was $506.70. What was the per cent?

4. The interest of a certain sum from March 7, 1895, to August 1, 1897, at $7\frac{1}{2}\%$, was $99.99. What was the principal?

5. The amount of a loan at $6\frac{2}{3}\%$ from Sept. 29, 1896, to May 5, 1900, was $232.50. What was the interest?

6. How much should be paid for the use of $937.50, from Nov. 25, 1894, to July 1, 1898, at $6\frac{2}{3}\%$?

7. A note of $320 was dated April 17, 1899. If the interest at $6\frac{1}{4}\%$ was $57.50, what was the date of payment?

8. $346.50 was borrowed Dec. 4, 1895, and on Feb. 24, 1898, the amount, which was $381.15, was paid. What was the rate?

9. A trader borrowed a certain sum for 3 yr. 4 mo. 15 da. at 4%. If he paid $43.74 interest, how much did he borrow?

10. I lent Mr. Smith some money on May 23, 1899, at 5%, and on Aug. 5, 1902, he paid the amount due, which was $565.50. What was the interest?

11. How much interest would be due on a note of $1638.40, from Dec. 29, 1897, to April 17, 1901, at $7\frac{1}{2}\%$?

12. I loaned $555 Sept. 27, 1895, at 7%, and received $720.76 on settlement. When was it paid?

13. The amount from Dec. 23, 1898, to June 11, 1900, at 3%, was $287.10, of which $12.10 was interest. What per cent was charged?

14. The interest for 3 yr. 20 da., at 6%, was $309.43. What was the principal?

15. The amount of $999, at $6\frac{2}{3}$%, was $1216.56. What was the time?

16. The interest of $819.20 for 3 yr. 1 mo. 15 da. was $128. What was the rate?

17. What principal will, in 4 yr. 4 mo. 8 da. at $7\frac{1}{2}$%, produce $551.25 interest?

18. What principal will, in 4 yr. 9 mo. 18 da. at $6\frac{2}{3}$%, amount to the same as $371.25 in 2 yr. 4 mo. 24 da., at $8\frac{1}{3}$%?

19. At the age of 39 I insured my life for $6000 on the endowment plan, the policy to be payable at the age of 50. Reckoning interest at 5% on the premiums paid, how much shall I lose if I live to receive the endowment, allowing $105 annual premium per $1000 of insurance?

PARTIAL PAYMENTS

181. 1. A *partial payment* is a payment of part of the amount due upon a note.

2. An *indorsement* is an acknowledgment of the receipt of a partial payment, written upon the back of the note. It specifies the date and the amount of the payment.

3. The following rule for computing interest upon notes and other obligations on which partial payments have been made has been adopted by the Supreme Court of the United States, and by almost all the states of the Union.

UNITED STATES RULE

When partial payments have been made, apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.

If the payment is less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal; and interest is to be computed on the balance as aforesaid. — KENT.

REMARK. This rule is founded upon the decision of Chancellor Kent. Its principle is, *that neither interest nor payment shall draw interest.*

$1250. PITTSBURG, PA., Jan. 25, 1892.

On demand, I promise to pay Sidney Smith, or order, Twelve Hundred and Fifty Dollars, with interest at 6%, for value received.

GEO. R. ELLIS.

Partial payments were indorsed upon the above note as follows: July 1, 1893, $477.50; July 25, 1894, $56.32; March 10, 1896, $75; January 28, 1897, $487.44. How much was due August 18, 1897?

OPERATION.

$1250		$477.50	1893	7	1
.086			1892	1	25
7500		.086	1	5	6
10000					
$107.500					
1250		$56.32	1894	7	25
$1357.50ø			1893	7	1
477.50		.064	1	0	24
$880					
.064					
3520		$75	1896	3	10
5280			1894	7	25
$56.32ø		.0975	1	7	15
880					
$936.32		$487.44	1897	1	28
56.32			1896	3	10
$880.		.053		10	18
.0975					
4400					
6160		?	1897	8	18
7920			1897	1	28
$85.80ø		.03⅓		6	20

$75

$880	
.053	
2640	
4400	
$46.64ø	
85.80	
880	
$487.44	$1012.44
$562.44	562.44
	$450.
	.03⅓
	1350
	150
	$15.00
	450
	$465.00

EXPLANATION. Find the difference of time between each two consecutive dates, as above, and find the interest of $1 for each period of time. Write each decimal multiplier on the left, opposite its corresponding period of time, and each payment opposite its corresponding date. The interest of $1250 for 1 yr. 5 mo. 6 da. at 6% is $107.50; the payment is $477.50. As the payment *exceeds* the interest, subtract the payment from the amount. The remainder, $880, is the *second principal*. The interest of $880 for 1 yr. 24 da. is $56.32; the payment, $56.32. As the payment *equals* the interest, subtract the payment from the amount. The remainder, $880, is the *third principal*. The interest of $880 for 1 yr. 7 mo. 15 da. is $85.80;

the payment, $75. As the payment is *less* than the interest, compute the interest for the next period of time. The interest of $880, the *fourth principal,* for 10 mo. 18 da. is $46.64; the payment, $487.44. As the sum of the payments ($75 + $487.44) *exceeds* the sum of the interests due ($85.80 + $46.64), subtract these payments from the amount. The remainder, $450, is the *fifth principal.* The interest of $450 for 6 mo. 20 da. is $15, and the amount due Aug. 18, 1897, is $465.

1. A note of $400 was dated Jan. 2, 1903. Interest 6%. Indorsed Jan. 2, 1904, $124; Jan. 2, 1905, $168. How much was due July 2, 1905?

2. A note of $450 was dated May 13, 1904. Interest 4%. Indorsed, Aug. 13, 1905, $172.50; Feb. 13, 1907, $193. How much was due Nov. 13, 1908?

3. A note of $480 was dated Mar. 15, 1905. Interest 5%. Indorsed, Oct. 21, 1905, $134.40; Sept. 15, 1906, $16.20. How much was due July 15, 1907?

4. A note of $525 was dated Sept. 19, 1902. Interest 7%. Indorsed, Dec. 1, 1903, $149.10; Jan. 7, 1905, $25. How much was due June 1, 1907?

5. A note of $687.50 was dated July 6, 1904. Interest $7\frac{1}{2}$%. Indorsed, Feb. 12, 1906, $182.50; Mar. 6, 1907, $147; Sept. 18, 1907, $119.50. How much was due Nov. 6, 1909?

6. A note of $725 was dated June 17, 1905. Interest 8%. Indorsed, May 11, 1906, $50; Dec. 26, 1907, $96.45; Feb. 11, 1909, $50; June 26, 1910, $240. How much was due Aug. 11, 1913?

7. A note of $480 was dated June 12, 1903. Interest 6%. Indorsed, Oct. 18, 1904, $93.88; Oct. 30, 1905, $91.35; Sept. 24, 1906, $129.44. How much was due Dec. 30, 1907?

8. A note of $1250 was dated Jan. 24, 1904. Interest 7½%. Indorsed, June 18, 1905, $501.25; Aug. 30, 1906, $209.20; June 18, 1908, $476.25. How much was due May 8, 1909?

9. A note of $640 was dated May 13, 1907. Interest 4½%. Indorsed, Oct. 7, 1908, $130.32; Sept. 13, 1909, $20; July 1, 1911, $247.65. How much was due Sept. 13, 1912?

10. A note of $1050 was dated Jan. 1, 1906. Interest 5%. Indorsed, April 19, 1907, $318.25; Jan. 1, 1909, $328; Nov. 25, 1910, $50. How much was due Sept. 13, 1911?

11. A note of $625 was dated May 7, 1910. Interest 6%. Indorsed, Nov. 19, 1911, $232.50; Aug. 1, 1913, $120.90; Aug. 25, 1914, $199. How much was due Jan. 1, 1915?

12. A note of $555 was dated May 11, 1904. Interest 7½%. Indorsed, July 23, 1905, $117.45; Aug. 17, 1906, $64; June 5, 1907, $18.50. How much was due Oct. 29, 1909?

13. A note of $956.25 was dated April 9, 1905. Interest 3⅓%. Indorsed, June 21, 1906, $144.50; Oct. 27, 1907, $175.75; Aug. 15, 1909, $42.75; Mar. 21, 1910, $389.25. How much was due Aug. 15, 1912?

14. A note of $666 was dated Nov. 14, 1904. Interest 4%. Indorsed, Sept. 29, 1905, $163.71; May 14, 1906, $69.99; Dec. 20, 1907, $30; Mar. 2, 1909, $103.75. How much was due Jan. 26, 1910?

15. A note of $787.50 was dated Jan. 1, 1909. Interest 4½%. Indorsed, Oct. 11, 1910, $50; Sept. 1,

1911, $30; June 19, 1912, $25; Nov. 29, 1912, $15. How much was due Feb. 19, 1915?

16. A note of $1280 was dated Aug. 7, 1908. Interest 5%. Indorsed, April 1, 1909, $361.60; Nov. 16, 1909, $350; April 1, 1911, $324; May 16, 1912, $286.50. How much was due Sept. 16, 1913?

17. A note of $1562.50 was dated July 23, 1909. Interest 6%. Indorsed, Aug. 17, 1910, $375; Oct. 29, 1911, $442.70; Aug. 17, 1912, $420; July 23, 1913, $156.50. How much was due Feb. 29, 1904?

18. A note of $1687.50 was dated July 6, 1910. Interest $6\frac{2}{3}\%$. Indorsed, May 30, 1911, $100; Feb. 12, 1912, $605; Mar. 18, 1913, $60; July 6, 1914, $407.25. How much was due Feb. 12, 1915?

19. A note of $1750 was dated Oct. 11, 1903. Interest 7%. Indorsed, Sept. 5, 1904, $110.25; May 17, 1905, $50; Sept. 5, 1906, $195; Oct. 11, 1907, $884.75. How much was due April 2, 1908?

20. A note of $1920 was dated Mar. 17, 1907. Interest $7\frac{1}{2}\%$. Indorsed, Nov. 5, 1907, $331.20; Oct. 23, 1908, $551.80; Nov. 17, 1909, $510; Aug. 23, 1910, $408.30. How much was due Sept. 5, 1911?

21. A note of $2160 was dated Jan. 1, 1900. Interest 8%. Indorsed, Dec. 8, 1900, $431.76; Jan. 13, 1902, $615.90; Jan. 24, 1903, $478.72; Nov. 20, 1903, $251.04. How much was due April 10, 1905?

22. A note of $720 was dated May 5, 1902. Interest $6\frac{1}{4}\%$. Indorsed, May 9, 1903, $40; Feb. 17, 1904, $30; Mar. 28, 1905, $40; March 11, 1906, $30; April 1, 1907, $125.75. How much was due April 25, 1908?

182. Business men usually settle notes and interest accounts payable in *one year or less* by the following rule:

MERCANTILE RULE

1. *Find the amount of the principal to the date of settlement.*

2. *Find the amount of each payment from the date of payment to the date of settlement.*

3. *Subtract the sum of the amounts of the payments from the amount of the principal.*

1. A note of $880 was dated Jan. 1, 1898. Interest 6%. Indorsed, April 1, 1898, $220; July 1, 1898, $220; Oct. 1, 1898, $220. What was due Jan. 1, 1899?

2. I borrowed $768.80, May 17, 1899, at 5%. I paid, June 23, 1899, $150; Sept. 5, 1899, $160; Nov. 17, 1899, $180; Jan. 29, 1900, $190. How much was due May 17, 1900?

3. A note of $660 was dated Aug. 15, 1901. Interest 7½%. Indorsed, Oct. 27, 1901, $100; Jan. 9, 1902, $100; March 21, 1902, $100; June 3, 1902, $100. How much was due Aug. 15, 1902?

4. A note of $1080.50 was dated July 1, 1903. Interest 8%. Indorsed, Aug. 16, 1903, $367; Nov. 16, 1903, $287.60; Feb. 16, 1904, $105; May 16, 1904, $261.50. How much was due July 1, 1904?

5. A note of $975 was dated Nov. 16, 1898. Interest 4%. Indorsed, Jan. 28, 1899, $175; March 4, 1899, $225; May 7, 1899, $150; July 1, 1899, $120; Sept. 4, 1899, $50. How much was due Nov. 16, 1899?

DISCOUNT

DEFINITIONS

183. *Discount* is a deduction made for the payment of money before it is due. There are two kinds, *bank discount* and *true discount.*

BANK DISCOUNT

184. 1. A *bank* is an institution chartered under the law, for the purpose of receiving and lending money, and issuing notes or bills which circulate as money.

2. *Business notes* are of two kinds, *negotiable* and *non-negotiable.*

3. A negotiable note is one that can be sold or transferred. It is made payable to the bearer, or to the order of the payee.

4. A non-negotiable note is made payable only to the person or persons named in the note.

5. A note is due and payable at the end of the time specified.

REMARK. In some states a note *matures*, or becomes *legally due* three days after the specified time. These are termed *days of grace*. In the accompanying problems *days of grace are not considered.*

6. *Bank discount* is the interest on a note for the number of days from the day of discount to the date upon which it is due.

7. The *proceeds* is the money received for the note when it is discounted.

REMARKS. 1. When the note *bears interest*, the discount is computed upon *the amount of the note at the time of its maturity.*

2. A note does not bear interest until after it is due, unless it is so stated.

3. Banks usually discount notes for short periods, seldom beyond three or four months.

CASE I

WHEN THE NOTE DOES NOT BEAR INTEREST

185. To find the bank discount and the proceeds when the face of the note, the time, and the rate are given.

$500. CINCINNATI, OHIO, Sept. 12, 1904.

Ninety days after date, I promise to pay Alfred Gates, or bearer, Five Hundred Dollars, for value received.

J. H. WILLIAMS.

Discounted Sept. 12, 1904, at 6%. Find the date when due, bank discount, and proceeds.

	Face	Time	Rate
	$500	90 da.	6%
OPERATION.	2).030		
	.015		
	500		
	$7.50	= bank dis.	

$500.00
7.50
$492.50 = proceeds.

EXPLANATION. The note is due in 90 days from Sept. 12, or Dec. 11, 1904. (Art. 69.) The interest of $1 for 90 days or 3 months at 6% is $0.015. The interest of $500 is $0.015 × 500 = $7.50, which is the bank discount, and the proceeds are $500 − $7.50 = $492.50.

RULE. 1. *Find the interest on the face of the note from the date of discount until the note is due. This will be the bank discount.*

2. *Subtract the discount from the face of the note. The remainder will be the proceeds.*

Find the date when due, bank discount, and proceeds of:

1. A note of $400, dated April 22, due in 33 days, discounted at 6%.

2. A note of $450, dated July 8, payable in 63 days, discounted at 4%.

3. A note of $540, dated March 19, payable in 93 days, discounted at 6%.

4. A note of $660, dated Feb. 10, 1904, due in 108 days, discounted at 7%.

5. A note of $725, dated Aug. 13, payable in 48 days, discounted at $7\frac{1}{2}$%.

6. A note of $360, dated Feb. 18, payable in 4 months, discounted at 6%.

7. A note of $555, dated Nov. 5, payable in 5 months, discounted at 4%.

8. A note of $1000, dated Sept. 7, due in 3 months, discounted at 5%.

9. A note of $750, dated May 26, payable in 2 months, discounted at 8%.

10. A note of $1638.40, dated Jan. 4, due in 100 days, discounted at $4\frac{1}{2}$%.

11. A grain dealer bought 1280 bu. of wheat at 0.87\frac{1}{2}$ a bushel, giving a note at 63 days in payment.

This note was immediately discounted at 10 %. What were the proceeds?

12. A merchant bought 240 bbl. of pork at $10.87½ a barrel. He sold it at $12.50 a barrel, taking in payment a 48-day note, which he discounted the same day at 7½ %. Find his gain on the transaction.

13. A cattle dealer sold 64 cows at $31.25 each, and received in payment a 90-day note, which he forthwith discounted at 7 %. How much did he gain if the cows cost him $25 apiece?

14. A contractor bought 25 mules at $45 each, but was obliged to sell them at $37.50 apiece, receiving in payment a 72-day note which he at once discounted at 6 %. What was his loss on the transaction?

15. A speculator bought 160 bbl. of beef at $11.25 a barrel. He sold it at $13.50 a barrel, taking in payment a note at 100 days, which he immediately discounted at 8 %. Find his gain on the transaction.

Find the bank discount and proceeds of:

16. A note of $550, dated June 16, 1903, due in 5 months, and discounted July 31, 1903, at 7 %.

17. A note of $1050, dated June 12, 1905, payable in 6 months, and discounted Aug. 14, 1905, at 7½ %.

18. A note of $312.80, dated March 10, 1906, payable in 7 months, and discounted July 27, 1906, at 6 %.

19. A note of $487.50, dated July 13, 1903, due in 10 months, and discounted Feb. 7, 1904, at 4½ %.

20. A note of $575, dated Feb. 21, 1907, due in 90 days, and discounted March 2, 1907, at 8 %.

21. A note of $720, dated Oct. 15, 1908, payable in 9 months, and discounted April 16, 1909, at 7%.

22. $666. BUFFALO, N.Y., June 7, 1904.

Six months after date, I promise to pay C. O. Edwards, or order, Six Hundred Sixty-six Dollars, for value received. WM. M. GRANDIN.

Discounted Aug. 21, 1904, at 5%. Find the proceeds.

23. $819.20. SPRINGFIELD, ILL., April 29, 1907.

One year after date, I promise to pay L. M. Brown, or order, Eight Hundred Nineteen $\frac{20}{100}$ Dollars, for value received. ROBERT MORRIS.

Discounted Dec. 31, 1907, at $7\frac{1}{2}$%. Find the proceeds.

24. $625.60. CINCINNATI, OHIO, Jan. 22, 1906.

Eight months after date, I promise to pay to the order of Israel Ludlow, Six Hundred Twenty-five $\frac{60}{100}$ Dollars, for value received. HENRY WOOD.

Discounted July 12, 1906, at $6\frac{1}{4}$%. Find the proceeds.

25. $972.80. PITTSBURG, PA., Aug. 28, 1909.

Five months after date, I promise to pay R. L. Murray, or bearer, Nine Hundred Seventy-two $\frac{80}{100}$ Dollars, for value received. JAMES DECAMP.

Discounted Sept. 15, 1909, at 10%. Find the proceeds.

CASE II

186. WHEN THE NOTE BEARS INTEREST

$1000. CLEVELAND, OHIO, Feb. 12, 1907.

Eight months after date, I promise to pay Geo. W. Stafford, or order, One Thousand Dollars, with interest at $7\frac{1}{2}$%, for value received. CHAS. MITCHELL.

Discounted July 8, 1907, at 6%. Find bank discount and proceeds.

Face	Time	Rate
$1000	8 mo.	7½%

OPERATION. 7½% = ⅛ of 12%.

8)$.08

.01

5

.05

1000

$50.00 = interest.

1000

$1050 = amount.

2)$.032 .016

.016 $16.80 = bank dis.

$1050.00

16.80

$1033.20 = proceeds.

EXPLANATION. The note is due Oct. 12, 1907. The amount of $1000 for 8 months at 7½% is $1050. The exact time of discount, from July 8 to Oct. 12, is 96 days, or 3 mo. 6 da. The bank discount of $1050 for 3 mo. 6 da. at 6% is $16.80. The proceeds are $1050 − $16.80 = $1033.20.

RULE. 1. *Find the amount of the note from its date until it is due.*

2. *Find the bank discount and proceeds of this amount.*

Find the bank discount and proceeds of:

1. A note of $750, dated April 13, 1905, payable in 5 months, bearing interest at 8%, and discounted June 15, 1905, at 8%.

2. A note of $720, dated June 24, 1896, due in 10 months, bearing interest at 10%, and discounted Jan. 14, 1906, at 6%.

3. A note of $625, dated Jan. 7, 1907, due in 9 months, bearing interest at 8%, and discounted June 21, 1907, at 6⅔%.

4. A note of $1000, dated April 15, 1904, due in 8 months, bearing interest at 6%, and discounted Oct. 4, 1904, at 7½%.

5. A note of $1500, dated May 8, 1908, due in one year, bearing interest at 6⅔%, and discounted Feb. 7, 1909, at 10%.

6. A note of $1280, dated Nov. 3, 1903, due Sept. 18, 1905, with interest at 8%, and discounted June 5, 1905, at 6%.

7. A note of $750, dated March 19, 1907, payable July 19, 1908, with interest at 7½%, and discounted April 2, 1908, at 10%.

8. A note of $1250, dated April 7, 1902, payable Nov. 13, 1903, with interest at 5%, and discounted Aug. 6, 1903, at 8%.

9. A note of $1125, dated May 10, 1906, due June 20, 1907, with interest at 6%, and discounted March 22, 1907, at 7½%.

10. A note of $1875, dated Aug. 6, 1903, due May 24, 1904, with interest at 9%, and discounted Feb. 6, 1904, at 8⅓%.

11. $1200. INDIANAPOLIS, IND., Oct. 18, 1909.

One year after date, I promise to pay to the order of Charles Gilbert, Twelve Hundred Dollars, with interest at 6¼%, for value received.

L. A. KNIGHT.

Discounted July 10, 1909, at 6%. Find the proceeds.

12. $1000. BOSTON, MASS., Dec. 22, 1905.

Nine months after date, I promise to pay J. B. King, or order, One Thousand Dollars, with interest at 8%, for value received.

R. M. WARD.

Discounted May 25, 1906, at $7\frac{1}{2}\%$. Find the proceeds.

13. $921.60. CHICAGO, ILL., March 14, 1910.

Ten months after date, I promise to pay Wm. Hayes, or bearer, Nine Hundred Twenty-One $\frac{60}{100}$ Dollars, with interest at 5%, for value received.

SAMUEL ADAMS.

Discounted Sept. 1, 1910, at 6%. Find the proceeds.

14. $960. BROOKLYN, N.Y., Aug. 5, 1907.

Nine months after date, I promise to pay to the order of Francis Arnold, Nine Hundred Sixty Dollars, with interest at $8\frac{1}{3}\%$, for value received.

JAMES MOORE.

Discounted Feb. 14, 1908, at $6\frac{2}{3}\%$. Find the proceeds.

15. $937.50. PHILADELPHIA, PA., Sept. 11, 1898.

On the seventeenth day of April, 1900, we promise to pay to the order of R. M. Stone, Nine Hundred Thirty-Seven $\frac{50}{100}$ Dollars, with interest at 5%, for value received.

JOHN RANDOLPH.

C. S. MARSHALL.

Discounted Jan. 17, 1900, at 8%. Find the proceeds.

CASE III

187. To find the face of the note, when the proceeds, time, and rate are given.

I wish to borrow $246.50 from a bank. For what sum, due 63 days hence, must I give a note, if the rate of discount is 8%?

Proceeds	Time	Rate
$246.50	63 da.	8%

OPERATION.

$$3)\overline{.021}$$
$$\overline{.007}$$
$$\overline{2}$$
$$\overline{.014}$$

$$1.000 - .014 = .986$$

$$.986)\overline{\$246.50}\ \$250$$

EXPLANATION. The bank discount of $1 for 63 days, at 8%, is $0.014; the proceeds of $1 are $1 − $0.014 = $0.986. The face of the note must be $246.50 ÷ .986 = $250.

RULE. *Divide the proceeds of the note by the proceeds of* $1 *for the given time at the specified rate. The quotient will be the face.*

REMARK. To find the *discount*, subtract the proceeds from the face of the note, or *from its amount when it bears interest.*

1. For what sum due 93 days hence must a note be given, that, when discounted at 6%, it will yield $393.80 as proceeds?

2. The proceeds of a note due in 3 months and discounted at 7% were $628.80. What was the face of the note?

3. I wish to borrow $472.80 from a bank. What must be the face of a note at 108 days, discounted at 5%, that will yield that sum?

4. For what sum must a 5-months note be drawn to realize $798.72 as proceeds, when discounted at 6%?

5. The proceeds of a 48-day note, discounted at 9%, were $555.75. What was the face of the note?

6. I desire to use $907.50 for 144 days. For what sum must a note be given to secure that amount if the rate of discount is 8%?

7. A bank paid $709.20 for a 4-months note, after discounting it at 4½%. What was the face of the note?

8. A note at 72 days was discounted, and the proceeds were $642.51. If the rate of discount was 5%, what was the face of the note?

9. The proceeds of a 100-day note discounted at 4½%, were $404.48. What was the discount?

10. A note due May 24, 1908, was discounted Jan. 25, 1908, at 7½%, and the proceeds were $1248. What was the discount?

11. A merchant bought a bill of goods amounting to $1303.50, giving in payment a note at 75 days, which was immediately discounted at 6%. If the note included the discount, what was its face?

12. I sold a lot of wheat at $1.20 a bushel, receiving in payment a note at 63 days, which was forthwith discounted at 6%. The proceeds of the note were $1484.25. How many bushels of wheat were sold?

13. A contractor bought beef for $12.50 a barrel. He sold it at 7½ ct. a pound, receiving in payment a note at 4 months, which he at once discounted at 6%. The proceeds of the note were $3763.20. Find the number of barrels sold, and his net profit.

TRUE DISCOUNT

188. 1. The *present worth* of a debt payable at a future time without interest is that sum of money which, at a specified rate for the given time, will *amount to the debt* when it becomes due.

2. *True discount* is the difference between a debt and its present worth; *it is the interest on the present worth from the time the debt is paid until it becomes due.*

189. To find the present worth and discount when the face of the note, the time, and the rate are given.

Find the present worth and the true discount of a debt of $507.50, due in 2 yr. 8 mo., money being worth 6%.

OPERATION

Debt	Time	Rate
$507.50	2 yr. 8 mo.	6%

12
2)32
.16
1.00
1.16)$507.50($437.50 = pres. worth.
464
435
348
870
812
580
580

$507.50
437.50
$70.00 = discount.

EXPLANATION. The amount of $1 for 2 yr. 8 mo at 6% is $1.16. The present worth of $507.50 is $507.50 ÷ 1.16 = $437.50, and the true discount is $507.50 − $437.50 = $70.

RULE. 1. *Divide the given debt by the amount of* $1 *for the given time at the specified rate. The quotient will be the present worth.*

2. *Subtract the present worth from the debt. The remainder will be the discount.*

REMARKS. 1. *When the note bears interest*, find the present worth of its *amount*.

2. When payments *without interest* are to be made at different periods, find the present worth of each payment. The sum of these will be the present worth of all.

3. Comparing *discount* with *interest*, it is evident that the *present worth* corresponds to the *principal*, the *debt* to the *amount*, and the *discount* to the *interest*. Therefore, the problems are usually solved in the same manner as those of Case V in Interest (p. 309).

1. Find the present worth and discount of a debt of $517.50, due in 2 years 6 months without interest, money being worth 6%.

2. Of a debt of $420, due in 2 yr. 4 mo. 24 da., money worth 5%.

3. Of a note of $606.30, due in 1 yr. 10 mo. 15 da., money worth 4%.

4. Of a note of $641, due in 3 yr. 1 mo. 15 da., money worth 8%.

5. What is the present worth of a debt of $703.75, due in 1 yr. 9 mo. 18 da., money worth 7%?

6. What is the true discount of a note of $816.75, due in 1 yr. 9 mo. 10 da., without interest, if money is worth $4\frac{1}{2}$%?

7. Find the true discount of a debt of $942.50, payable in 2 yr. 1 mo. 18 da., money being worth $7\frac{1}{2}\%$.

8. What is the present worth of a note of $852.50, due in 3 yr. 7 mo. 6 da., discounted at $6\frac{2}{3}\%$?

9. Find the present worth of a note of $962.56, payable in 2 yr. 1 mo. 6 da., discounted at $8\frac{1}{3}\%$.

10. What is the true discount of a note of $1181.25, due 3 yr. 5 mo. 18 da. hence, money being worth $7\frac{1}{2}\%$?

11. Find the difference between the interest and the true discount of $825, for 2 yr. 2 mo. 20 da. at $4\frac{1}{2}\%$.

12. What is the difference between the interest and the true discount of $725 for 3 yr. 2 mo. 12 da. at 5%?

13. How much more would be the interest than the discount of $1812.50, for 2 yr. 4 mo. 24 da. at $6\frac{2}{3}\%$?

14. I bought a bill of goods amounting to $656, and was offered $6\frac{1}{4}\%$ off for cash, or 4 mo. time without interest. If money was worth $7\frac{1}{2}\%$, how much did I gain by paying cash?

15. A merchant bought the stock in a store for $3250, and was offered 4% off for cash, or 6 mo. time without interest. How much did he lose by not paying cash, money being worth 8%?

16. Which is better, and how much, to buy a farm for $2580 at $8\frac{1}{3}\%$ discount for cash, or on 9 mo. time without interest, when money is worth 10%?

17. Find the present worth of a debt of $1474.20, of which $\frac{1}{3}$ is payable in 1 yr., $\frac{1}{3}$ in 2 yr., and $\frac{1}{3}$ in 3 yr., money being worth 4%.

18. What is the present worth of a debt of $741.75, $\frac{1}{3}$ to be paid in cash, $\frac{1}{3}$ in 1 year, and the remainder in 2 years, money worth $7\frac{1}{2}\%$?

19. Find the present worth of a debt of $956.34, $\frac{1}{3}$ being due in 9 mo., $\frac{1}{3}$ in 18 mo., and the remainder in 27 mo., discounted at $6\frac{2}{3}\%$.

20. Find the cash value of a note of $2125.20, of which $\frac{1}{4}$ is to be paid in 6, 12, 18, and 24 mo. respectively, and discounted at 10%.

21. What is the present value of a debt of $786.24, $\frac{1}{4}$ payable in cash, and $\frac{1}{4}$ in 8, 16, and 24 mo. respectively, money worth 6%?

22. A note of $1125, dated May 12, 1900, due Oct. 6, 1902, with interest at 5%, was sold for its present worth at 5%, Oct. 6, 1901. What were the present worth and the discount at the latter date?

23. A note of $2430, dated Sept. 25, 1899, due July 5, 1902, with interest at $4\frac{1}{2}\%$, was cancelled by payment of its present worth at 9%, Aug. 15, 1901. What were the present worth and the discount?

24. The true discount of a note dated Jan. 9, 1896, due Aug. 15, 1899, was $135. What would have been the true discount of the same note, Sept. 30, 1897, if money was worth $6\frac{2}{3}\%$?

25. On Sept. 4, 1896, the present worth of a note due Nov. 16, 1899, and bearing interest at $6\frac{1}{4}\%$, was $843.75. What would have been the true discount at $4\frac{1}{2}\%$ of the same note, Feb. 6, 1898?

EXCHANGE

DEFINITIONS

190. 1. *Exchange* is the method of making payments in distant places by means of drafts, or bills of exchange.

2. A *draft*, or *bill of exchange*, is a written order from one person to another for a specified sum of money.

REMARK. The person who signs the bill is called the *drawer*; the person to whom it is addressed is called the *drawee*; and the person to whom the money is to be paid, the *payee*.

3. A *sight* draft is one to be paid upon its presentation.

4. A *time* draft is one to be paid at a specified time after its date or presentation.

REMARK. Bills of exchange are negotiable the same as promissory notes. In some states three days of grace are granted on time drafts.

5. The *acceptance* of a draft is the drawee's agreement to pay it when it becomes due. This agreement is made by his writing "accepted" across the face of the bill, together with his name and the date. The bill then becomes an acceptance.

6. Exchange is of two kinds, *domestic*, or *inland*, and *foreign*.

DOMESTIC EXCHANGE

191. A domestic, or inland, bill is payable in the country in which it is drawn.

192. To find the cost or the face of a domestic bill of exchange.

(*a*) Find the cost of a draft on Cleveland for $750, payable in 33 days, exchange being at $\frac{3}{4}\%$ premium, and interest 6%.

OPERATION

$\$1 + \$.00\frac{3}{4} = \$1.0075$

$\$1 \times .0055 = .0055$

$\$1.002\emptyset$

750

50100

7014

$\$751.50\emptyset$

EXPLANATION. As exchange is at $\frac{3}{4}\%$ premium, the cost of $1 of the draft at sight would be $\$1.00\frac{3}{4}$. But since it is a time draft, it is entitled to interest for 33 days. The interest of $1 for 33 days at 6% is $0.0055; this, subtracted from $\$1.0075 = \$1.002$.

$\$1.002 \times 750 = \$751.50$, which is the cost of the draft.

RULE. *Multiply the face of the draft by the cost of $1 of exchange. The product will be the cost of the draft.*

(*b*) The cost of a draft on Brooklyn, due in 63 days, was $439.20. If exchange was at 1% discount, and interest 8%, what was the face of the draft?

OPERATION. $\$1 - \$.01 = \$0.99$

$\$1 \times .014 = .014$

$\$0.976)\$439.200(\$450$

3904

4880

4880

0

EXPLANATION. As exchange is at 1% discount, the cost of $1 of the draft at sight would be $0.99. But since it is a time draft, it is entitled to interest for 63 days. The interest of $1 for 63 days at 8% is $0.014; this, subtracted from $0.99 = $0.976. $439.20 ÷ .976 = $450, which is the face of the draft.

RULE. *Divide the cost of the draft by the cost of* $1 *of exchange. The quotient will be the face of the draft.*

1. Find the cost of a sight draft on Philadelphia for $720, at $1\frac{1}{2}\%$ premium.

2. Exchange being at $\frac{3}{4}\%$ discount, what must be paid for a sight draft on Louisville for $840?

3. What will be the cost of a sight draft on Minneapolis for $1000, if exchange is at $\frac{5}{8}\%$ premium?

4. What must be paid for a sight draft on Atlanta for $625, if exchange is at $\frac{1}{8}\%$ discount?

5. The cost of a sight draft on Baltimore was $645.60. If exchange was at $\frac{7}{8}\%$ premium, what was the face of the draft?

6. What was the face of a bill of exchange on Omaha that cost $842.35 at $\frac{9}{10}\%$ discount?

7. $976.80 was paid for a sight draft on Charleston at $1\frac{3}{4}\%$ premium. What was the face of the draft?

8. At $1\frac{3}{5}\%$ discount, $762.60 was charged for a sight draft on Memphis. For what amount was the draft drawn?

9. Find the cost of a 60-day draft on New Orleans for $750, exchange being at par, and interest 8%.

10. What must be paid for a bill of exchange on Galveston for $720, payable in 90 days, exchange at par, and interest 6%?

11. I paid $866.25 for a 48-day draft on St. Louis. If exchange was at par, and interest $7\frac{1}{2}\%$, what was the face of the draft?

12. The cost of a 36-day draft on Detroit was $928.70. If exchange was at 1% premium, and interest 6%, what was the amount of the draft?

13. A traveler bought a 45-day draft on San Francisco for $1080. How much did he pay for it, if exchange was at 1% discount, and interest 4%?

14. How large a draft on Boston could be bought for $1039.50, if the time were 54 days, exchange $\frac{1}{8}\%$ premium, and interest 8%?

15. A manufacturer in Minneapolis shipped 400 bbl. of flour to a merchant in Baltimore, receiving in payment a 50-day draft, which cost the sender $1556.10. If exchange was at $\frac{3}{8}\%$ premium, and interest $4\frac{1}{2}\%$, what was the price of the flour per barrel?

FOREIGN EXCHANGE

193. 1. A *foreign* bill of exchange is payable in a different country from the one in which it is drawn.

2. Foreign bills of exchange are usually drawn in sets of two or three *of the same date and tenor*, named respectively the *first*, *second*, and *third of exchange*. These are sent by different mails, in order to prevent loss or delay. When one is paid, the others are void.

3. Foreign bills of exchange are expressed in the money of the country on which they are drawn.

REMARK. The foreign exchange of the United States is principally with Great Britain, France, Germany, and Canada.

English Money

4 farthings = 1 penny . . . . *d*
12 pence = 1 shilling . . . *s.*
20 shillings = 1 pound . . . £.

Remark. The unit of English money is the *pound sterling.*
The unit of French money is the *franc*, marked fr.
The unit of German money is the *mark* of 100 *pfennige.*
Canadian money is expressed in *dollars* and *cents.*

4. The *par* of exchange is the value of the standard coin of one country compared with that of another.

Remark. The *commercial value* of foreign exchange varies, being sometimes above the par value, and sometimes below it. It is always quoted in gold.

The par value of the pound is $4.8665. Its commercial value varies from $4.83 to $4.90.

The par value of the franc is $0.193. Its commercial value is usually quoted at about 5.15 fr., equal to $1.

The par value of the mark is $0.238. The commercial quotations are always for four marks, and vary from $0.95 to $0.98.

194. To find the cost or the face of a foreign bill of exchange.

(*a*) What would be the cost in New York of a sight bill on Liverpool for £437 10*s.*, exchange being at $4.86?

OPERATION

£437.5
4.86
26250
35000
17500
$2126.250

Explanation. 10*s.* = £.5. As £1 costs $4.86, £437 5 will cost $4.86 × 437.5 = $2126.25.

(*b*) What must be the face of a bill on London that will cost $1240.25, exchange being at $4.84?

OPERATION

$4.84)$1240.25(£256.25
968 20
2722 5.00 *s.*
2420
3025
2904
1210
968
2420
2420

EXPLANATION. As £1 costs $4.84, it will require as many pounds to cost $1240.25 as $4.84 are contained times in $1240.25 = £256.25. £.25 = 5 *s.* Therefore the face of the bill must be £256 5 *s.*

1. Find the cost of a bill on London for £562 10 *s.*, exchange being at $4.86.

2. What must be paid for a bill on Glasgow for £409 12 *s.*, exchange being at $4.85?

3. How large a bill on Liverpool can be bought for $1512.50, exchange being at $4.84?

4. What was the face of a bill on Edinburgh that cost $2287.50 when exchange was at $4.88?

5. What will be the cost of a bill on Paris for 2472 francs, exchange being at 5.15 fr. to $1?

6. How large a bill on Marseilles will cost $1875 if exchange is at 5.16 fr.?

7. Find the face of a bill on Berlin that cost $1404 when exchange was at $0.975 for 4 marks.

8. Find the cost of a bill on Hamburg for 5280 marks when exchange is at $0.96.

RATIO AND PROPORTION

RATIO

195. 1. *Ratio* is the relation of one number to another of the *same kind.*

2. The ratio of two numbers is expressed by the quotient arising from the division of the first by the second.

Thus, the ratio of 10 to 5 is 2, because $10 \div 5 = 2$.

3. The *sign* of a ratio is a colon (:) written between the numbers.

Thus, 10 : 5 is read *the ratio of* 10 *to* 5.

REMARK. A ratio may also be expressed in the form of a fraction by writing the first number for the numerator and the second for the denominator. Thus, 10 : 5 is the same as $\frac{10}{5}$.

4. The *terms* of a ratio are the two numbers compared. The first term, or *dividend*, is called the *antecedent;* the second term, or *divisor*, is called the *consequent;* and the two together are called a *couplet.*

REMARK. A ratio can not exist between two numbers of *different* kinds, because *one can be no part of the other.*

5. Ratios are of two kinds, *simple* and *compound.*

6. A *simple* ratio is the ratio of two numbers; as, 12 : 3.

7. A *compound ratio* is the product of two or more simple ratios.

Thus, $\left\{\begin{matrix} 9:4 \\ 8:5 \\ 10:6 \end{matrix}\right. = \frac{9}{4} \times \frac{8}{5} \times \frac{10}{6}$.

196. To find the ratio when the antecedent and consequent are given.

(*a*) What is the ratio of \$250 to \$75?

OPERATION. \$250 ÷ \$75 = $3\frac{1}{3}$.

EXPLANATION. The antecedent is the dividend and the consequent the divisor. \$250 divided by \$75 = $3\frac{1}{3}$.

(*b*) What is the ratio of $\frac{4}{5}$ to $\frac{8}{9}$?

OPERATION. $\frac{4}{5} \div \frac{8}{9} = \frac{4}{5} \times \frac{9}{8} = \frac{9}{10}$.

(Explanation same as preceding.)

RULE. *Divide the antecedent by the consequent. The quotient will be the ratio.*

REMARK. When the terms are compound numbers they must be reduced to the same denomination.

What is the ratio of:

1. 117 to 13?
2. 343 to 49?
3. 196 to 441?
4. 253 to 368?
5. 650 to 117?
6. 150 to $9\frac{3}{8}$?
7. $68\frac{3}{4}$ to 55?
8. $8\frac{4}{5}$ to $12\frac{3}{8}$?
9. $34\frac{17}{28}$ to $35\frac{5}{8}$?
10. $88\frac{8}{9}$ to $41\frac{2}{3}$?
11. $18\frac{3}{4}$ to $234\frac{3}{8}$?
12. 70.3 to 38?
13. 22.4 to 25.6?
14. 58.24 to 9.1?
15. 118.4 to $12\frac{4}{5}$?
16. $3\frac{3}{10}$ to .0625?
17. 633.6 to $12\frac{3}{8}$?
18. 8.775 to 32.5?

19. $104\frac{1}{6}$ to $266\frac{2}{3}$?

20. 93.75 to $16\frac{2}{3}$?

21. 1.25 to 40?

22. 64 to .625?

23. $888\frac{8}{9}$ to $1066\frac{2}{3}$?

24. bu. qt. pt. 12 4 1 to pk. qt. pt. 1 2 1?

25. gal. pt. gi. 71 1 1 to qt. pt. gi. 2 1 3?

26. T. cwt. lb. 7 16 25 to cwt. lb. 6 25?

27. yd. ft. in. 3 2 3 to 1 rod?

28. da. hr. 4 12 to wk. da. 1 3?

29. A. sq. rd. 2 $17\frac{1}{2}$ to A. sq. rd. 2 40?

30. 1 ℔ to 7 ʒ 1 ʒ 1 ℈ 16 gr.?

197. To find the antecedent when the ratio and consequent are given.

The consequent is 6 and the ratio $8\frac{1}{3}$. Find the antecedent.

OPERATION. $6 \times 8\frac{1}{3} = 50$.

EXPLANATION. The consequent is the divisor and the ratio the quotient. Therefore the antecedent is 6 multiplied by $8\frac{1}{3} = 50$.

RULE. *Multiply the consequent by the ratio. The product will be the antecedent.*

Find the antecedent:

1. Con. 28, ratio 7.
2. Con. 39, ratio 15.
3. Con. 64, ratio $31\frac{1}{4}$.
4. Con. 43.75, ratio 12.8.
5. Con. 343, ratio $\frac{6}{7}$.
6. Con. $68\frac{3}{4}$, ratio $1\frac{9}{11}$.
7. Con. $\frac{93}{169}$, ratio $\frac{33}{62}$.
8. Con. .91, ratio $28\frac{4}{7}$.
9. Con. 2.7, ratio .37.
10. Con. 56.25, ratio $.333\frac{1}{3}$.
11. rd. yd. ft. Con. 2 2 1, ratio $7\frac{1}{2}$.
12. bu. pk. qt. Con. 15 2 4, ratio 64.

198. To find the consequent when the antecedent and ratio are given.

The antecedent is $62\frac{1}{2}$, the ratio 5. Find the consequent.

OPERATION. $62\frac{1}{2} \div 5 = \frac{\overset{25}{\cancel{125}}}{2} \times \frac{1}{\cancel{5}} = 12\frac{1}{2}$.

EXPLANATION. The antecedent is the dividend and the ratio the quotient. Therefore the consequent is $62\frac{1}{2}$ divided by $5 = 12\frac{1}{2}$.

RULE. *Divide the antecedent by the ratio. The quotient will be the consequent.*

Find the consequent:

1. Ant. 133, ratio 19.
2. Ant. 576, ratio 24.
3. Ant. 99.9, ratio 37.
4. Ant. $44\frac{4}{9}$, ratio $56\frac{1}{4}$.
5. Ant. 729, ratio .6.
6. Ant. $.83\frac{1}{3}$, ratio $6\frac{1}{4}$.
7. Ant. $\frac{45}{64}$, ratio $9\frac{3}{8}$.
8. Ant. 25, ratio .04.
9. Ant. .04, ratio 25.
10. Ant. 56, ratio 6.4.
11. Ant. 1728, ratio 180.
12. Ant. .375, ratio $\frac{15}{16}$.

199. To find the value of a compound ratio.

Find the value of the compound ratio $\begin{cases} 8:5. \\ 10:4. \end{cases}$

OPERATION. $\frac{\overset{2}{\cancel{8}}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{4}} = 4.$

EXPLANATION. The ratio of 8 to 5 is $\frac{8}{5}$, of 10 to 4 is $\frac{10}{4}$. Therefore $\frac{8}{5} \times \frac{10}{4} = 4$. But the same result is obtained by dividing the product of the antecedents by the product of the consequents, which reduces the ratio to its simplest form at once.

RULE. *Divide the product of the antecedents by the product of the consequents. The result will be the value of the compound ratio.*

Find the value of the following compound ratios:

1. 18 : 15.
   25 : 12.

2. 8 : 21.
   36 : 25.
   35 : 24.

3. 105 : 51.
   69 : 65.
   68 : 56.
   52 : 46.

4. 45 men : 35 men.
   21 days : 18 days.
   104 rods : 39 rods.

5. 76 horses : 63 horses.
   81 acres : 51 acres.
   49 days : 48 days.
   $68 : $133.

6. 75 tons : 96 tons.
   119 mi. : 37 mi.
   $111 : $153.
   64 da. : 35 da.

7. 9.1 : 87.
   144 : 10.5.
   1.16 : 1.43.
   99 : 192.

8. 93 oxen : 117 oxen.
   85 acres : 135 acres.
   54 days : 62 days.
   $455 : $340.

9. $\frac{16}{25} : \frac{28}{45}$.
   $\frac{27}{40} : \frac{32}{75}$.
   $\frac{64}{63} : \frac{54}{49}$.

10. $3\frac{3}{5} : 5\frac{5}{8}$.
    $12\frac{1}{2} : \frac{8}{15}$.
    $41\frac{2}{3} : 6\frac{1}{4}$.

SIMPLE PROPORTION

200. 1. A *proportion* is an equality of ratios.

Thus, 9 : 15 = 3 : 5 is a proportion.

2. A proportion may be written in two ways.

1st. By placing a double colon (::) between the ratios.

Thus, 9 : 15 :: 3 : 5 is read, 9 *is to* 15 *as* 3 *is to* 5.

2d. By placing the sign of equality between the ratios.

Thus, 9 : 15 = 3 : 5 is read, *the ratio of* 9 *to* 15 *is equal to the ratio of* 3 *to* 5.

3. A proportion may be either *simple* or *compound.*

4. A *simple proportion* is an equality of two simple ratios.

Thus, $3 : 4 :: 9 : 12$ is a simple proportion.

5. A *compound proportion* is an equality of two ratios, either or both of which may be compound.

Thus, $\left\{\begin{matrix} 3:4 \\ 5:6 \end{matrix}::\begin{matrix} 20:32, \\ \end{matrix}\right.$ and $\left\{\begin{matrix} 4:7 \\ 5:8 \end{matrix}\right\} :: \left\{\begin{matrix} 9:14 \\ 10:18, \end{matrix}\right.$

are compound proportions.

6. Every proportion includes *four terms*. The first and fourth terms are called the *extremes;* the second and third terms are called the *means;* while each of the four terms is called a *proportional.*

7. When the second and third terms are *equal*, each is called a *mean proportional* between the other two.

Thus, in the proportion $3 : 9 :: 9 : 27$, 9 is a mean proportional between 3 and 27, and 3, 9, and 27 are in *continued* proportion, while 27 is called the *third* proportional to 3 and 9.

201. The solution of problems in proportion depends upon the following principles:

1st. *In every proportion, the product of the extremes is equal to the product of the means.*

2d. *If the product of the extremes is divided by either of the means, the quotient will be the other mean.*

3d. *If the product of the means is divided by either of the extremes, the quotient will be the other extreme.*

For, in the proportion $9 : 15 :: 3 : 5$, $9 \times 5 = 15 \times 3$. Therefore, if the product of two terms of the *same name* is divided by either of the other terms, the quotient will be the remaining term.

202. To find any term of a proportion, when three terms are given.

1. $6 : 9 :: (\ ) : 15$?

OPERATION $\dfrac{\overset{2}{\cancel{6}} \times \overset{5}{\cancel{15}}}{\underset{3}{\cancel{9}}} = 10.$

EXPLANATION. The extremes are 6 and 15. Their product divided by 9 (one of the means) gives 10 (the other mean).

2. $\left.\begin{matrix} 8 : 15 \\ 9 : 12 \end{matrix}\right\} :: 14 : (\ )$?

OPERATION. $\dfrac{\overset{5}{\cancel{15}} \times \overset{3}{\cancel{12}} \times \overset{7}{\cancel{14}}}{\underset{4}{\cancel{8}} \times \underset{3}{\cancel{9}}} = 35.$

EXPLANATION. The means are 15, 12, and 14. Their product divided by 8×9 (one of the extremes), gives 35 (the other extreme).

REMARK. When a mean and an extreme contain a common factor, it should be canceled as above (Art. 82).

Find the missing term in the following:

1. $18 : 39 :: 24 : (\ )$?
2. $21 : (\ ) :: 15 : 35$?
3. $(\ ) : 36 :: 63 : 27$?
4. $57 : 45 :: (\ ) : 75$?
5. $12\frac{1}{2} : 16\frac{2}{3} :: 51 : (\ )$?
6. $56\frac{1}{4} : (\ ) :: 83\frac{1}{8} : 444$?
7. $(\ ) : 14\frac{2}{5} :: 43\frac{3}{4} : 93\frac{1}{3}$?
8. $\frac{9}{112} : 6\frac{3}{10} :: (\ ) : 19\frac{3}{5}$?
9. $125 : 14\frac{1}{16} :: 8\frac{8}{9} : (\ )$?
10. 3 gal. 3 qt. : 9 gal. 1 qt. 1 pt. :: $66\frac{2}{3} : (\ )$?
11. 1 bu. 3 pk. 6 qt. 1 pt. : 7 qt. 1 pt. :: $37\frac{1}{2} : (\ )$?
12. $38 : 63 :: \left\{\begin{matrix} 24 : 42 \\ 57 : (\ ) \end{matrix}\right.$?
13. $\left.\begin{matrix} (\ ) : 69 \\ 91 : 105 \end{matrix}\right\} :: 39 : 115$?
14. $\left.\begin{matrix} 15 : (\ ) \\ 16 : 27 \end{matrix}\right\} :: \left\{\begin{matrix} 50 : 18 \\ 52 : 65 \end{matrix}\right.$?
15. $\left.\begin{matrix} 6.8 : 45 \\ 36 : 3.8 \end{matrix}\right\} :: \left\{\begin{matrix} 1.02 : .95 \\ (\ ) : 7.5 \end{matrix}\right.$?

16. $\left.\begin{matrix} 10.8 : :93 \\ 40 : 6\frac{3}{4} \end{matrix}\right\} :: \left\{\begin{matrix} 8\frac{1}{8} : 15.5 \\ 1.28 : (\ )\ ? \end{matrix}\right.$

203. Proportion is sometimes called "*the rule of three*," because any one of the four terms can be found if the other three are given. Two of these three terms are always of the same denomination, but the remaining term must be of the denomination required in the answer. This number is usually written for the *third term* of the proportion, while the arrangement of the other two terms depends upon the conditions of the question.

Therefore, *the solution of a problem by proportion depends upon the proper arrangement of the terms.*

204. (*a*) If a company of soldiers consume $600 worth of provisions in 4 weeks, in what time would they consume $1350 worth?

FIRST OPERATION

$600 : $1350 :: 4 weeks : ( )?

$$\begin{array}{r} 1350 \\ \underline{\quad\ 4} \\ 600\,)\,5400\,(9 \text{ weeks.} \\ \underline{5400} \end{array}$$

SECOND OPERATION

$600 : $1350 :: 4 weeks : ( )?

$$\frac{\cancel{1350} \times \cancel{4}}{\cancel{600}} = 9 \text{ weeks.}$$

EXPLANATION. By the conditions of the question, $600 must have the same ratio to $1350, as the time occupied in consuming $600 worth of provisions has to the required time. Therefore, the statement is, $600 : $1350 :: 4 weeks : to the required number of weeks. To find the fourth term, it is necessary to multiply together the two means, 1350 and 4, and divide the product by the given extreme, 600. In the first operation, the entire work is performed; in the second the work is shortened by cancellation.

(*b*) If 9 tons of hay cost $108, how much should 15 tons cost?

FIRST OPERATION

9 tons : 15 tons :: \$108 : ( ) ?

$$\begin{array}{r} \$108 \\ 15 \\ \hline 540 \\ 108 \\ 9)\$1620 \\ \$180 \end{array}$$

SECOND OPERATION

9 tons : 15 tons :: \$108 : ( ) ?

$$\frac{15 \times 108}{9} = 180.$$

EXPLANATION. As the denomination of the answer is to be *dollars*, the *third term* is \$108. It is evident that 15 tons will cost *more* than 9 tons. Therefore, the *second term* is 15 tons, and the *first term* 9 tons. Dividing the product of the means, 15 and 108, by the given extreme, 9, the result is 180, which is the *fourth term*, or answer.

(*c*) If 8 carpenters can build a house in 75 days, in what time could 12 carpenters build a similar house?

OPERATION. 12 car. : 8 car. :: 75 da. : ( ) ?

$$\frac{2 \quad 25}{\cancel{8} \times \cancel{75}} = 50.$$
$$\cancel{12}$$
$$3$$

EXPLANATION. As the denomination of the answer is to be *days*, the *third term* is 75 days. It will require *less* time for 12 carpenters than for 8 carpenters. Therefore, the *second term* is 8 carpenters, and the *first term* 12 carpenters. Multiplying 8 by 75, and dividing by 12, the result is 50 days, which is the *fourth term*.

RULE. 1. *Write for the third term the number which is of the same denomination as that required in the answer.*

2. *If from the conditions of the question, the answer should be greater than the third term, write the greater of the other two numbers for the second term, and the less for the first, but if the answer should be less than the third term, write the less number for the second term, and the greater for the first.*

3. *Multiply together the second and third terms, and divide the product by the first. The quotient will be the fourth term, or answer.*

1. If a man can plow 15 acres in 4 days, how many acres can he plow in 10 days?

2. If 12 tons of coal cost $39, how much would 21 tons cost?

3. If a workman can build 45 rods of fence in 8 days, in how many days could he build 75 rods?

4. The freight on a quantity of merchandise was $37.50 for 225 mi. At the same rate what amount should have been paid on the same quantity for 165 mi.?

5. Find the cost of 15,625 feet of lumber at $12.80 per 1000 feet.

6. If a tree 87.5 ft. high, casts a shadow 122.5 ft. long, how long a shadow will be cast at the same time by a chimney 62.5 ft. high?

7. A merchant failing in business owes $10,800, and has property worth $6750. How much can he pay to B, to whom he owes $560?

8. If a steamer consumes 14.4 bu. of coal in sailing 25.6 mi., how much would it consume in sailing 88.8 mi.?

9. A reservoir can be filled by 9 pipes in 3.5 hr. If 2 of the pipes were closed, in what time could the others fill it?

10. If a yardstick held upright casts a shadow 4.32 ft. long, what is the height of a tree, whose shadow at the same time is 99 ft. long?

11. If $\frac{5}{8}$ of a barrel of wine costs $31\frac{1}{4}$, how much should be paid for $\frac{3}{4}$ of a barrel?

12. If a loaf of bread weighs $12\frac{1}{2}$ oz. when flour is $\$4\frac{1}{5}$ a barrel, how much should it weigh when flour is $\$5\frac{1}{4}$ a barrel?

13. If $37\frac{1}{3}$ is $5\frac{3}{5}$ times a certain number, what is $6\frac{3}{4}$ times the same number?

14. $62\frac{1}{2}$ is $93\frac{3}{4}\%$ of a number. What is $66\frac{2}{3}\%$ of the same number?

15. How many barrels of flour could be bought for $\$113\frac{2}{5}$, if $18\frac{3}{4}$ bbl. cost $\$67\frac{1}{2}$?

16. A wheel has 40 cogs; a larger wheel working in it has 64 cogs. In how many rotations of the larger wheel will the smaller one gain 75 rotations?

17. If 15 dozen eggs are worth $\$2.70$, how much are $\frac{5}{6}$ dozen worth?

18. A and B built a fence. A worked $8\frac{1}{3}$ days, and earned $\$12\frac{1}{2}$. If B earned $\$10$, how many days did he work?

19. If $9\frac{3}{8}$ bbl. of cider cost $\$52\frac{1}{2}$, how many barrels could be bought for $\$37\frac{4}{5}$?

20. If 12.8 acres produce 1248 bu. of potatoes, how many bushels should $16\frac{2}{3}$ acres produce?

21. If property assessed at $\$2250$ pays a tax of $\$40.50$, what should be the value of property upon which the tax is $\$49.50$?

22. The taxable property of a village is listed at $\$187,500$. A's property is assessed at $\$3250$, and his tax is $\$58.50$. What is the entire amount of tax?

23. If $15\frac{7}{9}$ acres yield 248.5 bu. of wheat, how many acres should yield 525 bu.?

24. What is the value of 3 lb. 1 oz. 10 pwt. of silver, if 4 lb. 6 oz. are worth $64.80?

25. The capital stock of a mining company is $250,000, upon which a dividend of $15,625 was declared. If A received $275, what was the value of his stock?

26. How many bushels of wheat at $0.87½ a bushel should be exchanged for 218 bu. 3 pk. of corn at $0.45 a bushel?

27. If a man can perform a journey in 18¾ da., what part of it can he complete in 5⅝ da.?

28. A travels 41⅔ mi. to B's 56¼ mi. How far will B have gone when A has traveled 400 mi.?

29. A lot 68.75 ft. front is worth $2200. At the same rate, what is the value of a lot 47.5 ft. front?

30. Find the cost of 18,750 lb. of hay at $12.80 a ton.

31. A drover bought 108 hogs at $6.25 each. Had they cost $6.75 apiece, how many could he have bought for the same money?

32. A dealer bought 2750 bu. of corn at 44 ct. a bushel, but having lost 330 bu. by fire, at what price per bushel must he sell the remainder to sustain no loss?

33. A is pursuing B, who is 48 rods ahead. A walks 20 rods, while B walks 17 rods. How far must A walk to overtake him?

34. If the interest of a sum of money is $144 for 3 yr. 2 mo. 12 da., what would be the interest of the same sum for 2 yr. 2 mo. 20 da. at the same rate?

35. If 56.25 yd. of ingrain carpet 1 yd. wide will cover a floor, how many yards of Brussels carpet ⅞ yd. wide would be required?

PARTNERSHIP

205. 1. A *partnership* is an association of two or more persons for the transaction of business. Each of these is called a *partner*, while the association itself is styled a *firm* or *house*.

2. The *capital* or *stock* is the money or property furnished by the partners.

SIMPLE PARTNERSHIP

206. In *simple partnership* the capital of each partner is employed for the same period of time.

A, B, and C entered into partnership. A contributed $400; B, $500; C, $600. They gained $750. What was each one's share of the gain?

FIRST OPERATION

$400 $\frac{400}{1500} = \frac{4}{15}$. $\frac{4}{15}$ of $750 = $200, A's share.

$500 $\frac{500}{1500} = \frac{5}{15}$. $\frac{5}{15}$ of $750 = $250, B's share.

$600 $\frac{600}{1500} = \frac{6}{15}$. $\frac{6}{15}$ of $750 = $300, C's share.

$1500

EXPLANATION. Their entire capital is $1500, of which A's part is $\frac{400}{1500}$ or $\frac{4}{15}$; B's, $\frac{500}{1500}$ or $\frac{5}{15}$; and C's, $\frac{600}{1500}$ or $\frac{6}{15}$. Then, A's gain is $\frac{4}{15}$ of $750 = $200; B's gain is $\frac{5}{15}$ of $750 = $250; C's gain is $\frac{6}{15}$ of $750 = $300.

SECOND OPERATION

$400 $400 × .5 = $200 = A's gain.
$500 $500 × .5 = $250 = B's gain.
$600 $600 × .5 = $300 = C's gain.

$1500)$750.0(.5
750 0

EXPLANATION. Their entire capital is $1500. The gain is $750. $750 ÷ $1500 = 50% or .5. $400 × .5 = $200; $500 × .5 = $250; $600 × .5 = $300.

RULE. 1. *Take such a part of the entire gain or loss as each partner's capital is part of the whole capital.*

2. *Divide the amount of gain or loss by the amount of capital, and multiply each partner's capital by the quotient.*

REMARK. These principles may also be applied in dividing any sum or number into parts having a given ratio to each other, as in Bankruptcy, etc.

1. A and B in partnership gain $450. A's capital is $400, B's $500. Find each one's share of the gain.

2. A and B were partners in a speculation. A's capital was $450, B's $750. They lost $640. How much did each lose?

3. A's stock was $480, B's $640, C's $800. They sold out for $3120. How much did each receive?

4. A contributed $625, B $750, C $875. On dissolving partnership they had but $1800. Find the loss of each.

5. Divide 512 into two parts that shall have the ratio of 7 to 9.

6. Divide 540 into two parts that shall have the ratio of 8.4 to 13.2.

7. Divide 735 into two parts in the ratio of $\frac{3}{5}$ to $\frac{5}{8}$.

8. Divide 1012.5 into three parts that shall have the ratios of 6.8, 7.5, and 8.2.

9. Divide 393.75 into three parts that shall be to each other as $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$.

10. Divide $1066\frac{2}{3}$ into three parts having the ratios of $\frac{7}{12}$, $\frac{8}{15}$, and $\frac{11}{20}$.

11. C, D, and E bought 285 acres of land. C paid $1350, D $1525, E $1875. How many acres belonged to each?

12. E and F rented a tract of land for $375. E's part was 88 rd. long and 50 rd. wide. F's part, 75 rd. long by 48 rd. wide. How much did each pay?

13. A father bequeathed his property to three daughters as follows: to the eldest $2160, to the second $1920, and to the third $1680. The property was sold for $5040. How much did each receive?

14. An estate valued at $10,000 was inherited by four heirs in the ratios of their ages, which were respectively 21, 18, 15, and 12 years. After debts amounting to $1750 were paid, how much did each receive?

15. A's lot is 50 ft. front, B's 60 ft., C's 75 ft., D's 90 ft., and E's 100 ft. The cement sidewalk along their property costs $180. How much must each pay?

16. Two political clubs, numbering respectively 185 and 215 members, chartered a train for $500. They were afterward joined by another club of 225 members. How much did the third club pay?

17. A cargo of wheat cost $9660 at $87\frac{1}{2}$ ct. a bushel. It was bought by four merchants whose shares were in the ratios of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{7}{10}$. How many bushels belonged to each?

18. A furnished 1080 bu. of wheat worth $0.62\frac{1}{2}$ a bushel, B 1500 bu. of corn worth 59 ct. a bushel, C 1440 bu. of barley worth $41\frac{2}{3}$ ct. a bushel. They lost $360. How much did each lose?

19. Three teamsters received $77 for hauling. The first hauled 8 loads of 1250 lb. each, daily for 5 days; the second 4 loads of 1500 lb. each, daily for 7 days; the third 6 loads of 1000 lb. each, daily for 8 days. How much did each receive?

20. A furnished 750 bu. of wheat at $87\frac{1}{2}$ ct. a bushel, B 660 bu. of barley at $56\frac{1}{4}$ ct. a bushel, C 756 bu. of corn at $62\frac{1}{2}$ ct. a bushel. They gained $480, and dissolved partnership. How much money belonged to each?

21. Smith, Jones, and Robinson bought a tract of land for $18,000. Smith's part was 160 acres, Jones' 170 acres, while Robinson's share cost him $5625. How many acres did he buy?

BANKRUPTCY

207. 1. A *bankrupt* is one who has failed in business, or who is unable to pay his debts.

2. The *creditors* are the persons to whom the bankrupt is indebted.

3. An *assignee* is a person appointed to settle the estate of a bankrupt by converting his property into cash for the benefit of the creditors.

1. A bankrupt's liabilities are $10,240 and his assets $7680. How much can he pay on each dollar of his indebtedness?

2. A merchant failing in business owes $9875. His stock sells for $8690. What per cent can he pay his creditors?

3. A bankrupt's debts are $22,500, and his property is worth $19,125. How much will a creditor receive to whom he owes $1280?

4. A merchant becoming insolvent fails for $12,800. His property amounts to $11,200. How much will a creditor lose to whom he owes $1920?

5. A bankrupt's liabilities are $17,250 and his assets $12,995. If the assignee is allowed $575, how much will a creditor receive whose claim is $487.50?

6. A wholesale merchant fails for $56,250. His stock sells for $40,000, and the assignee's charges are $2500. How much will a creditor lose whose claim is $1875?

7. A manufacturer failing owes A $2560, B $3120, C $3680, D $4320. His stock and machinery sell for $10,000, and the assignee's charges are $595. How much will A, B, C, and D each receive?

8. A retail merchant who had failed had assets amounting to $7812.50. He owed $440 to B, who received $275. What was the amount of the failure?

9. A bank becoming insolvent had assets valued at $248,000. A owned 48 shares ($100 each) and received $3720. What was the capital stock?

INVOLUTION

DEFINITIONS

208. 1. A *power* of a number is the product obtained by taking the number as a factor a certain number of times.

2. The *first power* is the *number itself.* It is the root from which the other powers are derived.

3. The *second power* is the product obtained by taking the number *twice* as a factor. It is also called the *square*, for the area of a square is the product of *two equal factors.* Thus, 7×7, or 49, is the *second power*, or *square*, of 7.

4. The *third power* is the product obtained by taking the number *three times* as a factor. It is also called the *cube*, because the volume of a cube is the product of *three equal factors.* Thus, $5 \times 5 \times 5$, or 125, is the *third power*, or *cube*, of 5.

5. The higher powers are termed successively the *fourth power, fifth power, sixth power*, etc. Thus, the fourth power of 3 is $3 \times 3 \times 3 \times 3 = 81$. The fifth power of $\frac{1}{3}$ is $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{243}$. The sixth power of 3 is $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$, etc.

6. The *exponent* is a small figure written on the right of the number and slightly above it. It indicates how

many times the number is to be taken as a factor. Thus 4^2 indicates the square of 4; 6^3 indicates the cube of 6; 3^5, the fifth power of 3; etc.

7. *Involution* is the process of finding a power of a number.

209. To find any power of a number.

What is the fourth power of 12?

SOLUTION. $12 \times 12 \times 12 \times 12 = 20{,}736$.

EXPLANATION. To raise a number to the fourth power is to obtain a product in which the number is taken four times as a factor. Then $12^4 = 20{,}736$.

RULE. *Take the number as a factor as many times as there are units in the exponent of the power to which it is to be raised.*

The following tables of squares and cubes should be memorized:

The square of

1 is 1.	5 is 25.	9 is 81.	13 is 169.	17 is 289.
2 is 4.	6 is 36.	10 is 100.	14 is 196.	18 is 324.
3 is 9.	7 is 49.	11 is 121.	15 is 225.	19 is 361.
4 is 16.	8 is 64.	12 is 144.	16 is 256.	20 is 400.

The cube of

1 is 1.	4 is 64.	7 is 343.	10 is 1000.
2 is 8.	5 is 125.	8 is 512.	11 is 1331.
3 is 27.	6 is 216.	9 is 729.	12 is 1728.

REMARK. The work of raising numbers to high powers may be shortened by multiplying together powers of the same number. For instance, if the *third power* of a number *is multiplied* by the *fourth power* of the same number, the result will be the *seventh power*. Thus, $5^3 \times 5^4 = 5^{3+4} = 5^7$.

Find the following:

1. Second power of 99.
2. Third power of 39.
3. Fourth power of 45.
4. Fifth power of 3.2.
5. Sixth power of 1.5.
6. Second power of 62.5.
7. Third power of $12\frac{1}{2}$.
8. Fourth power of $6\frac{2}{3}$.
9. Fifth power of $\frac{7}{8}$.
10. Sixth power of $1\frac{1}{3}$.

Find the square of:

11. 789.
12. 31.25.
13. $68\frac{3}{4}$.
14. $\frac{49}{60}$.
15. $92\frac{25}{36}$.
16. $24\frac{24}{25}$.
17. $85\frac{5}{7}$.
18. .707.
19. 9.99.

Find the cube of:

20. 37.
21. 128.
22. 202.
23. $18\frac{3}{4}$.
24. $\frac{24}{25}$.
25. 12.3.
26. 1.01.
27. $33\frac{1}{3}$.
28. $7.14\frac{2}{7}$.

Find the fourth power of:

29. 21.
30. 49.
31. 5.5.
32. .36.
33. $\frac{8}{9}$.
34. $6\frac{1}{4}$.

Find the fifth power of:

35. 48.
36. 7.5.
37. $16\frac{2}{3}$.

EVOLUTION

DEFINITIONS

210. 1. A *root* of a number is one of the equal factors which produce that number.

2. The *name* of the root depends upon the number of equal factors. Thus, the *square root* of a number is one of *two* equal factors; the *cube root* is one of *three* equal factors; the *fourth root* is one of *four* equal factors, etc.

3. The *radical sign*, $\sqrt{\ }$, is a symbol placed before a number, to indicate that a root is to be found.

4. The *index* of the root is a small figure written above the radical sign, to indicate the number of the root. Thus, $\sqrt[3]{125}$, the cube root of 125; $\sqrt[4]{81}$, the fourth root of 81, etc. When no index is written above the radical sign, the square root is understood: as, $\sqrt{49} = 7$.

REMARK. A root is sometimes indicated by a fractional exponent. Thus, $64^{\frac{1}{2}}$ and $64^{\frac{1}{3}}$ mean the same as $\sqrt{64}$ and $\sqrt[3]{64}$.

5. A *perfect power* is a number whose exact root can be found. Thus, 100, .125, $\frac{16}{81}$ are perfect powers, for $\sqrt{100} = 10$; $\sqrt[3]{.125} = .5$; $\sqrt[4]{\frac{16}{18}} = \frac{2}{3}$.

6. An *imperfect power*, or *surd*, is a number whose exact root can not be found. Thus, $\sqrt{10} = 3.16227+$.

7. *Evolution* is the process of finding a *root* of a number, or of resolving a number into *equal factors*.

SQUARE ROOT

211. To find the number of figures in the square root.

1. Since $\sqrt{1}$ is 1, and $\sqrt{100}$ is 10, the square roots of all squares *between* 1 and 100 must consist of *one figure*. As $\sqrt{100}$ is 10, and $\sqrt{10000}$ is 100, the square roots of all squares *between* 100 and 10000 must consist of *two figures*. Similarly, as $\sqrt{10000}$ is 100, and $\sqrt{1000000}$ is 1000, the square roots of all squares *between* 10000 and 1000000 must consist of *three figures*. Thus, it is shown that *two* additional figures in a number will require *one* additional figure in its square root. Therefore,

2. If a number is pointed off into periods of *two figures each*, the number of periods will be equal to the number of figures in the square root.

RULE. *Place a dot over the units' figure and also over every alternate figure on the left and on the right of units.*

REMARK. When there is an *odd number* of decimal places in the given number, *a cipher must be annexed before pointing it off into periods.*

212. To extract the square root of a number.

(*a*) The surface of a square pane of glass is 625 square inches. What is the length of its sides?

OPERATION

$\dot{6}\dot{2}\dot{5}$ (25 in.
4
45)225
225

EXPLANATION. As the area of a square is the product of *two equal factors*, the *square root* of 625 is the length of the sides.

Point off 625 into two periods by placing a dot over 5 and over 6 (Art. 211). By the table (Art. 209) the largest square in 6 is 4, and its square root is 2. Write 2 on the right for the first figure of the root, and subtract its square (4) from 6. Write the remainder (2) beneath, and to it annex the next period (25). Double the part of the root already found (2), and write the

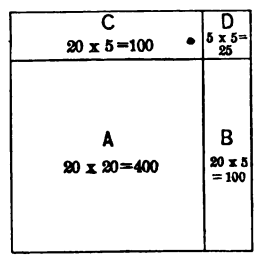
result (4) on the left of 225 for a trial divisor. Divide 22 by 4, and write the quotient (5) for the second figure of the root, and also write it on the right of the trial divisor (4). This forms the complete divisor 45. Multiply 45 by 5. The product is 225, which, subtracted from 225, leaves no remainder. Therefore, the square root of 625 is 25.

PRINCIPLE I. *In every square number, the first period contains the square of the units, the second period the square of the tens, the third period the square of the hundreds*, etc.

PRINCIPLE II. *The square of a number composed of* **tens** *and* **units** *is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

GEOMETRICAL ANALYSIS

As 625 consists of *three* figures, its square root will contain *two* figures (Art. 219, 2), and the left period (6 hundreds) must contain the square of the tens. The greatest square in 6 hundreds is 4 hundreds, and its square root is 2 tens, or 20, which is the side of the square A in the figure. Then 20×20, or 400 sq. in. = the area of the square A. Subtracting 400 sq. in. from 625 sq. in., the remainder is 225 sq. in. Now, in order to preserve the form of a square, both the length and the width of the square A must be increased *equally*. Therefore it will require two rectangles, B and C, each 20 in. long, to be added to two sides of the square A. Then, as their combined length is 20×2, or 40 in., and their area *nearly* 225 sq. in., their width must be $225 \div 40 = 5$ in. The length of the sides of the small square D is also 5 in. Multiplying the entire length of B, C, and D (45 in.) by their width (5 in.) = 225 sq. in. Subtracting 225 sq. in. from 225 sq. in. leaves no remainder. Therefore the side of a square containing 625 sq. in. is 25 in., and the square root of 625 is 25.



(*b*) Extract the square root of 1174.444.

OPERATION

$$
\begin{array}{r}
\dot{1}\dot{1}7\dot{4}.\dot{4}\dot{4}\dot{4}0(34.27+ \\
9 \\
\hline
64)274 \\
256 \\
\hline
682)1844 \\
1364 \\
\hline
6847)48040 \\
47929 \\
\hline
111
\end{array}
$$

EXPLANATION. Annex a decimal cipher, and point off the number into periods by placing a dot over the units' figure, and also over the alternate figures on the left and on the right of units (Art. 211, Rule). Extract the root as in Example 1. As there is a remainder of .0111, 1174.444 is not a perfect square, and its square root is 34.27+.

REMARK. The operation may be continued indefinitely by annexing periods of decimal ciphers.

(*c*) Extract the square root of $\frac{225}{256}$.

SOLUTION. $\sqrt{\frac{225}{256}} = \frac{15}{16}$.

EXPLANATION. The square root of the numerator, 225, is 15; the square root of the denominator, 256, is 16. Then, the square root of $\frac{225}{256}$ is $\frac{15}{16}$.

(*d*) Extract the square root of $\frac{3}{4}$.

SOLUTION. As both terms are not perfect squares, reduce $\frac{3}{4}$ to a decimal, .75. Extracting the square root of .75 to seven decimal places, the result is .8660254+.

RULE. 1. *Begin at units and separate the given number into periods of two figures each.*

2. *Find the greatest square in the first period on the left, and write its root on the right for the first figure of the root. Subtract the square of this number from the period, and to the remainder annex the next period for a dividend.*

3. *Double the part of the root found and write it on the left of the dividend for a trial divisor. Excluding the right hand figure of the dividend, divide the remaining figures by this divisor and write the quotient for the second figure of the root, and also annex it to the trial divisor for the complete divisor.*

4. *Multiply the complete divisor by the second figure of the root; subtract the product from the dividend, and to the remainder annex the next period for another dividend.*

5. *Double the part of the root already found for another trial divisor, and proceed as before until all the periods have been used.*

REMARKS. 1. When a cipher occurs in the root annex a cipher to the trial divisor, bring down another period, and proceed as before.

2. When the given number is not a perfect square annex periods of decimal ciphers, and continue the operation as far as desirable.

3. When a fraction is in its lowest terms its square root is the square root of both terms. If both terms are not perfect squares, reduce the fraction to a decimal and extract the square root of the decimal.

Extract the square root of:

1. 961.	**5.** 15129.	**9.** 138384.
2. 2401.	**6.** 60516.	**10.** 166464.
3. 7569.	**7.** 88804.	**11.** 230400.
4. 10201.	**8.** 110889.	**12.** 308025.

13.	389376.	21.	.071824.	29.	$\frac{15625}{65536}$.
14.	531441.	22.	.001369.	30.	$\frac{104976}{108241}$.
15.	767376.	23.	.00093636.	31.	$7766\frac{1}{64}$.
16.	998001.	24.	1.002001.	32.	$3086\frac{34}{81}$.
17.	1562500.	25.	316406.25.	33.	$10485\frac{19}{25}$.
18.	3515625.	26.	15239.9025.	34.	$\frac{4851}{11264}$.
19.	57.76.	27.	.00038416.	35.	14.4.
20.	10.4976.	28.	$\frac{6604}{9801}$.	36.	.625.

213. To extract the square root of a perfect square by factoring.

Extract the square root of 1225.

OPERATION

$$\begin{array}{r|l} 5 & 1225 \\ 5 & 245 \\ 7 & 49 \\ \hline 7 & \end{array}$$

$5 \times 7 = 35$

EXPLANATION. The prime factors of 1225 are 5, 5, 7, 7. Then the square root of 1225 is $5 \times 7 = 35$.

RULE. 1. *Resolve the given number into its prime factors.*

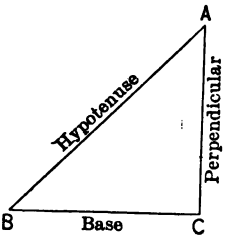
2. *Take one of each pair of equal factors and find their product.*

Extract the square root of:

1.	729.	5.	5625.	9.	60.84.
2.	1521.	6.	4356.	10.	$14 \times 22 \times 77$.
3.	1936.	7.	9604.	11.	$21 \times 24 \times 56$.
4.	3969.	8.	10404.	12.	$30 \times 55 \times 56$.

214. 1. A *triangle* is a figure having three sides and three angles.

2. A *right-angled triangle* is a triangle which has one right angle. Thus, ABC is a right-angled triangle.



3. The *base* of triangle is the side upon which it stands, as BC.

4. The *perpendicular* is the side which forms a right angle with the base, as AC.

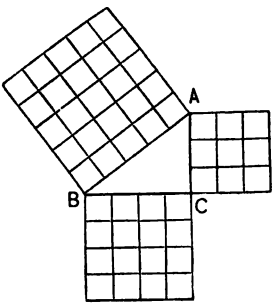
5. The *hypotenuse* is the side opposite the right angle, as AB.

The truth of the following principles has been established by geometry:

PRINCIPLE I. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the base and the perpendicular.*

PRINCIPLE II. *The square of either side of a right-angled triangle is equal to the square of the hypotenuse diminished by the square of the other side.*

The smallest integers that can exactly express the sides of a right-angled triangle are 3, 4, and 5. In the triangle ABC, the base is 4 inches; the perpendicular AC, 3 inches; and the hypotenuse AB, 5 inches. Then the square described on AB contains 25 square inches, and the sum of the squares described



on BC and AC contain $(16 + 9) = 25$ square inches. Hence,

To find the hypotenuse.

RULE. *Add the square of the base to the square of the perpendicular and extract the square root of the sum.*

To find the base or the perpendicular.

RULE. *Subtract the square of the given side from the square of the hypotenuse and extract the square root of the difference.*

REMARK. The diagonal of a rectangular solid is equal to *the square root of the sum of the squares of its three dimensions.*

A room is 20 ft. long, 12 ft. wide, and 9 ft. high. What is the length of its diagonal?

SOLUTION. $20^2 = 400$; $12^2 = 144$; $9^2 = 81$. $(400 + 144 + 81) = 625$, and $\sqrt{625} = 25$. Therefore the length of the diagonal is 25 ft.

1. The base of a right-angled triangle is 21 ft., the perpendicular 28 ft. Find the hypotenuse.

2. The hypotenuse of a right-angled triangle is 65 ft., the base, 33 ft. Find the perpendicular.

3. A rope 68 ft. long extends from the eaves of a house 32 ft. high to the base of a building directly opposite. How wide is the street?

4. A boy is flying a kite which is 224 yd. above the ground. From where the boy stands to a point directly beneath the kite it is 207 yd. How long is the string?

5. A rectangular farm is 119 rd. wide and 120 rd. long. What is the distance between two opposite corners?

6. A mast 48 ft. high stands in the center of a vessel. A rope 50 ft. long extends from the top of the mast to the side of the deck. What is the width of the deck?

7. A tree was broken by the wind in such a way that the top struck the ground at a distance of 75 ft. from the foot of the tree. If the broken part was 85 ft. long, how high was the tree at first?

8. A skiff in crossing a river 494 yd. wide, landed at a point on the opposite side 192 yd. below the place from which it started. How far did it go in a straight line?

9. What distance would be saved by walking along the diagonal instead of two adjacent sides of a rectangle 84 rd. long and 80 rd. wide?

10. A rectangular pile of wood is 56 ft. long, 8 ft. wide, and 7 ft. high. What is the length of its diagonal?

11. An assembly room is 74 ft. 3 in. long, 49 ft. 6 in. wide, and 16 ft. 6 in. high. What is the distance between its opposite upper and lower corners?

12. A rectangular block of marble is 80 in. long, 48 in. wide, 36 in. high. What is the difference between its longest and shortest diagonals?

215. To find the side of a square when its area is given.

RULE. *Extract the square root of the area.*

1. The floor of a square room requires 27 sq. yd. 4 sq. ft. of carpet. What is the length of its sides?

2. The area of a square field is 57 acres, 96 sq. rd. Find the cost of the fence surrounding it at $62\frac{1}{2}$ ct. a rod.

3. The length of a rectangular pasture is 72 rods, and its area is 22 acres 80 square rods. How much more fence does it require than a square field of equal area?

4. A owns a square field containing 15 acres 100 square rods. B owns a square field whose area is $2\frac{1}{4}$ times as great. How many more rods of fence does B's field require than A's?

5. One square farm contains 68 acres 145 square rods. Another square farm contains 48 acres 64 square rods. What is the length of one side of a square tract equal in area to both?

6. A, B, and C, each own a square farm. The area of A's is 31 acres 81 square rods; of B's, 58 acres 129 square rods, while the area of C's is half as great as that of the other two combined. How many rods of fence will inclose their respective farms?

7. A rectangular field is twice as long as it is wide, and its area is 12 acres 33.125 square rods. What is the value of the fence surrounding it at 80 ct. a rod?

SIMILAR FIGURES

216. *Similar figures* are those having the *same form* as squares, circles, etc.

PROPOSITION. *Similar figures are to each other as the squares of their like dimensions;* therefore,

PRINCIPLE I. *The ratio of the areas of two similar figures is equal to the square of the ratio of their like dimensions.*

PRINCIPLE II. *The ratio of the like dimensions of two similar figures is equal to the square root of the ratio of their areas.*

(*a*) How many circles 3 feet in diameter will be equal in area to a circle 15 feet in diameter?

SOLUTION 1. The ratio of 15 to 3 is 5. $5^2 = 25$. *Ans.* 25 circles.
SOLUTION 2. $3^2 : 15^2 :: 1 : 25$.

(*b*) The area of a triangular lot, whose base is 45 ft., is 270 sq. yards. What is the base of a similar lot whose area is 2430 square yards?

SOLUTION. The ratio of 2430 to 270 is 9. $\sqrt{9} = 3$. $45 \times 3 = 135$. *Ans.* 135 ft.

1. How many pipes 4 inches in diameter will discharge as much water as one pipe 2 feet in diameter?

2. A rectangular lot is 30 rd. wide, and its area is 9 acres. What would be the area of a similar lot 50 rd. wide?

3. A reservoir was filled by an 8-inch pipe in 2 hr. 15 min. In what time could a 12-inch pipe have filled it?

4. A rectangular field is 30 rods in length, by 25 rods in width. What are the dimensions of a similar field containing 75 acres?

5. The fence around a square lot cost $120 at 8 ct. a rod. How much would it cost to fence a square lot 9 times as large, at $83\frac{1}{3}$ ct. a rod?

6. A flower garden is 50 ft. in length, and its area is 250 sq. yd. Another garden is similar in shape, with an area of 160 sq. yd. How much longer is the gravel walk around the first, than that around the second?

7. The cost of plowing a rectangular field 50 rd. long, by 32 rd. wide, was $2.40 per acre; and the entire cost of plowing a similar field at the same rate, was $150. What were its dimensions?

8. The base of a right-angled triangle is 45 ft., the perpendicular 60 ft., the hypotenuse 75 ft., and its area is 1350 sq. feet. The area of a similar triangle is 7350 sq. feet. What are its dimensions?

9. A circular reservoir is 200 ft. in diameter, and its capacity is 120,000 barrels. What is the diameter of another circular reservoir of the same depth, whose capacity is 46,875 barrels?

10. A rectangular farm is 80 rd. long, with an area of 32 acres. What would be the value of a similar farm 48 rd. wide, at $87.50 an acre?

REVIEW OF SQUARE ROOT

217. 1. $(\sqrt{244.140625} + \sqrt{351.5625}) \div \sqrt{156.25} =$ what?

2. $\sqrt{74.8225} - \sqrt{506.25} \div \sqrt{87.890625} =$ what?

3. A boy is flying a kite which is 165 yards above the ground. From where he holds the string to a point directly beneath the kite it is 280 yards. What is the length of the string?

4. How much more would the fence cost around a lot 72.25 rd. long by 42.25 rd. wide, at $62\frac{1}{2}$ ct. a rod, than that around a square lot of the same area?

5. The capacity of a circular pond 144 ft. in diameter is 25,600 barrels. What is the diameter of a circular pond of the same depth whose capacity is 19,600 barrels?

6. Two vessels sail from the same port, one going west at the rate of 160 mi. per day, the other south at the rate of 168 mi. per day. If the surface of the water were level, how far apart would they be at the end of a week?

7. A farm is in the shape of a rectangle 108 rd. long, and its area is 50.625 acres. How much more fence would it require than a square farm of equal area?

8. A line 60 ft. 3 in. long extends from the eaves of a house to the base of a building directly opposite. If the street is 52 ft. 3 in. wide, how high is the house?

9. The flue of a chimney 40 ft. high is 30 by 32 inches. What is the length of the longest straight line that can be stretched inside of it?

10. A man exchanged a square lot containing 12.65625 acres, for a lot of the same area whose width was .81 of its length. How much more fence did the latter lot require than the former?

11. What distance would be saved by walking along the diagonal instead of two sides of a rectangular lot 55 rd. long, with an area of 16.5 acres?

12. A rope 65 ft. long extends from the top of a centre-pole 60 ft. high, to a stake in the circumference of a circus ring. What is the diameter of the ring?

13. A rectangular lot is 1.25 times as long as it is wide, and its area is 8 acres. What was the cost of the fence surrounding it at $83\frac{1}{3}$ ct. a rod?

14. What is the greatest distance between any two points in a hall 14 ft. high, 42 ft. wide, and 63 ft. long?

15. The width of a rectangular field is 80% of its length, and its area is 18 acres. If the fence around it cost $180, what was the cost per rod?

16. A farm 64 rods wide cost $3000 at $75 per acre. What are the dimensions of a farm similar in shape that cost $1800 at $80 per acre?

17. The extreme terms of a mean proportion are $5\frac{5}{8}$ and $13\frac{11}{18}$. What is one of the mean terms?

18. A reception room is 68 ft. long, 36 ft. wide, and 15 ft. high. What is the shortest distance that a spider can crawl by floor and wall, from a southeast lower corner to a northeast upper corner?

19. Two towers, 90 ft. and 108 ft. high respectively, are standing on the same level. A line 112.5 ft. long attached to a peg driven in the ground between them will just reach the top of either. How far apart are the towers?

20. A flagstaff is 10 inches in circumference, and 30 ft. high. A rope fastened to the top winds around it once every 24 inches. How long is the rope?

21. The diagonal of the floor of a square parlor is 33 ft. 9 in. long. Find the cost of covering the floor with Brussels carpet at $1.60 a yard.

CUBE ROOT

218. To find the number of figures in the cube root.

1. Since $\sqrt[3]{1}$ is 1 and $\sqrt[3]{1000}$ is 10, it is evident that the cube roots of all the cubes *between* 1 and 1000 must consist of *one figure.*

2. As $\sqrt[3]{1000}$ is 10 and $\sqrt[3]{1000000}$ is 100, the cube root of all cubes *between* 1000 and 1000000 must consist of *two figures.*

Similarly, as $\sqrt[3]{1000000}$ is 100 and $\sqrt[3]{1000000000}$ is 1000, the cube root of all cubes *between* 1000000 and 1000000000 must consist of *three figures.* Thus it is shown that *three* additional figures in a number will require *one* additional figure in its cube root. Therefore,

3. If a number be pointed off into periods of *three figures each,* the number of periods will be equal to the number of figures in the cube root. Hence,

To point off a number into periods for the extraction of the cube root.

RULE. *Place a dot over the units' figure, and also over every third figure on the left and on the right of units.*

REMARKS. 1. The first period on the left of an integer or of a mixed decimal will frequently contain but one or two figures.

2. When the number of figures in a decimal number is not divisible by *three,* annex the necessary number of ciphers before pointing off the number into periods.

219. To extract the cube root of a number.

(*a*) The solid contents of a cube are 15,625 cubic inches. What is the length of one edge?

OPERATION. $15\dot{6}2\dot{5}$(25 in.

		8
$2^2 \times 300$	= 1200	7625
$2 \times 5 \times 30$	= 300	
5^2	= 25	
	1525	7625

EXPLANATION. As the solid contents of a cube are the product of *three equal factors*, the cube root of 15,625 is the length of the edges. Point off 15,625 into two periods by placing a dot over 5 (units) and 5 (thousands) (Art. 218, Rule). By the table (Art. 209), the largest cube in 15 is 8 and its cube root is 2. Write 2 on the right for the first figure of the root, and subtract its cube (8) from 15. Write the remainder (7) beneath, and to it annex the next period (625). Square the part of the root already found (2), multiply it by 300, and write the result (1200) on the left of 7625 for a trial divisor. Divide 7625 by 1200, and write the quotient (5) for the second figure of the root. Multiply 2 by 5, and that product by 30; also square 5. The sum of the three products, 1200 + 300 + 25 is 1525, which forms the complete divisor. Multiply 1525 by 5. The product is 7625, which, subtracted from 7625, leaves no remainder. Therefore the cube root of 15,625 is 25.

GEOMETRICAL ANALYSIS

PRINCIPLE I. *In every number which is a perfect cube the first period contains the cube of the units, the second period contains the cube of the tens, the third period contains the cube of the hundreds,* etc.

PRINCIPLE II. *The cube of a number composed of tens and units is equal to the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

As 15,625 consists of *five* figures, its cube root will contain *two* figures, *tens* and *units* (Art. 218, 2), and the left period (15 thousands) must contain the cube of the tens. The greatest cube in 15 thousands is 8 thousands, and its cube root is 2 tens or 20, which is the edge of the cube A (Fig. 1). $20 \times 20 \times 20$, or 8000 cubic inches = the solid contents of the cube A. Subtracting 8000 cubic inches from 15,625 cubic inches, the remainder is 7625 cubic inches.

In order to preserve the form of a cube, the length, width, and height of the cube A must be increased *equally*. It will require three square blocks, B, C, and D (Fig. 2), to be added to three sides of A. Their entire area is $20^2 \times 3$, or 1200 square inches; and as their solid contents are *nearly* 7625 cubic inches, their thickness must be $7625 \div 1200 = 5$.

Referring to Fig. 3, it is seen that another addition is required of three rectangular blocks, EFG, of the same length and thickness as the blocks BCD. Therefore each of these rectangular blocks is 20 inches long and 5 inches square. There still remains the small cube H (Fig. 4), whose edges are also 5 inches in length. The solid contents of the three blocks B, C, D are $20^2 \times 5 \times 3 = 6000$ cubic inches; of the three rectangular solids E, F, G, $5^2 \times 20 \times 3 = 1500$ cubic inches; and of the small cube H, $5^3 = 125$ cubic inches. $6000 + 1500 + 125 = 7625$ cubic inches. Subtracting 7625 cubic inches from 7625 cubic inches leaves no remainder. Hence, the edge of a cube which contains 15,625 cubic inches is 25 inches.

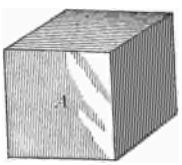


FIG. 1.

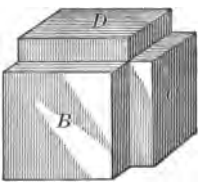


FIG. 2.

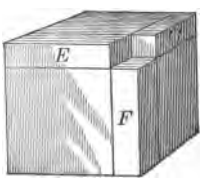


FIG. 3.

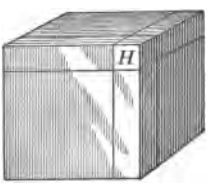


FIG. 4.

(*b*) Extract the cube root of 34012.54.

OPERATION. $\dot{3}4\dot{0}1\dot{2}.5\dot{4}0(32.4+$

			27
$3^2 \times 300$	$= 2700$		7012
$3 \times 2 \times 30$	$= 180$		
2^2	$= 4$		
	2884		5768
$32^2 \times 300$	$= 307200$		1244540
$32 \times 4 \times 30$	$= 3840$		
4^2	$= 16$		
	311056		1244224
			316

EXPLANATION. Annex a decimal cipher and point off the number into periods by placing a dot over the units' figure, and also over the third figure on the left and on the right of units (Art. 218, Rule). Extract the cube root as in Ex. 1. As there is a remainder of .316, 34012.54 is not a perfect cube, and its cube root is 32.4+.

REMARK. The operation may be continued indefinitely by annexing periods of decimal ciphers.

(*c*) Extract the cube root of $\frac{4096}{35937}$.

SOLUTION. $\sqrt[3]{\frac{4096}{35937}} = \frac{16}{33}$.

EXPLANATION. The cube root of the numerator 4096 is 16; the cube root of the denominator 35,937 is 33. Therefore the cube root of $\frac{4096}{35937}$ is $\frac{16}{33}$.

(*d*) Extract the cube root of $\frac{7}{8}$.

SOLUTION. As both terms are not perfect cubes, reduce $\frac{7}{8}$ to a decimal, .875. Extracting the cube root of .875 to five decimal places, the result is .95646+.

RULE. 1. *Begin at units, and separate the given number into periods of three figures each.*

2. *Find the greatest cube in the first period on the left, and write its root on the right for the first figure of the*

root. Subtract the cube of this number from the period, and to the remainder annex the next period for a dividend.

3. *Square the part of the root found, multiply the product by* 300, *and write the result on the left of the dividend for a third divisor. Divide the dividend by this divisor, and write the quotient for the second figure of the root.*

4. *Multiply the part of the root previously found by the last figure, and that product by 30; also, square the last figure in the root, and add the two products to the trial divisor for the complete divisor.*

5. *Multiply the complete divisor by the last figure in the root; subtract the product from the dividend, and to the remainder annex the next period for another dividend.*

6. *Square the part of the root now found, multiply the product by 300, find the next figure of the root as before, and so continue the operation until all the periods have been used.*

REMARKS. 1. When a cipher occurs in the root, annex two ciphers to the trial divisor, and proceed as before.

2. When the number is not a perfect cube, annex periods of decimal ciphers, and continue the operation as far as desirable.

3. When a fraction is in its *lowest terms*, its cube root is the cube root of the terms. If both terms are not perfect cubes, reduce the fraction to a decimal, and extract the cube root of the decimal.

Extract the cube root of:

1. 13824.	7. 681472.	13. 469097433.
2. 32768.	8. 804357.	14. 527514112.
3. 97336.	9. 1030301.	15. 926859.375.
4. 132651.	10. 15438249.	16. .059319.
5. 300763.	11. 94818816.	17. .000166375.
6. 421875.	12. 150568768.	18. .997002999.

19. 1.003003001.
20. 7.29.
21. $\frac{21952}{91125}$.
22. $\frac{1088823}{1488877}$.
23. $\frac{1331000}{1367631}$.
24. $\frac{588245}{625000}$.
25. $8201\frac{145}{216}$.
26. $3814\frac{357}{512}$.
27. $83740\frac{15}{64}$.

28. $\sqrt[3]{63 \times 147 \times 125} = ?$
29. $\sqrt{99.8001} - \sqrt[3]{970.299} = ?$
30. $\sqrt[6]{4826809} =$ what?
31. $\sqrt[6]{85766121} =$ what?
32. $\sqrt[9]{1953125} =$ what?
33. $\sqrt[3]{2^{12} \times 3^6 \times 5^3} =$ what?
34. $\sqrt{.390625} + \sqrt[3]{19.683} - \sqrt{\frac{2809}{1600}} = ?$
35. $\sqrt[3]{912.673} - \sqrt{19\frac{21}{400}} + \sqrt{.455625} = ?$
36. $\sqrt[4]{923521} + \sqrt[3]{614.125} - \sqrt{39\frac{49}{784}} = ?$

220. To find the edge of a cube when its solid contents are given.

RULE. *Extract the cube root of the solid contents.*

1. The solid contents of a cubical box are 5832 cubic inches. What is the depth of the box?

2. The pedestal of a monument is a cubical block of granite containing 35,937 cu. in. What is the height of the block?

3. 1728 cubical starch boxes, each 1 ft. 3 in. long, were piled in the form of a cube. What was its width?

4. The inside measurements of a rectangular box are 108 in. by 64 in. by 54 in. What would be the depth of a cubical bin of equal capacity?

5. What is the edge of a cube whose solid contents are 25 times as great as those of a rectangular solid $18\frac{3}{4}$ in. long, $16\frac{2}{3}$ in. wide, 16 in. high?

6. A river 660 miles long has an average width of 1980 ft. and a depth of 9 ft. What would be the depth of a cubical pit that would exactly contain all the water in the river?

SIMILAR SOLIDS

221. *Similar solids* are those having the same form, but differing in size, as cubes, spheres, etc.

PROPOSITION. *Similar solids are to each other as the cubes of their like dimensions.*

PRINCIPLE I. *The ratio of the solid contents of two similar solids is equal to the cube of the ratio of their like dimensions.*

PRINCIPLE II. *The ratio of the like dimensions of two similar solids is equal to the cube root of the ratio of their solid contents.*

(*a*) How many marble balls 2 inches in diameter will be equal in volume to a marble ball 12 inches in diameter?

SOLUTION 1. The ratio of 12 to 2 is 6. $6^3=216$. Ans. 216 balls.

SOLUTION 2. $2^3:12^3::1:x$; or, $8:1728::1:216$.

(*b*) If a pyramid 5 ft. high contains 15 cu. ft. what is the height of a similar pyramid whose solid contents are 405 cu. ft.?

SOLUTION. The ratio of 405 to 15 is 27. $\sqrt[3]{27}=3$. $5\times3=15$. Ans. 15 ft.

1. A cistern 5 ft. in diameter contains 750 gallons of water. What would be the contents of a similar cistern 10 ft. in diameter?

2. A rectangular block of wood is 18 in. long, 12 in. wide, and 9 in. thick. What are the dimensions of a similar block, whose solid contents are 64 times as great?

3. A pyramid 8 ft. high contains 96 cu. ft. What are the solid contents of a similar pyramid 20 ft. high?

4. A reservoir 140 ft. in length holds exactly 40,960 bbl. of water. What is the capacity of a similar reservoir $87\frac{1}{2}$ ft. in length?

5. A pyramid is 25 ft. square at the base, 37 ft. 6 in. high, and its solidity is 7812.5 cu. ft. What are the dimensions of a similar pyramid whose solidity is 500,000 cu. ft.?

6. A block of granite 6 ft. 3 in. long, 5 ft. wide, and 4 ft. 2 in. thick cost $468.75. What would be the dimensions of a similar block costing $240?

7. The interior of a rectangular bin is 4 times as long and $1\frac{3}{4}$ times as wide as it is deep. If its capacity is 448 cubic feet, what are the dimensions of a similar bin whose capacity is 45 bushels?

REVIEW OF CUBE ROOT

222. 1. A cubical solid contains 54,872 cu. in. What is its height?

2. A party of speculators purchased a tract of land for $110,592. If the number of men, the number of acres each received, and the number of dollars paid per acre were all equal, how many acres did each receive?

3. A rectangular pile of wood is 162 ft. long, 24 ft. wide, and 12 ft. high. If it were piled in the form of a cube, how high would it be?

4. The capacity of a cubical box is 11 cu. ft., 675 cu. in. What is the depth of the box?

5. How many half-inch cubes could be placed in a cubical box $12\frac{1}{2}$ in. deep?

6. The capacity of a cubical pit is 46,656 cu. in. What is the depth of another cubical pit whose capacity is 27 times as great?

7. A granite base for a monument is cubical in form, and its solid contents are 15,625 cubic inches. What would be the cost of polishing four of its faces at \$0.012 a sq. in.?

8. The diameter of a globe is 18 in. What is the diameter of a globe whose solid contents are $15\frac{5}{8}$ times as great?

9. A cubical block of granite contains 35,937 cu. in. Find the cost of polishing four of its faces at \$1.20 a sq. ft.

10. The cost of polishing four sides of a cubical block of marble, at 20 ct. a sq. ft., was \$5. What were its solid contents?

11. A rectangular piece of granite is $4\frac{1}{2}$ in. square, and 16 in. high. How much would it cost to polish five faces of a cubical block of granite whose solid contents are $42\frac{2}{3}$ times as great, at \$1.25 a sq. ft.?

12. A marble column is square at the ends, and its length is $4\frac{1}{2}$ times its width. If its solid contents are 36 cu. ft., what is the area of its surface?

13. What is the depth of a cubical bin that will exactly contain 141.12 bu. of wheat, allowing 2150.4 cu. in. to the bushel?

14. The solid contents of a rectangular box are $7\frac{1}{2}$ cubic feet. If its length, width, and depth inside have the ratio of 12, 8, and 5 respectively, what are its dimensions?

15. A hewn log is square at the ends, and its length is twelve times its width. If the entire area of its surface is 112.5 sq. ft., what are its solid contents and its dimensions?

16. A rectangular reservoir is five times as wide as it is deep, and three times as long as it is wide. If 253,125 cu. ft. of earth were removed in excavating it, what are its dimensions?

17. A pile of wood 160 ft. long, 12 ft. wide, and 8 ft. high was sold at \$4 a cord. What are the dimensions of a similar pile that would realize \$202.50 at the same rate?

MENSURATION

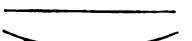
SURFACES

223. 1. *Mensuration* treats of the measurement of lines, surfaces, and solids.

2. A *line* has length, without breadth or thickness.

3. Lines are either straight or curved.

4. A *straight line* is a line that does not change its direction.



5. A *curved line* is a line that changes its direction at every point.



6. *Parallel lines* are everywhere equally distant.

7. An *angle* is the opening between two lines which diverge from a common point. This point is called the *vertex* of the angle.

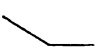
8. A *right angle* is an angle formed by joining two straight lines perpendicular to each other.



9. An *acute angle* is an angle less than a right angle.



10. An *obtuse angle* is an angle greater than a right angle.



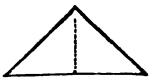
11. A *surface* has length and breadth, but not thickness.

12. A *plane surface* is a flat surface, as a ceiling.

13. A *curved surface* is a surface no part of which is plane.

14. A *plane figure* is one that represents a plane surface, bounded by lines either straight or curved.

15. A *polygon* is a plane figure bounded by straight lines called its *sides*. The *perimeter* of a polygon is the sum of all its sides.



16. A *triangle* is a plane figure having three sides.

REMARK. Any side of a triangle may be called its base. The perpendicular dropped upon the base from the opposite angle is called the *altitude* of the triangle.

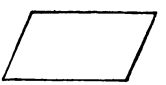
17. A *quadrilateral* is a figure bounded by four straight lines. There are three kinds of quadrilaterals; the *trapezium*, the *trapezoid*, and the *parallelogram*.



18. A *trapezium* is a quadrilateral, no two sides of which are parallel.

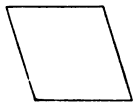


19. A *trapezoid* is a quadrilateral, two of whose sides are parallel.

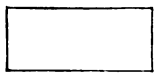


20. A *parallelogram* is a quadrilateral whose opposite sides are equal and parallel.

REMARK. Any side of a parallelogram may be called its base. The perpendicular dropped upon the base from the opposite side is called the *altitude* of the parallelogram.



21. A *rhombus* is a parallelogram whose sides are all equal, and whose angles are not right angles.

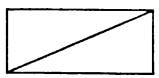


22. A *rectangle* is a parallelogram, all of whose angles are right angles.

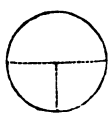
23. A *square* is a rectangle, all of whose sides are equal.



24. A polygon having five sides is called a *pentagon;* one having six sides, a *hexagon;* one having eight sides, an *octagon.*



25. A *diagonal* of a polygon is a straight line joining the vertices of two angles not adjacent.



26. A *circle* is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *center.*

27. The *circumference* of a circle is the curved line that bounds it.

28. The *diameter* of a circle is a straight line passing through the center, and terminating at each end in the circumference. The *radius* of a circle is a straight line extending from the center to the circumference. It is equal to half the diameter.

224. To find the area of a parallelogram.

RULE. *Multiply the base by the altitude.*

1. Find the difference in area between one garden 45 yd. square, and two others, each 44 yd. long and 23 yd. wide.

2. How many acres are there in a square field whose sides are $62\frac{1}{2}$ rd. in length?

3. How many acres are there in a rhombus, each side of which measures 80 rd. and the perpendicular between two sides, 68.75 rd.?

4. The area of a square baseball park is 7 A. 105 sq. rd., and it is surrounded by a close board fence 8 ft. high. Find the value of the lumber at $12.50 per thousand feet.

5. A stable is 75 ft. long, and the rafters on each side are 16 ft. 8 in. in length. What would be the cost of covering the roof with slates 10 in. by 12 in. at $2.50 per 100, allowing one half for overlapping?

6. How many boards 12 ft. long and 15 in. wide would be required to inclose the sides of a barn 48 ft. 9 in. long, 37 ft. 6 in. wide, and 18 ft. high to the eaves?

7. In a school building there are twenty-four blackboards, each 26 ft. 8 in. long, by 3 ft. 9 in. wide. Find their value at $31\frac{1}{4}$ ct. a square foot.

8. A roll of paper 250 yd. long, and 2 ft. 8 in. wide was cut into posters $12\frac{1}{2}$ by 16 in., which were sold at $0.33\frac{1}{3}$ per hundred. What was their value?

9. A parlor 20 ft. 3 in. long requires 50 yd. of Brussels carpet $\frac{3}{4}$ yd. wide. What is the width of the floor?

10. A box of glass contains 50 sq. ft. How many boxes would be used to glaze a factory having 75 windows, with two sashes to the frame, and 6 panes of glass, each 8 in. by 12 in. to the sash?

11. A rectangular block of marble 5 ft. long, 3 ft. 4 in. wide, and 2 ft. 6 in. thick, was sawn into slabs, each 30 in. long, 20 in. wide, and 3 in. thick. Allowing 10% for waste, what was the value of the slabs at $5 apiece?

12. A square room contains $30\frac{1}{4}$ sq. yd. of carpet. If its sides were 18 inches longer, how many square yards would it require?

13. A rectangular pasture is 25 rd. wide by 36 rd. long. What is the distance around a square field whose area is $6\frac{1}{4}$ times as great?

14. A rectangular meadow is $1\frac{1}{2}$ times as long as it is wide, and it requires 160 rd. of fence. What is its area?

15. A owns a rectangular farm containing 50 A. 100 sq. rd., whose width is to its length as 25 is to 36. B owns a square farm of equal area. If the fence around B's farm cost $180, what should have been the cost of that around A's, at the same rate?

225. To find the area of a trapezoid.

RULE. *Multiply half the sum of the parallel sides by the altitude.*

1. A floor is 25 ft. long, and its two ends are respectively 16 ft. and 20 ft. wide. What is its area?

2. Find the area of a field in the form of a trapezoid, the parallel sides being 72 rd. and 56 rd. in length, and 75 rd. apart.

3. A city lot is 68 ft. wide in front, 52 ft. in the rear, and 150 ft. deep. Find its value at 50 ct. a square foot.

4. A lot is bounded by four streets. Two of them are parallel, measuring 250 ft. and 234 ft. along the lot, and lying 180 ft. apart. Find the value of the lot at $31.25 a square rod.

5. A field in the shape of a trapezoid measures 104 rd. and 88 rd. along its parallel sides, and $62\frac{1}{2}$ rd. in width. What is its value at $\$66\frac{2}{3}$ per acre?

6. The floor of an assembly room is 42 ft. wide at one end, 48 ft. at the other, and its area is 330 sq. yd. What is its length?

7. A meadow in the form of a trapezoid is 50 rd. wide, and its area is 18 A. 120 sq. rd. If one of its parallel sides is 55 rd. long, what is the length of the other?

8. A town lot is 150 ft. deep, and its area is 1125 sq. yd. If one of its parallel sides is 25% longer than the other, what is the length of each?

9. A city lot in the form of a trapezoid cost $3750, at 60 ct. a square foot. If its parallel sides were 48 ft. and 52 ft. in width, what was its length?

10. A farm in the shape of a trapezoid 80 rd. long cost $1875 at $62.50 an acre. If one of its parallel sides was $12\frac{1}{2}\%$ shorter than the other, what were their respective lengths?

11. A lot 35 rd. long is in the form of a trapezoid 31 rd. wide at one end, and 17 rd. at the other. How far from the wider end must a fence be placed straight across, that the two parts may be of equal area?

NOTE. The square root of half the sum of the squares of the parallel sides will be the length of the dividing line (25 rd.). Multi-

plying the area of the wider half by 2, and dividing the product by the sum of its parallel sides (31 + 25) gives the distance required.

12. A, owning a farm in the shape of a trapezoid 88 rd. in length, with parallel sides respectively 49 rd. and 71 rd. in width, sold half of it to B. How far from the narrower end must a fence between them be built straight across?

13. A pasture in the form of a trapezoid 31 rd. wide at one end, 49 rd. at the other, and 72 rd. in length, was divided into two equal lots by a fence parallel to the ends. What per cent of the length of the longer lot was the length of the shorter?

226. To find the area of a triangle.

1st. When the base and altitude are given.

RULE. *Multiply the base by the altitude, and take half the product.*

2d. When the three sides are given.

RULE. 1. *From half the sum of the three sides subtract each side separately.*

2. *Multiply together the half sum and the three remainders, and extract the square root of the product.*

1. The base of a triangle is 48 ft., and its altitude 75 ft. Find its area.

2. A triangle has a base of 495 ft., and an altitude of 352 ft. What is its area?

3. Find the value of a triangular farm 120 rd. long and 96 rd. wide, at $62.50 an acre.

4. A lot is in the form of a triangle 133 ft. 4 in. long and 112 ft. 6 in. wide. What is its value at 75 cents a square yard?

5. A meadow in the form of a right-angled triangle cost $495, at $60 an acre. If its base was 55 rods long, what was the length of the other sides?

6. The sides of a triangular field are 56, 90, and 106 rods. Find its value at $80 an acre.

7. The sides of a triangular lot are 68, 60, and 32 rods respectively. If it produces 80 bu. of potatoes per acre, how much is the crop worth at 50 ct. a bushel?

8. A triangular garden measures 45, 108, and 117 ft. along its sides. If each square yard produces 2 qt. of strawberries, what is their value at $8\frac{1}{3}$ ct. a quart?

9. The sides of a triangular lot are 104.4, 75.6, and 72 ft. Find its value at $62\frac{1}{2}$ ct. a square yard.

10. The edges of a triangular sail are $24\frac{1}{4}$, $16\frac{1}{4}$, and 18 ft. in length. How much did it cost at 50 ct. a square yard?

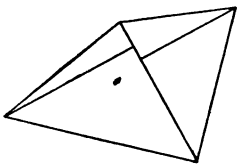
11. Two lawns are in the form of right-angled triangles. The base of the first is 126 ft., hypotenuse 174 ft.; the perpendicular of the second is 153 ft., hypotenuse 185 ft. Find the difference in the cost of sodding them at 20 ct. a square yard.

12. A line 85 ft. long attached to a peg driven in the ground between two trees whose heights are 68 and 77 ft. will just reach the top of either. How far apart are the trees?

13. A flag pole 55 ft. high stands at one of the vertices of an equilateral triangle. A rope 73 ft. long

attached to the top of the pole will exactly reach either of the other vertices. What is the area of the triangle?

227. To find the area of a trapezium.



RULE. *Multiply a diagonal by half the sum of the perpendiculars drawn to it from the vertices of the two opposite angles.*

1. What is the area of a trapezium whose diagonal is 150 ft., and the perpendiculars to the diagonal from the opposite angles, respectively 51 ft. and 45 ft.?

2. The diagonal of a lot in the form of a trapezium is 264 ft., and the perpendiculars on the diagonal are 74 ft. and 87 ft. How many square rods does it contain?

3. The diagonal of a parlor in the shape of a trapezium is 30 ft., and the perpendiculars to this diagonal are 10 ft. and 11 ft. How much would it cost to carpet the floor at $0.75 a square yard?

4. A baseball park is in the form of a trapezium whose diagonal is 880 ft. The perpendiculars to the diagonal are 420 ft. and 471 ft. What is its area?

5. A man paid $0.25 a square foot for a city lot in the shape of a trapezium whose diagonal was 125, and the perpendiculars on this diagonal from the vertices of the opposite angles, 37 ft. and 27. How much did the lot cost?

6. A tract of land is in the form of a trapezium whose diagonal is 192 rd., and the perpendiculars to it from the opposite angles are 61 rd. and 67 rd. Find its value at $50 an acre.

7. A lot is bounded by four streets, and its sides are respectively 432, 126, 360, and 270 ft. in length, the last two lying at right angles. What is the area of the lot?

8. $60 an acre was paid for a farm in the shape of a trapezium whose sides were respectively 154, 72, 102, and 136 rd. in length, the first two forming a right angle. What was the cost of the farm?

228. To find the circumference of a circle when the diameter is given.

RULE. *Divide the circumference by* 3.1416.

REMARK. The ratio of the circumference to the diameter is very nearly $3\frac{1}{7}$.

1. The diameter of a circle is 250 ft. What is its circumference?

2. The diameter of a circular fish pond is 75 ft. Find its circumference.

3. The diameter of a circular race course is 1250 ft. What is the distance around it?

4. The circumference of a circus ring is 183.26 ft. Find the greatest distance across it.

5. The roundhouse of a railway is 471.24 ft. in circumference. What is the distance between two points in the circumference on opposite sides of its center?

6. A carriage wheel travels 1 mi. 37 rd. in making 375 revolutions. What is the diameter of the wheel?

7. The fly wheel of an engine is 6 ft. 8 in. in diameter. What distance would it travel in making 3000 revolutions?

8. A boy attached a string to a peg in a playground, and with the other end ran around the peg. If the greatest distance that he could travel in making the circuit was 314.16 ft., what was the length of the string?

9. The diameter of a circular fair ground is 1680 ft. 8 in. What is the distance around it?

10. What length of rope would enable a horse to graze over a circle whose circumference is 209.44 ft.?

11. A rope 93 ft. 9 in. long extends from the top of a flagstaff 56 ft. 3 in. high, to the edge of a circular inclosure. What is the distance around the inclosure?

12. A pole stands in the center of a circular tent whose circumference is 235.62 ft. A line 42.5 ft. long extends from the top of the pole to a stake in the circumference of the tent. How high is the pole?

229. 1. **To find the area of a circle when the diameter is given.**

RULE. *Multiply the square of the diameter by* .7854.

2. To find the diameter of a circle when the area is given.

RULE. *Divide the area by* .7854, *and extract the square root of the quotient.*

1. Find the area of a circle whose diameter is 45 ft.

2. The diameter of a circle is 165 ft. What is its area?

3. The radius of a circle is 12.5 rd. What is its value at 80 ct. a square rod?

4. The circular area under the dome of a cathedral is 235.62 ft. in circumference. How much would it cost to

cover the floor with marble tiles at $3.20 per square yard?

5. A horse is tied to a stake by a rope 37 ft. 6 in. long. Over what area can he graze?

6. A cow is tied by a rope to a post at one corner of a rectangular lot. If the rope is 25 ft. long, over what area can she graze?

7. The area of a circular drill ground is 12 A. 43.5 sq. rd. What is its diameter?

8. The area of a circular fair ground is 70 A. 109.76 sq. rd. What is the greatest distance across it?

9. $1963.50 was paid for sodding the circular space of an amphitheater, at $0.16 a square yard. What was its circumference?

10. The wheels of a bicycle are 2 ft. 6 in. in diameter. If each makes 200 revolutions in a circuit around a track, what is the area of the circle?

11. The area of the space inside of a circular tent is $872\frac{2}{3}$ sq. yd. If the center pole is 37.5 ft. high, how long a rope will just reach from the top of the pole to the edge of the circular space?

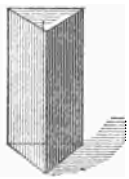
12. A flag pole 125 ft. high stands in the center of a circular park. A line 325 ft. long will exactly reach from the top of the pole to a point in the circumference of the inclosed space. What is the area of the park?

13. A circular tower whose base is 75 ft. in diameter, stands in the center of a circular inclosure whose circumference is 2591.82 ft. What is the area of the space outside of the tower?

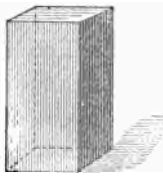
SOLIDS

230. 1. A *solid* or *volume* is a figure that has length, breadth, and thickness.

2. A *prism* is a solid whose bases are equal polygons, parallel to each other, and whose sides are parallelograms.



TRIANGULAR PRISM



QUADRANGULAR PRISM

REMARK. Prisms are named from the form of their bases, as *triangular*, *quadrangular*, etc.

3. A *right prism* is a prism whose sides are perpendicular to its bases.

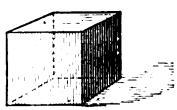
4. The *altitude* of a prism is the perpendicular distance between its bases.

5. The *convex surface* of a prism is the sum of the areas of its sides.



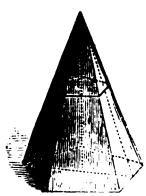
6. A *parallelopipedon* is a prism whose bases are parallelograms.

7. A *right parallelopipedon* is a solid all of whose faces are rectangles.



8. A *cube* is a solid having six equal square faces.

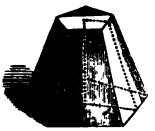
9. A *pyramid* is a solid whose base is a polygon and whose faces are triangles that meet at a common point called the *vertex* of the pyramid.



PYRAMID

10. A *right pyramid* is a pyramid all of whose faces are equal.

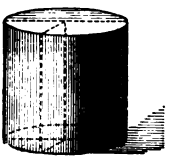
11. The *altitude* of a pyramid is the perpendicular distance between the vertex and the base.



FRUSTUM

12. The *slant height* of a pyramid is the distance from the vertex to a point in the middle of each side of the base.

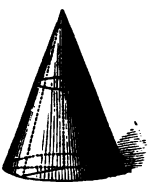
13. The *frustum* of a pyramid is the part which remains after cutting off the top by a plane parallel with the base.



CYLINDER

14. A *cylinder* is a circular solid of uniform diameter whose bases are equal parallel circles.

15. The *convex surface* of a cylinder is the area of its curved surface.



CONE

16. A *cone* is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the *vertex*.

17. The *frustum* of a cone is the part which remains after cutting off the top by a plane parallel with the base.



FRUSTUM

18. A *sphere* is a solid having a curved surface, every point of which is equally distant from a point within called the *center*.

19. The *diameter* of a sphere is a straight line passing through the center, and terminating at each end in the surface.



SPHERE

231. 1st. To find the convex surface of a right prism.

RULE. *Multiply the perimeter of the base by the altitude.*

2d. To find the convex surface of a cylinder.

RULE. *Multiply the circumference of the base by the altitude.*

3d. To find the entire surface of a prism or of a cylinder.

RULE. *Add the areas of the two bases to the convex surface.*

1. Find the surface of a rectangular prism 30 in. high, standing on a base 15 in. square.

2. Find the surface of a right prism 25 in. high, standing on a triangular base whose sides are 9 in., 12 in., and 15 in.

3. What is the surface of a right prism 20 in. high, standing on a triangular base whose sides are 8 in., 15 in., and 17 in.?

4. What is the convex surface of a right prism 4 ft. high, standing on a base whose perimeter is $37\frac{1}{2}$ inches?

5. Find the convex surface of a cylinder 40 in. in diameter, with an altitude of 6 ft. 3 in.

6. What is the entire surface of a cylinder 25 inches in diameter and 50 inches long?

7. Find the entire surface of a cylinder 5 ft. high and 20 inches in diameter.

232. To find the volume of a prism or of a cylinder.

RULE. *Multiply the area of the base by the altitude.*

1. How many cubic inches are in a prism 16 in. high, standing on a base $4\frac{1}{2}$ in. square?

2. A right prism 3 ft. high stands on a triangular base whose sides are 12 in., 16 in., and 20 in. What are its solid contents?

3. A granite column cost $250 at $5 a cubic foot. If its base is 18 in. by 24 in., what is its height?

4. Find the solid contents of a cylinder 2 ft. 1 in. in diameter and 6 ft. high.

5. A cylinder is 8 ft. 4 in. high and 4 ft. 2 in. in diameter. What is its volume?

6. One of the columns of a temple is 3 ft. 4 in. in diameter and its volume is 327.25 cu. ft. What is its altitude?

233. 1st. To find the convex surface of a right pyramid.

RULE. *Multiply the perimeter of the base by the slant height, and take half of the product.*

2d. To find the convex surface of a cone.

RULE. *Multiply the circumference of the base by the slant height, and take half of the product.*

3d. To find the entire surface of a pyramid or of a cone.

RULE. *Add the area of the base to the convex surface.*

4th. To find the convex surface of the frustum of a pyramid or of a cone.

RULE. *Multiply the sum of the perimeters or the circumferences of the two bases by the slant height, and take half of the product.*

5th. To find the entire surface of a frustum.

RULE. *Add the areas of the two bases to the convex surface.*

1. Find the convex surface of a right pyramid whose base is 18 in. square, and slant height 5 ft.

2. What is the entire surface of a pyramid whose base is 3 ft. 6 in. square, and slant height 9 ft.?

3. Find the convex surface of a cone whose slant height is 16 in., and circumference of base $12\frac{1}{2}$ in.

4. What is the entire surface of a cone whose base is 20 in. in diameter, and slant height 50 in.

5. Find the convex surface of the frustum of a pyramid whose bases are respectively 6 ft. square and 4 ft. square, and whose slant height is 12 ft.

6. What is the entire surface of the frustum of a pyramid, whose slant height is 30 in., the side of the upper base being 16 in., and of the lower base 20 in.

7. Find the convex surface of the frustum of a cone whose bases are respectively 15 ft. and 9 ft. in circumference, and whose slant height is 12 ft. 6 in.

8. Find the entire surface of the frustum of a cone whose slant height is 16 ft. 8 in., and whose bases are respectively 10 ft. and 5 ft. in diameter.

234. 1. To find the volume of a pyramid or of a cone.

RULE. *Multiply the area of the base by one third the altitude.*

2. To find the volume of the frustum of a pyramid or of a cone.

RULE. *To the sum of the areas of the two bases, add the square root of the product of those areas, and multiply the result by one third of the altitude.*

REMARK. The square root of the product of the areas of the two bases is a mean proportional, or *the area of a mean base between them.*

1. Find the volume of a pyramid 45 ft. high, standing on a base 15 ft. square.

2. The altitude of a pyramid is 6 ft., and its base is a triangle whose sides are 18 in., and 30 in. What is its volume?

3. A pyramid stands on a base 75 ft. square, and its volume is 6875 sq. yd. What is its altitude?

4. Find the volume of a cone whose base is 7 ft. 6 in. in diameter, and its altitude 24 ft.

5. Find the volume of a cone whose base is 39.27 ft. in circumference, and its altitude 48 ft.

6. The radius of the base of a cone is 12.5 in., and its volume is 9 cu. ft. 156 cu. in. What is its altitude?

7. Find the volume of the frustum of a square pyramid, the sides of whose bases are 9 ft., and 15 ft., and whose altitude is 18 ft.

8. Find the volume of the frustum of a triangular pyramid, upper base 3 ft., 4 ft., and 5 ft., lower base 12 ft., 16 ft., and 20 ft., altitude 45 ft.

9. The bases of the frustum of a cone are respectively 25 ft. and 15 ft. in diameter, and its altitude is 54 ft. What are its solid contents?

10. The upper and lower bases of the frustum of a square pyramid are 4 ft. and 9 ft., altitude 18 ft. What is the volume of a similar frustum whose mean base is 12 ft. sq.?

235. To find the surface of a sphere.

RULE. *Multiply the square of the diameter by* 3.1416.

1. Find the surface of a sphere whose diameter is 20 in.

2. Find the surface of a sphere whose radius is 22.5 in.

3. A ball surmounting the spire of a church is 37.5 in. in diameter. What was the cost of gilding it at 4 mills a square inch?

4. How much rubber cloth would be required for 750 footballs, each 10 in. in diameter?

5. How many globes 2.25 in. in diameter would equal in surface a globe 27 in. in diameter?

236. To find the volume of a sphere.

RULE. *Multiply the cube of the diameter by* .5236.

1. The diameter of a sphere is 30 inches. What is its volume?

2. If a cubic inch of copper weighs 5 oz., what is the weight of a solid ball of copper 10 inches in diameter?

3. How many marbles .6 inch in diameter would be equal in volume to a marble ball 9 inches in diameter?

4. The volume of a globe is 1 cu. ft. 29.15 cu. in. What is its diameter?

5. The surface of a sphere is 13 sq. ft. 91.5 sq. in. What is its volume?

6. A granite globe contains 2 cu. ft. 732.8 cu. in. Find the cost of polishing its surface at $1.50 a square foot.

APPLICATIONS OF MENSURATION

237. 1. The unit of measure for plastering, house-painting, paving, paper-hanging, etc., is the *square foot* or the *square yard.*

2. Glazing is measured by the *square foot,* or by the *pane.*

3. Flooring, roofing, etc., are measured by the *square yard,* or by the *square* of 100 square feet.

4. Stone cutting is measured by the *square foot.*

1. A rectangular court is 33 ft. 4 in. wide, by 67 ft. 6 in. long. Find the cost of paving it with bricks, each 8 in. long by 4 in. wide, at $10 per thousand.

2. A square ball park containing $5\frac{5}{8}$ acres is surrounded by a close board fence 8 ft. high. What would be the cost of whitewashing both sides of the fence at 3 ct. a square yard?

3 A school building has three floors, with eight rooms on each floor. There are five windows in every room, all 6 ft. high, 3. ft. 4 in. wide. How many boxes of glass, of 50 sq. ft. each, would be required to glaze the windows?

4. Find the cost of dressing 8 stone sills, each 3 ft. 9 in. long, 1 ft. 3 in. wide, and 9 in. high, at $16\frac{2}{3}$ ct. a square foot.

5. A circular fish pond is 120 ft. in diameter, and 4 ft. 6 in. deep. How much did the excavation cost at $0.18\frac{3}{4}$ a cubic yard?

6. A croquet ground is 80 ft. long and 65 ft. wide. A walk 2 ft. 6 in. wide extends around the inside of its

borders. Find the total cost of sodding the inclosed space at 22½ ct. a square yard, and of cementing the walk at $1.35 a square yard.

7. A parlor is 18 ft. long, 16 ft. 8 in. wide, and 12 ft. high. What was the cost of papering the walls and ceiling at 15 ct. a square yard, after deducting a fireplace 4 ft. 6 in. by 3 ft., two doors, each 7 ft. 6 in. by 3 ft., and two windows, each 6 ft. by 3 ft.

8. A cistern is in the form of a cylinder 10 ft. in diameter, and 12 ft. 6 in. deep. Find the cost of cementing the floor and wall at 25 ct. a square yard.

9. A rectangular piece of granite is 22½ in. long, 13⅓ in. wide, and 3¾ in. thick. How much would it cost to polish four sides of a cubical block of granite whose volume is twenty-four times as great, at $1.25 a square foot?

10. A barn is 50 ft. long, 32 ft. wide, and 15 ft. high to the eaves. The roof rises 12 ft. above the eaves. Find the total cost of the lumber inclosing the barn at $17.50 per thousand feet, and of the roof at $2.50 per square.

Board Measure

238. 1. *Board measure* is used for measuring lumber, timber, planks, joists, etc.

2. A *board foot* is 1 foot long, 1 foot wide, and 1 inch thick. Hence, a *cubic foot* is equal to 12 *board feet.*

1st. To find the contents of a board in board feet.

Rule. *Multiply the length in feet by the width in inches, and divide the product by* 12.

2d. To find the contents of a plank, joist, etc., in board feet.

RULE. *Multiply the length in feet by the width in inches, and that product by the thickness in inches, and divide the result by* 12.

REMARK. When a board tapers uniformly, its *mean width* is half the sum of the two ends.

1. How many feet are in a 2-inch oak plank 13 ft. 6 in. long, by 1 ft. 4 in. wide?

2. A squared walnut log is 22 ft. 6 in. long, and 1 ft. 6 in. square. What is its value at $0.048 a foot?

3. A lot 32 rd. wide and 48 rd. long is surrounded by a fence four boards high, each board being 6 in. wide. Find the cost of the lumber at $17.50 per 1000 ft.

4. How many planks 8 ft. long, 9 in. wide, and 2 in. thick could be sawed from a log 24 ft. long and 18 in. square, allowing $\frac{1}{9}$ for waste in sawing?

5. How many feet are in a 3-inch plank 15 ft. long, 17 in. wide at one end, and 13 in. at the other?

6. A hewed log is 18 ft. 9 in. long, 18 in. wide at one end, 14 in. at the other, and 15 in. thick. What is its value at $3\frac{1}{3}$ ct. a foot?

MASONS' AND BRICKLAYERS' WORK

239. 1. Stone-masons' work is usually measured by the *perch*, which contains 24.75 cubic feet.

2. Brickwork is generally estimated by the *thousand bricks*.

1. How many perches are in a pile of stone 82 ft. 6 in. long, 13 ft. 6 in. wide, and 2 ft. 8 in. high?

2. A cellar is 6 ft. 9 in. deep, and 40 ft. long by 30 ft. wide inside. If the walls are 1 ft. 8 in. thick, how many perches of stone were used in constructing them?

3. A cellar is 7 ft. 4 in. deep, and 33 ft. long, by 24 ft. wide outside. If the walls are 18 in. thick, what is the value of the foundation at $3.75 a perch?

4. A space 123 ft. long by 75 ft. wide is surrounded by a brick wall 6 ft. high and 1 ft. thick. Allowing 20 bricks to the cubic foot, what was their cost at $8.33\frac{1}{3}$ per thousand?

5. A chimney is 45 ft. high, 5 ft. 4 in. by 2 ft. 8 in. outside, and 4 ft. by 16 in. inside. How much did the brick cost at $12.50 per thousand, allowing 20 bricks to the cubic foot?

6. A solid brick tower is 16 ft. square at the base, 4 ft. square at the top, and its height is 150 ft. Estimating 20 bricks to the cubic foot, how much did they cost at $11.25 per thousand?

7. A cellar $5\frac{1}{2}$ ft. deep is $\frac{3}{4}$ as wide as it is long, outside measurement, and its walls are 18 in. thick. If the excavation cost $22 at 25 ct. a cubic yard, what is the value of the foundation at $2.50 a perch?

8. The stone for a retaining-wall cost $5760 at $4.80 a perch. If the wall was 7 ft. 6 in. thick at the base, 4 ft. 6 in. at the top, and 22 ft. 6 in. high, what was its length?

Measurement by Bushels or Gallons

240. 1. To find the number of bushels.

Rule. *Find the contents in cubic inches, and divide by* 2150.4.

2. To find the number of gallons.

RULE. *Find the contents in cubic inches, and divide by* 231.

REMARKS. 1. Since 1728 cu. in. = 1 cu. ft., and 2150.4 cu. in. = 1 bu., 1 cu. ft. = 1728 ÷ 2150.4 = $\frac{4}{5}$ bu. Therefore, when it is required to reduce cubic feet to bushels, *multiply by* 45, *and divide the product by* 56.

2. Since .7854 ÷ 231 = .0034, the work of multiplying by .7854 and dividing the product by 231 may be shortened by multiplying at once by .0034.

3. The barrel of 31½ gallons and the hogshead of 63 gallons are used in measuring the capacity of cisterns, vats, tanks, etc.

4. A cubic foot of pure water weighs 1000 oz. Avoirdupois.

1. A bin is 7 ft. long, 4 ft. wide, and 2 ft. 6 in. deep. How many bushels will it hold?

2. A crib is 17 ft. 6 in. long, 8 ft. wide, and 6 ft. high. What is its capacity in bushels?

3. A box car is 28 ft. long and 6 ft. 8 in. wide in the clear. How many bushels of grain will fill it 3 ft. deep?

4. A transfer wagon 10 ft. 6 in. long, 5 ft. 4 in. wide, and 2 ft. 6 in. deep is loaded with wheat. What is the value of the wheat at 80 ct. a bushel?

5. How much grain will fill a cylindrical vessel 3 ft. 4 in. in diameter and 8 ft. deep? How many gallons of water?

6. A vat is 10 ft. 6 in. long, 5 ft. 6 in. wide, and 3 ft. 6 in. deep. How many barrels of wine will it hold?

7. A tank is 11 ft. long, 4 ft. wide, and 3 ft. 6 in. deep. Find its capacity in cu. ft., gal., or bu.

MISCELLANEOUS PROBLEMS

241. 1. I bought lemons at $12\frac{1}{2}$ cents a dozen. Had I received 8 lemons fewer for the same money, they would have cost $2\frac{1}{2}$ cents more per dozen. How many dozen were bought?

2. The product of two numbers is 25, and the less is to the greater as 4 is to 9. Find the G. C. D. and the L. C. M. of the numbers.

3. The surface of a cube is 6 sq. ft. 73.5 sq. in. What is the surface of a cube whose volume is 4.096 times as great?

4. What sum invested in U. S. 3% bonds at $102\frac{1}{4}$, brokerage $\frac{1}{4}$%, would yield a semi-annual income of $500 in currency when gold is at $6\frac{2}{3}$% premium?

5. A wall 2 ft. 8 in. thick, and 7 ft. 6 in. high cost $1800 at $37\frac{1}{2}$ ct. a cubic foot, but its length had been measured by a pole 9 ft. $8\frac{1}{4}$ in. long, instead of 10 ft. as it was supposed to be. Find the true length and value of the wall.

6. The L. C. M. of two numbers is 2500, and their G. C. D. is $6\frac{1}{4}$. If one of them is .64 as great as the other, what are the numbers?

7. A house, lot, and stable cost such a sum of money, that, if the interest of it at $7\frac{1}{2}$% for 2 yr. 9 mo.

10 da. were put upon interest for 2 yr. 9 mo. 10 da. at $4\frac{1}{2}\%$ it would *amount* to $1125. If the lot cost $37\frac{1}{2}\%$ as much as the house, and the stable 60% as much as the lot, what was the cost of each?

8. A tree $87\frac{1}{2}$ ft. high was broken in such a way that the top struck the ground 35 ft. from the foot of the tree. How long was the part broken off?

9. If the cost of an article had been $6\frac{1}{4}\%$ less, the gain by its sale would have been $7\frac{1}{2}\%$ more. Find the per cent of gain.

10. $2\frac{2}{3}\left\{\frac{6\frac{3}{10}+2\frac{2}{15}}{12\frac{4}{5}-5\frac{5}{6}}\times\frac{22\frac{2}{24}\frac{3}{4}-7\frac{1}{8}}{9\frac{2}{36}\frac{5}{6}+3\frac{1}{12}}\div\frac{4\frac{4}{8}\div\frac{4}{45}}{16\frac{2}{3}\times 3\frac{3}{4}}\right\}=$ what?

11. A rope 51 ft. long is wound about a flagstaff 45 ft. high, passing around it once for every 30 inches of its height. What is the circumference of the flagstaff?

12. A, B, and C went hunting. C killed 20 birds, B and C together killed 2.75 times as many as A, and $62\frac{1}{2}\%$ of the sum of A's and C's equaled .9375 of B's number. How many did A and B each kill?

13. A trader bought a number of sheep. 10 were killed, and he sold $41\frac{2}{3}\%$ of the remainder at a loss of $6\frac{1}{4}\%$, receiving $300, which was $62\frac{1}{2}\%$ less than all cost. At what price per head should he have sold the others to realize a clear profit of 25% on the entire transaction?

14. A railway company built four side tracks, 6002 ft. 6 in., 4401 ft. 10 in., 3344 ft. 3 in., and 5030 ft. 8 in. long respectively. If they used the greatest length of rail possible to allow an exact number, what was the value of all the rails at $3.33\frac{1}{3}$ apiece?

15. If a certain number be multiplied by its cube root, the square root of the product will be .046656. What is the number?

16. A grocer with one straight cut 10 inches in length took off $\frac{1}{6}$ of the circumference of a cheese. If the part cut off weighed 1.51 lb., what was the weight of the entire cheese?

17. The time past noon, $+12\frac{1}{2}\%$ of the time to midnight, is equal to 25% of the time past midnight. What is the time of day?

18. There is coal on the dock, and coal is running on at a uniform rate. 8 men can clear the dock in 2 hours, but 12 men could clear it in 40 minutes. How long would it take 10 men? 7 men? 18 men?

19. If the cost of an article had been $4\frac{1}{6}\%$ less, the gain by its sale would have been 5% more. Find the per cent of gain.

20. A fly wheel 10 ft. 6 in. in diameter is attached to a shaft whose end rests upon the floor. In describing the circumference of a circle around the point upon which the end of the shaft rests, the wheel revolves $3\frac{4}{7}$ times. What is the length of the shaft and the area of the circle?

21. Five cyclists start together from a line to ride around a track $\frac{5}{6}$ of a mile in circumference, on wheels 8 ft. 4 in., 9 ft. 2 in., 12 ft. 6 in., 13 ft. 4 in., and 14 ft. 8 in. in circumference respectively, each wheel making one revolution per second. In what time will all first arrive together at the starting place, and how far will the third have traveled?

22. B has $2712.50 in gold. If he has $.83\frac{1}{3}$ as many quarter eagles as gold dollars, .8 as many half eagles as quarter eagles, .75 as many eagles as half eagles, and $.66\frac{2}{3}$ as many double eagles as eagles, how many coins has he altogether?

23. $\dfrac{5\frac{3}{5}(6\frac{5}{8}+6\frac{2}{3}\div 1\frac{1}{33})}{6\frac{3}{10}(17\frac{1}{20}-2\frac{13}{16}\times 1\frac{17}{27})}\div\dfrac{12\frac{1}{12}(40\frac{5}{6}-2\frac{7}{9}\times 7\frac{1}{2})}{19\frac{1}{3}(13\frac{5}{6}+6\frac{1}{15}\div\frac{14}{45})}$ = what?

24. The area of a circle is $218\frac{1}{6}$ sq. yd. Find the area and the altitude of the largest equilateral triangle that can be inscribed in it.

25. An assembly room is 74 ft. 3 in. long, 49 ft. 6 in. wide, and 16 ft. 6 in. high. What is the length of the longest straight line that can be stretched inside?

26. The true discount of a note (without interest) dated July 13, 1904, due Sept. 25, 1907, was $281.25, money being worth $6\frac{1}{4}\%$. What would have been the true discount of the same note Nov. 10, 1905, if money were worth $6\frac{2}{3}\%$?

27. If $18\frac{3}{4}\%$ of the loss equals .0225 of the selling price, how much would be received for 80 acres of land that cost $43.75 per acre?

28. A's land includes three separate tracts. The first is a right-angled triangle whose perpendicular is $87\frac{1}{2}\%$ longer than its base. The second is a rectangle whose width is 60% of its length, and whose area is equal to that of the triangle, while the third is a square equal in area to $83\frac{1}{3}\%$ of the others combined. If their total area is 22 acres, how many rods of fence does each tract require?

29. If the cost of an article had been $16\frac{2}{3}\%$ more, the loss by its sale would have been $12\frac{1}{2}\%$ more. Find the per cent of loss.

30. .333 of the difference in the contents of two cisterns is 1248.75 gal., and the contents of the smaller are to those of the larger as 3.75 is to 5.25. How many gallons does each contain?

31. By discounting a 4 months' note at $7\frac{1}{2}\%$ per annum, I paid \$0.75 above true interest. What was the face of the note?

32. A waterman can row $6\frac{3}{4}$ miles with the current in the same time that he can row $4\frac{1}{2}$ miles against it. But if the rate of the current were increased .375 mile per hour, he could row only .6 as rapidly in coming up the stream as when going down. Find his rate per hour of rowing in still water, and also the rate of the current.

33. Two circles, each 100 ft. in diameter, are so constructed that a point in the circumference of each touches the center of the other. What is the area of the space inclosed between their centers?

34. $93\frac{3}{4}\%$ of a certain number, multiplied by $83\frac{1}{3}\%$ of the same number, is 1.44 less than the square of $87\frac{1}{2}\%$ of the number. What is the number?

35. A tree 35 ft. high stands in the center of a square lot. A line 125 ft. long just reaches from the top of the tree to a peg at one of the corners of the lot. What is the area of the lot?

36. Portland, Maine, is in longitude 70° 15′ west, and Portland, Oregon, in longitude 122° 40′ west. Two travelers started at noon of the same day, A from Port-

land, Me., B from Portland, Or. When they met, the true time was 1 hr. 45 min. P.M., but by B's watch it was 40 sec. after 12 M. What was it by A's watch, if both had kept correct time?

37. I bought eggs at 25 cents a dozen. Had I received 5 more eggs for the same money, they would have cost 1 cent per dozen less. How many dozen were bought?

38. $\dfrac{22\frac{1}{2}(11\frac{3}{10}+2\frac{1}{6}\times 4\frac{2}{5}-21\frac{7}{18}\div\overline{2\frac{7}{12}+2\frac{11}{20}})}{9\frac{3}{8}(20\frac{4}{9}+8\frac{8}{15}\div 3\frac{1}{3}\times 1\frac{1}{25}+\frac{4}{5}\times\overline{3\frac{3}{4}\times 8\frac{1}{3}}-25\frac{25}{36})}$ = what?

39. I bought 75 bbl. of pork for 10% less than its value, and sold it for $6\frac{2}{3}$% more than its value. Had I paid $16\frac{2}{3}$% more, and sold it for $12\frac{1}{2}$% less, I should have lost $105. What was the cost per barrel?

40. I bought goods at 20, $16\frac{2}{3}$, and 4% off, and the net price exceeded the discounts by $630. If I sold them at 80% of the list price, what was the selling price and the per cent of gain?

41. By discounting a note at 6% per annum, I made $6\frac{1}{4}$% on the investment. What was the time?

42. The difference of two numbers is 119.75, and the square of their sum is 59,902.5625. What is the cube root of their product?

43. A reservoir is $3\frac{1}{3}$ times as wide as it is deep, and $2\frac{1}{4}$ times as long as it is wide. If 21,093.75 tons of water are required to fill it, what are its dimensions in feet?

44. What is the greatest number that will divide 2870, 3563, and 4454, and leave the same remainder after each division?

45. B has four square tracts of land. The perimeter of the first is .5 mile, and its area is .64 as great as that of the second. The area of the second is .5 as great as that of the third and fourth combined. If the perimeter of the third tract is 136 rods, what are those of the second and fourth?

46. A tax of $57,262.50 was levied in a district containing 1350 voters, each of whom was subject to a poll tax of $0.75. John Russell's real estate was assessed at $3000, his personal property at $750, and he paid one poll. His tax bill was $113.40, including a penalty of 20%. If the value of all the personal property was 20% of the yalue of the real estate, what was the amount of each?

47. If the cost of an article had been 8% more, the loss by its sale would have been 6% more. Find the per cent of loss.

48. A note was dated July 6, 1903, bearing interest at $7\frac{1}{2}\%$. On Feb. 12, 1905, $33\frac{1}{3}\%$ of the amount then due was paid. On Oct. 12, 1906, $66\frac{2}{3}\%$ of the amount then due was paid, and on Jan. 2, 1909, the balance then due, amounting to $306.25 was paid. What was the face of the note?

49. A lodge room is 60 ft. long, 43 ft. wide, and 16 ft. 6 in. high. What is the shortest distance that a spider can crawl by floor and wall, from a northeast lower corner to a southwest upper corner?

50. .8 of the cube of a certain number divided by the cube of .8 of the same number, is equal to .09,765,625 of the cube root of the square of the number. What is the number?

51. B's income is $93\frac{3}{4}\%$ of A's, and C's is $88\frac{8}{9}\%$ of B's. A contracts an annual debt equal to $12\frac{1}{2}\%$ of his income, and B an annual debt equal to $11\frac{1}{9}\%$ of his income, while C lives upon $62\frac{1}{2}\%$ of his, and at the end of five years lends A and B enough to pay their debts, and has $1000 left. What is the income of each?

52. There are three cubes, of which the solid contents of the second are 216 times as great as those of the first, and the surface of the third is $1\frac{7}{9}$ times as great as that of the second. If the surface of the first is $\frac{1}{6}$ sq. ft., what is the surface of a cube equal in volume to that of the three combined?

53. An express train ran from A to B, traveling $\frac{7}{15}$ of the distance at the rate of $33\frac{1}{3}$ miles an hour. An accident then delayed it 20 min., and it completed the remainder of the trip at a rate 10% slower than before, arriving one hour late. What is the distance from A to B?

54. I paid $16,200 for bonds at $7\frac{1}{2}\%$ premium, and sold them at 2% decline, brokerage $\frac{1}{2}\%$ on each operation. I invested the net proceeds in stock at $93\frac{1}{2}$, and sold it at $6\frac{3}{4}\%$ advance, brokerage $\frac{1}{4}\%$ on each operation. What was the net gain or loss on the whole transaction?

55. Two buildings, each 45 ft. long and 30 ft. high, are situated upon an alley, and stand directly opposite. If the longest straight line that can be stretched between them is 55 ft. in length, what is the width of the alley?

56. What is the square root of the sum of the cubes of all the integers from 1 to 1414 inclusive?

57. A merchant imported wine for which he paid $2.50 a gallon. 4% was lost by leakage, $16\frac{2}{3}\%$ of the remainder was spoiled, and a duty of 25% was charged upon the balance. If he sold it at a profit of 20% on the entire cost, what was the selling price per gallon?

58. A speculator sold two farms at the same price, gaining $27\frac{1}{2}\%$ on one, and losing 15% on the other. If his net gain was $120, what was the cost of each farm?

59. A lot is in the form of a right-angled triangle whose base is 20 rods, and area 3 acres. What are the dimensions of a similar lot whose area is 6.75 acres?

60. A rectangular bin will hold exactly 168 bu. 3 pk. of wheat. If its length, breadth, and depth inside have the ratios of $16\frac{2}{3}$, $11\frac{2}{3}$, and 5, what are its dimensions?

61. $$\frac{1.125}{20.25}\left\{\frac{89.32+37.2425}{141.7-113.575}\times\frac{.31575\div.842}{.3125\times.72}+\frac{.9375\times.128}{6.032-5.9552}+\frac{33.75}{70.3125}\div\frac{5.12}{6.4}\right\}=?$$

62. The lower base of the frustum of a pyramid is 33 ft. 9 in. square, the upper base 15 ft. square, and its altitude is 50 ft. What are the corresponding dimensions of a similar frustum whose volume is 15,960 cubic feet?

63. A stable is 80 ft. long, 45 ft. wide, and 20 ft. high to the eaves. The roof rises 14 ft. above the eaves, and projects 1 foot beyond the walls. Find the total cost of the lumber inclosing the stable at $25 per M, and of the roof at $3.50 per square.

64. If the cube root of a certain number is divided by the fourth root of the same number, the square root of the quotient will be 2. What is the number?

65. A man dying, left $31,500 to be divided among his widow, son, and daughter, on condition that, if the son died, the widow should have $\frac{4}{7}$ of the money; but if the daughter died, the widow should have $\frac{4}{5}$ of it. Both son and daughter survived. How much did each heir receive?

66. The solid contents of a rectangular block of granite are 32 cu. ft. If the dimensions are as $4\frac{1}{2}$, $13\frac{1}{2}$, and $20\frac{1}{4}$, what are its dimensions, and the length of its diagonal?

67. I exchanged 5% bonds at 117 for 4% bonds at 91, thereby increasing my income by $5. What was the par value of each kind of bonds?

68. A grain merchant has four bins whose capacities are respectively $113\frac{3}{4}$, $193\frac{3}{8}$, $111\frac{9}{16}$, and $145\frac{1}{32}$ bu. How many times could the fourth bin be filled from the smallest quantity of grain that could be exactly measured in each?

69. A rectangular farm has an area of 62 A. 80 sq. rd., and its width is to its length as 1 is to 1.5625. Another farm is similar in shape, with an area of 22 A. 80 sq. rd. Find the difference in the cost of fencing the farms at $0.625 a rd.

70. The area of a circle described by one revolution of a cone about its apex is 132.7326 sq. ft. If its altitude is 6 ft., what is its volume?

71. B had $56\frac{1}{4}\%$ more money than A, and C 24% less than B. B lent 20% of his money to C, C then lent $16\frac{2}{3}\%$ of his money to A, after which A lent 5% of his money to B. If B then had $\$93.60$ more than C, how much less than C had A?

72. The product of six numbers is 53,760. Two of them are equal, the others are unequal. What is the sum of the numbers?

73. A reservoir is $2\frac{2}{3}$ times as long as it is wide, and $3\frac{3}{4}$ times as wide as it is deep. If its exact capacity is 9375 tons of water, what are the dimensions of a similar reservoir whose capacity is 4800 tons?

74. I bought 150 bbl. of flour for $6\frac{1}{4}\%$ less than its value, and sold it for $12\frac{1}{2}\%$ more than its value. I invested the proceeds in 50 bbl. of pork at 8% more than its value, and sold it for 12% less than its value, thereby losing $\$12.50$ on the whole transaction. Find the cost per barrel of the flour, and the selling price per barrel of the pork.

75. The distance from one of the corners of the base of a square pyramid to its apex is 162.5 ft., and the area of the base is 50 sq. rd. If the granite composing it weighs 200 lb. per cubic foot, what is the weight of the pyramid?

76. A capitalist bought New York 5's at 106, brokerage $\frac{1}{4}\%$, and U. S. 4's at $112\frac{3}{8}$, brokerage $\frac{1}{8}\%$. The latter cost him $\$1406.25$ less than the former, and yielded him $\$325$ less income. How much was invested in each kind of bonds, and what was the amount of brokerage?

77. The width of a rectangular solid is $\frac{3}{4}$ of its length, its height is $\frac{5}{6}$ of its width, and its entire surface is 40 sq. ft. 140 sq. in. Find the surface of a cube whose volume is $\frac{9}{10}$ as great. Also, find the volume of a globe whose surface is .5236 as great as that of the cube.

78. A dealer bought apples at \$2.40 a barrel. He sold 40% of them so that .125 of the loss equaled .025 of the selling price, and the remainder so that .16 of the gain equaled .0576 of the selling price. If he cleared \$32.50 on the entire transaction, how many barrels were sold?

79. If an octagon is inscribed in a circle 50 ft. in diameter, what will be the length of one of its sides, and also the length of a perpendicular dropped from the center of the circle to a point in the center of one of the sides of the octagon?

80. $$\left\{ \frac{\sqrt[3]{6\{4^5 \div (\frac{2}{3})^8\}}}{\sqrt{\frac{9}{10}(\sqrt[3]{250} \times \sqrt[3]{256})}} \div \frac{\sqrt[3]{\frac{125}{16}(3^6 \div 2^5)}}{\sqrt{\frac{75}{4}(4096^{\frac{1}{2}} - 4096^{\frac{1}{3}})}} + \frac{\sqrt{3(\sqrt{18} \times \sqrt{8})}}{\sqrt[3]{8\{(\frac{5}{8})^2 + (1\frac{1}{4})^2\}}} \times \frac{\sqrt[4]{12 \times 75 - 125 \div \frac{5}{11}}}{4\sqrt[6]{288 \div \frac{8}{15} + 210 \times \frac{9}{10}}} \right\}^{\frac{1}{2}} = ?$$

81. The ratios of the edges of three cubical blocks of marble are as 3, 4, and 5, while the surface of a cubical block of the same kind of marble equal in volume to that of the three combined is $13\frac{1}{2}$ sq. ft. If a cubic foot of marble weighs 192 lb., what is the weight of the respective blocks?

82. In the preceding problem, had the ratios been as 1, 6, and 8, what would have been the respective weights of the first three blocks?

83. If 5 pineapples are equal in value to 72 peaches, 20 peaches to 48 pears, 32 pears to 75 apples, 36 apples, to 25 apricots, and 24 apricots to 80 plums, how many pineapples could be bought for 75 cents if 1250 plums cost $1?

84. How many acres are there in a square field containing as many sq. rods as are equal to the number of boards in the fence surrounding it, if the boards are 7 ft. 4 in. long, and the fence is four boards high?

85. The amount of a certain principal at simple interest for 3 yr. 2 mo. 12 da. at a certain rate is $1162.50, and for $\frac{3}{4}$ of the time at $\frac{8}{9}$ of the rate, it is $1087.50. Find the principal and the rates.

86. The areas of the upper and lower bases of the frustum of a square pyramid are respectively 50 sq. ft. and 200 sq. ft., its altitude 12 ft., and its weight 140 tons. If the pyramid were entire, what would be its weight?

87. A note dated April 7, 1908, was made payable July 13, 1912, with interest at $7\frac{1}{2}\%$. On April 23, 1910, the true discount of the note, at $4\frac{1}{2}\%$ was $67.50. What was the face of the note, and what would have been the bank discount at 8%, April 4, 1912?

88. The extreme terms of a mean proportion are 15.625 and 22.5. What is one of the other terms?

89. If 6 acres will pasture 18 horses 4 weeks, and 8 acres will pasture 20 horses 6 weeks, how many acres would 15 horses require for 12 weeks, the grass growing uniformly? Also, how many horses could be pastured on 20 acres for 8 weeks?

90. The width of a rectangular block of marble is $1\frac{1}{2}$ times its height, its length is twice its width, and the length of its diagonal is 35 inches. Find the cost of polishing its entire surface at 20 cents a sq. foot.

91. A farm is in the shape of a trapezoid 62 rd. wide at one end, 34 rd. at the other, with an area of 35 A. How far from the wider end should a fence be built straight across, that the two parts may be of equal area?

92. A lot is in the form of a trapezium whose sides are respectively 48, 14, 30, and 40 rods in length, the last two lying at right angles. How much more fence does it require than a square lot whose area is $\frac{8}{13}$ as great, and what is the difference in the value of the lots at \$50 an acre?

93. I bought a farm on 4 mo. credit. At the end of 9 mo. I sold it, giving the buyer the choice of 5 mo. time without interest, or $3\frac{1}{2}\%$ discount for cash. He chose the former, thereby losing \$5. My net gain was \$625. Money being worth 8%, what was the original value of the farm?

94. $41\frac{2}{3}\%$ of the product of two numbers is 3125, and the quotient of the greater divided by the less is 1.6875. What are the numbers?

95. I bought stock at 101, and sold it at $105\frac{3}{8}$, brokerage $\frac{3}{8}\%$ on each operation. I invested the net proceeds in 5% bonds at $87\frac{1}{2}$, which I exchanged for 4% bonds at 75, thereby increasing my income \$40. What was my net gain on the first operation?

96. Five rectangular solids of different sizes have each a diagonal of 57.75 inches. What are their exact dimensions?

ANSWERS

Art. 12. — **1.** 867. **2.** 1098. **3.** 13,017. **4.** 16,306. **5.** 19,307. **6.** 231,490. **7.** 293,658. **8.** 311,111. **9.** 327,415. **10.** 300,244. **11.** 243,125. **12.** 400,000. **13.** 1,028,979. **14.** 1,892,314. **15.** 1,000,000. **16.** 1,080,905. **17.** 1,879,109. **18.** 3,381,591. **19.** 35,367. **20.** 4,200,030. **21.** 810,900. **22.** 1,605,043. **23.** 1,071,512. **24.** 320 pounds. **25.** 1587 barrels. **26.** $10,000. **27.** 43,875 bushels. **28.** 165,890 bricks. **29.** 18,069 barrels. **30.** 50,000 acres. **31.** 35,893 gallons. **32.** 99,999 men. **33.** 53,764. **34.** 6017 pupils. **35.** 69,696 bushels. **36.** $673,125. **37.** 66,465 sq. miles. **38.** $50,000. **39.** 210,000 pounds. **40.** 13,471,840. **41.** 999,999. **42.** $1,250,000. **43.** 251,748. **44.** 512,425 feet. **45.** 1,482,009 pounds. **46.** 1,387,654 sq. feet. **47.** 637,295 gallons. **48.** 640 acres. **49.** 15,218 bushels; 694,936 cents. **50.** $18,125. **51.** 13,025. **52.** 118,768. **53.** 68,885. **54.** 945,343. **55.** 12,744,972. **56.** 3,683,414.

Art. 18. — **1.** 415. **2.** 4321. **3.** 3062. **4.** 3333. **5.** 2456. **6.** 5647. **7.** 10,099. **8.** 10,875. **9.** 7345. **10.** 12,908. **11.** 10,101. **12.** 41,976. **13.** 5625. **14.** 18,750. **15.** 108,627. **16.** 24,854. **17.** 5386. **18.** 530,865. **19.** 716,249. **20.** 4,516,438. **21.** 23,875. **22.** 562,870. **23.** 320,698. **24.** 556,677. **25.** 4,016,924. **26.** 123,456. **27.** 1,756,980. **28.** 3,067,859. **29.** 1,907,305. **30.** 39,480. **31.** $1250. **32.** 4593 miles. **33.** 20,850 bushels. **34.** $39,876. **35.** 2547 pupils. **36.** 8485 bushels. **37.** $60,375. **38.** 138,024 barrels. **39.** 152,871 pounds. **40.** 107,420 sq. miles. **41.** 287,563 letters. **42.** 138,074 ties. **43.** 535,827 acres. **44.** 739,839. **45.** 3,154,946 bales.

Art. 19. — **1.** 1000. **2.** 1575. **3.** 3750. **4.** 28,739. **5.** 12,345. **6.** 7932. **7.** 35,279. **8.** 102,050. **9.** 160 acres. **10.** 52 years. **11.** $25,000. **12.** 8045 volumes. **13.** 10,000 bushels.

Art. 22. — **1.** 1046. **2.** 1386. **3.** 2468. **4.** 1780. **5.** 4704. **6.** 6125. **7.** 7872. **8.** 21,249. **9.** 19,413. **10.** 31,300. **11.** 123,365. **12.** 294,936. **13.** 439,166. **14.** 596,784. **15.** 888,885. **16.** 30,760 pounds.

Art. 23. — **1.** 2130. **2.** 4872. **3.** 12,802. **4.** 21,068.

5. 31,111. **6.** 42,772. **7.** 60,198. **8.** 73,500. **9.** 88,688. **10.** 98,901. **11.** 160,000. **12.** 605,885. **13.** 1,167,828. **14.** 1,882,824. **15.** 3,217,500. **16.** 4,051,875. **17.** 5,317,326. **18.** 7,228,488. **19.** 8,668,676. **20.** 3,200,000. **21.** 2,522,520. **22.** 6,847,456. **23.** 12,085,164. **24.** 22,128,603. **25.** 36,450,000. **26.** 3,035,725. **27.** 7,004,792. **28.** 13,525,500. **29.** 12,800,000. **30.** 67,048,164. **31.** 600 packages. **32.** $2160. **33.** 10,080 pounds. **34.** 8112 bushels. **35.** 45,600 cents. **36.** $28,925. **37.** 27,756 gallons. **38.** 171,360 sheets. **39.** 146,016 miles. **40.** $105,400. **41.** 130,733 pounds. **42.** 525,600 minutes. **43.** 8,528,345 pages. **44.** 4,571,136 gallons. **45.** 204,984 ems. **46.** 385,875 cents. **47.** 500,000 hills. **48.** 1,900,800 cents. **49.** 564,480 pounds. **50.** 10,707,552 cents. **51.** 1,115,775 cu. ft. **52.** 51,200,000 peas. **53.** 18,817,920 sq. ft. **54.** 28,753,920 cents. **55.** 31,250 pages; 62,500 columns; 4,000,000 lines; 100,000,000 letters.

Art. 25. — **1.** 43,740. **2.** 259,200. **3.** 675,000. **4.** 3,456,000. **5.** 38,352,000. **6.** 99,750,000. **7.** 100,000,000. **8.** 112,500,000. **9.** 281,187,500. **10.** 111,111,000. **11.** 500,000 cents. **12.** 1,382,400 gr. **13.** 10,000,000 cu. ft. **14.** 25,920,000 sq. ft. **15.** 14,976,000 cents.

Art. 32. — **1.** 423. **2.** 318. **3.** 315. **4.** 475. **5.** 514. **6.** 654. **7.** $653\frac{7}{8}$. **8.** $745\frac{1}{2}$. **9.** 24,784. **10.** 20,374. **11.** $18,741\frac{3}{5}$. **12.** $42,737\frac{5}{8}$. **13.** 45,132. **14.** 54,321. **15.** 62,500. **16.** $61,728\frac{3}{10}$. **17.** 72,135. **18.** 82,060. **19.** 906,875. **20.** $345,698\frac{5}{8}$. **21.** 415,263. **22.** 625,625. **23.** $708,090\frac{1}{8}$. **24.** 876,543. **25.** $101,010\frac{1}{10}$. **26.** 128 quarts. **27.** 987 yards. **28.** $1485. **29.** 2160 pounds. **30.** 5326 balls. **31.** 6250 bills. **32.** $3125. **33.** 8764 bricks. **34.** 15,625. **35.** 12,500 seconds. **36.** $45,639. **37.** 38,726 passengers. **38.** 76,543. **39.** 62,785 revolutions. **40.** 25,248 boards. **41.** 1625 bricks. **42.** 5364 sacks. **43.** 4550 cents. **44.** 1389 beans. **45.** 216 pounds. **46.** 2096 horses. **47.** 3125 cents. **48.** 1350 miles. **49.** 1024 horses. **50.** 3967 boxes.

Art. 33. — **1.** 123. **2.** 246. **3.** $354\frac{7}{17}$. **4.** 412. **5.** 543. **6.** 625. **7.** $734\frac{6}{25}$. **8.** 856. **9.** 928. **10.** 1024. **11.** 2160. **12.** $3471\frac{25}{48}$. **13.** $4375\frac{49}{55}$. **14.** 5625. **15.** 6161. **16.** 7654. **17.** $8029\frac{72}{73}$. **18.** 9687. **19.** 8524. **20.** 7906. **21.** 6789. **22.** 8070. **23.** 9876. **24.** $10,101\frac{100}{101}$. **25.** 750. **26.** 852. **27.** 978. **28.** 1024. **29.** 1280. **30.** 2658. **31.** 3745. **32.** 4736. **33.** 5625. **34.** 6346. **35.** 7875. **36.** 8739. **37.** 5280. **38.** 5786. **39.** 6666. **40.** 15,625. **41.** 475 pounds. **42.** 352 rails. **43.** 398 bushels. **44.** 647 cubic feet. **45.** $1375. **46.** 256 days. **47.** 743. **48.** 1625 times. **49.** 1280 sacks. **50.** $750. **51.** 863. **52.** 216 pounds. **53.** 4750 square miles. **54.** 1947 pounds. **55.** 359 days. **56.** 250 square miles. **57.** 2175 cents. **58.** 7462 rails. **59.** 4736. **60.** $85. **61.** 647 miles. **62.** 512 pounds. **63.** 313 days. **64.** $1475. **65.** 576 Troy pounds. **66.** $45. **67.** 1575 bricks. **68.** 45 cents. **69.** 425 boxes. **70.** 196

cents. **71.** 250 acres. **72.** 144 packages. **73.** 24 volumes. **74.** 68 laborers. **75.** 99 cords.

Art. 34. — **1.** 72. **2.** 123. **3.** 512. **4.** 168. **5.** 3825 bushels. **6.** 14,040 miles. **7.** 391 acres. **8.** 365 days. **9.** 216. **10.** $12,000.

Art. 35. — **1.** $125\frac{6}{10}$. **2.** $243\frac{49}{100}$. **3.** $367\frac{485}{1000}$. **4.** $406\frac{2817}{10000}$. **5.** $5386\frac{24}{100}$. **6.** $6940\frac{718}{1000}$. **7.** $1788\frac{3}{40}$. **8.** $3387\frac{113}{250}$. **9.** $5318\frac{1418}{1800}$. **10.** $691\frac{1783}{3000}$. **11.** 5840. **12.** 1776. **13.** 5460. **14.** 629.

Art. 36. — **1.** $1. **2.** 999 bushels. **3.** 8000. **4.** 128 acres. **5.** 50,623. **6.** 480 cents. **7.** 1001 sheep. **8.** 48 hours. **9.** 324. **10.** 105 days. **11.** 19 days. **12.** 75. **13.** 99. **14.** 1125 cents. **15.** 135 days. **16.** 24 sacks. **17.** 25,344 miles. **18.** XLV. **19.** $1820. **20.** $20. **21.** $11,250. **22.** 486. **23.** $1925. **24.** 365 barrels. **25.** Nothing. **26.** A, $6250; B, $18,750. **27.** 456. **28.** 351 rows. **29.** 13,200 sq. yards. **30.** 70,200 bushels.

Art. 37. — **1.** 275. **2.** 159. **3.** 144. **4.** 40. **5.** 1. **6.** 100. **7.** 1444. **8.** 11. **9.** 0. **10.** 75. **11.** 0. **12.** 1. **13.** 16.

Art. 40. — **1.** 1200 cents. **2.** $15. **3.** 680 m. **4.** 94 ct. 7 m. **5.** $156.25. **6.** 48,000 m. **7.** $34.567. **8.** 4375 ct. **9.** 516,000 m. **10.** 60,000 ct. **11.** $62.50. **12.** 100,101 m. **13.** 48,048 m. **14.** $550.50. **15.** 7,812,500 ct. **16.** $3906.25. **17.** 606,606 m. **18.** $2020.202. **19.** 125,050 m. **20.** $1000.

Art. 41. — **1.** $100. **2.** $303.38. **3.** $222.572. **4.** $1220. **5.** $3514.70. **6.** $7020. **7.** $2500. **8.** $2930.75. **9.** $1581.20. **10.** $10,505.05. **11.** $3750. **12.** $24,651.73. **13.** $8146.25. **14.** $7248.75. **15.** $50,000.

Art. 42. — **1.** $163.87. **2.** $100.90. **3.** $419.76. **4.** $656.25. **5.** $357.85. **6.** $75.75. **7.** $445.50. **8.** $487.125. **9.** $186.43. **10.** $567.89. **11.** $396.25. **12.** $495.40. **13.** $3352.85. **14.** $540.50. **15.** $216.20. **16.** $7.50. **17.** $260.75. **18.** $679.55. **19.** $1560.60. **20.** $12,500.

Art. 43. — **1.** $159.375. **2.** $731.52. **3.** $1500. **4.** $3457.125. **5.** $5625. **6.** $9203.04. **7.** $2000. **8.** $1843.20. **9.** $123.456. **10.** $15,000. **11.** $1440. **12.** $367.50. **13.** $13,200. **14.** $393.75. **15.** $540. **16.** $483.84. **17.** $19,008. **18.** $48. **19.** $5000. **20.** $2011.50. **21.** $1555.20. **22.** $3240. **23.** $216. **24.** $172.80. **25.** $380.16. **26.** $630. **27.** $1350. **28.** $8985.60. **29.** $7654.50. **30.** $1872. **31.** $1531.25. **32.** $600. **33.** $2764.80. **34.** $14,400. **35.** $160,000. **36.** $1058.40. **37.** $18,895.68. **38.** $4321.80. **39.** $3283.20. **40.** $139,276.80.

Art. 44. — **1.** $24.35. **2.** $18.75. **3.** $15.06. **4.** $2.345. **5.** $3.125. **6.** $5.625. **7.** $9.999. **8.** $2.016. **9.** $8.375. **10.** $390.625. **11.** 144. **12.** 392. **13.** 444. **14.** 3125. **15.** 750. **16.** 625. **17.** 3125. **18.** 640. **19.** 1250. **20.** 1234. **21.** 225 bushels. **22.** 250 gallons. **23.** $56.25. **24.** $4.875. **25.** 72 turkeys. **26.** 216 miles. **27.** $1.875. **28.** $0.225. **29.** 1760 yards. **30.** 5760

feet. **31.** $0.325. **32.** $0.625. **33.** 1296 pounds. **34.** 640 cattle. **35.** $0.075. **36.** 42 gallons. **37.** $1.125. **38.** 250 bales. **39.** $1.65. **40.** 25 acres; 25 bushels.

Art. 45. — **1.** $168.75. **2.** $2000. **3.** $1064.09. **4.** $1980.50. **5.** 24 barrels. **6.** $182.65. **7.** $1.50. **8.** $1843.20. **9.** $1.10. **10.** $500. **11.** 45 gallons. **12.** 144 barrels. **13.** 245 bushels. **14.** 80 barrels. **15.** 138 sheep. **16.** Gained $100. **17.** 40 gallons. **18.** $68.75. **19.** $50. **20.** $420. **21.** $3250. **22.** 1225 loads. **23.** 53 bushels. **24.** $19.50. **25.** 9 hours. **26.** 42 gallons. **27.** $0.40. **28.** 45. **29.** $4500. **30.** $480. **31.** $10,025.01. **32.** 296 pounds. **33.** 35 boxes. **34.** $5355. **35.** 84 barrels. **36.** $96.32. **37.** 25 cows. **38.** 24 acres. **39.** C, $26.25; D, $21. **40.** $0.25. **41.** $555. **42.** $33.80. **43.** $0.95. **44.** 69. **45.** $1260. **46.** 96 days. **47.** Nothing.

Art. 46. — **1.** $21.35. **2.** $360.75. **3.** $10.39. **4.** $532.90. **5.** $300.

Art. 51. — **1.** 185 pt. **2.** 721 pt. **3.** 1286 qt. **4.** 4825 pt. **5.** 7960 qt. **6.** 31,625 pt. **7.** 312 bu. **8.** 781 bu. 1 pk. **9.** 466 bu. 3 pk. 2 qt. **10.** 999 bu. 3 pk. 7 qt. 1 pt. **11.** $23.04. **12.** $8.75. **13.** $40. **14.** 7 bu. 3 pk. 2 qt. **15.** 128 boxes. **16.** 67 bu. 2 pk. **17.** 150 barrels. **18.** $345.60. **19.** 18 cans. **20.** 400 baskets; 125 bushels.

Art. 52. — **1.** 19,800 oz. **2.** 12,345 lb. **3.** 304,188 oz. **4.** 96,875 lb. **5.** 500,000 oz. **6.** 18 cwt. 75 lb. **7.** 49 T. 1 cwt. 25 lb. **8.** 17 cwt. 16 lb. 15 oz. **9.** 3 T. 4 cwt. 5 lb. 6 oz. **10.** 31 T. 5 cwt. **11.** $13.50. **12.** $108. **13.** 30 loads. **14.** 15 tons. **15.** $487.50. **16.** $536.25. **17.** 6732 tons. **18.** 480 boxes. **19.** $675. **20.** 125 boxes; 15 cwt.

Art. 53. — **1.** 4796 gr. **2.** 5932 pwt. **3.** 293,175 gr. **4.** 51,976 pwt. **5.** 1,234,567 gr. **6.** 76 lb. 4 oz. 4 pwt. **7.** 50 oz. **8.** 12 lb. 11 oz. 10 pwt. 9 gr. **9.** 49 lb. 11 oz. 19 pwt. 23 gr. **10.** 111 lb. **11.** $360. **12.** 3 oz. **13.** $140,400. **14.** 12 lb. 6 oz. **15.** $750. **16.** 900 medals. **17.** 84 cases. **18.** $150. **19.** 10 lb. 5 oz. **20.** $375.

Art. 54. — **1.** 3430 ℈. **2.** 5151 gr. **3.** 50,000 gr. **4.** 144,310 gr. **5.** 654,321 gr. **6.** 11 ℥ 1 ℨ 1 ℈ 11 gr. **7.** 45 ℔. **8.** 5 ℔ 4 ℥ 3 ℨ 2 ℈ 1 gr. **9.** 49 ℔ 11 ℥ 7 ℨ 2 ℈ 19 gr. **10.** 33 ℔. **11.** $5.85. **12.** $7.20. **13.** 1 ℔ 8 ℥ 2 ℨ 1 ℈ 10 gr. **14.** $75. **15.** 1 ℔ 2 ℥ 3 ℨ. **16.** $6480. **17.** 2,000,000 pellets. **18.** 10 ℥. **19.** 24 capsules.

Art. 55. — **1.** 250 pt. **2.** 2250 gi. **3.** 2793 pt. **4.** 15,624 gi. **5.** 34,567 gi. **6.** 90 gal. 2 qt. 1 pt. **7.** 45 gal. **8.** 12 hhd. **9.** 15 hhd. 43 gal. 1 qt. 1 gi. **10.** 87,696 gi. **11.** $86.40. **12.** $50. **13.** $20. **14.** $28.80. **15.** $75. **16.** $1008. **17.** $20.48. **18.** 50 gal. **19.** $1.20.

Art. 56. — **1.** 10,250 rd. **2.** 9750 in. **3.** 63,348 in. **4.** 25 mi. **5.** 33 mi. **6.** 1562 yd. 1 ft. 6 in. **7.** 99 mi. 99 rd. **8.** 1728 yd. 3 in. **9.** 35,964 in. **10.** 50,000 rd. **11.** 40 minutes. **12.** $5280. **13.** $16,500. **14.** $30,720. **15.** $26,400. **16.** 13,200 revolutions. **17.** 5 miles. **18.** 7 ft. 6 in.

Art. 57. — **1.** 7980 sq. rd. **2.** 38,520 sq. in. **3.** 375,625 sq. rd. **4.** 129,599 sq. in. **5.** 1,089,000 sq. ft. **6.** 324 acres. **7.** 28 sq. yd. 7 sq. ft. 29 sq. in. **8.** 4 sq. mi. **9.** 13 acres. **10.** 2 sq. mi. 320 A. **11.** $178.20. **12.** $2625. **13.** $480. **14.** $82,500. **15.** $187.50. **16.** 14,080 sq. yd. **17.** 80 acres. **18.** 1125 sq. yd. **19.** $630. **20.** 40 acres.

Art. 58. — **1.** 80 sq. yd. **2.** 6 acres. **3.** $13.86. **4.** $1750. **5.** $344.25. **6.** $8.50. **7.** $256.50. **8.** $624. **9.** $30,800. **10.** $760.50.

Art. 59. — **1.** 15 ft. **2.** 150 ft. **3.** 27 ft. **4.** 45 rd. **5.** 42 ft. **6.** $1250. **7.** $63. **8.** $143.50. **9.** 3 miles. **10.** 30 ft. **11.** $35. **12.** $3200. **13.** 11,520 bricks. **14.** $0.75. **15.** 1500 sods. **16.** $177.60. **17.** 320 tiles. **18.** $75. **19.** 1152 posters. **20.** $210. **21.** 320 boards. **22.** $50,000. **23.** 384 panes. **24.** 10 A. 80 sq. rd. **25.** $1687.50. **26.** $184.32. **27.** 64 bu. **28.** $60. **29.** $543. **30.** $14.40.

Art. 61. — **1.** 3096 cu. ft. **2.** 470,460 cu. in. **3.** 250,000 cu. in. **4.** 456 cu. yd. 25 cu. ft. **5.** 1,000,000 cu. in. **6.** 64 cu. yd. **7.** 99 C. 127 cu. ft. **8.** 5 cu. yd. 25 cu. ft. 500 cu. in. **9.** 33 cords. **10.** 9 cords. **11.** 208 cu. yd. **12.** 25 cords. **13.** 24 headstones. **14.** $1600. **15.** 108 boards. **16.** 1000 blocks. **17.** 288 gallons. **18.** 108 cakes. **19.** 72 bushels. **20.** 75 posts. **21.** 90 cans. **22.** 1008 boxes. **23.** 5 feet. **24.** 10 days. **25.** $4800.

Art. 62. — **1.** 10,000 min. **2.** 416,500 sec. **3.** 788,645 sec. **4.** 527,040 min. **5.** 7,862,400 sec. **6.** 6 da. 23 hr. 59 min. 59 sec. **7.** 5 leap years. **8.** 313 days. **9.** 52 wk. 1 da. **10.** 31,556,926 sec. **11.** $767.50. **12.** 10 da. 10 hr. **13.** 50 miles. **14.** 26 hr. 40 min. **15.** 24 hours. **16.** 15 acres. **17.** 80 days. **18.** 4 hours. **19.** 3 days. **20.** 10,125 bu.

Art. 64. — **1.** $100.50. **2.** $668.25. **3.** $16.80. **4.** 85 barrels. **5.** $4.80. **6.** $0.10. **7.** 15 lb. **8.** Nothing. **9.** $1012.50. **10.** $10,800. **11.** $810. **12.** $225. **13.** $2250. **14.** $144.

Art. 65. — **1.** $28.80. **2.** $38.40. **3.** $60. **4.** $21.60. **5.** $900. **6.** $139.968. **7.** $144. **8.** $115.20. **9.** $103.68. **10.** 100 cubes. **11.** $1000. **12.** 315 dozen. **13.** 5400 cards. **14.** $626.40. **15.** 45 miles. **16.** $25. **17.** $11.55. **18.** $67.20. **19.** 12 lb. **20.** 6 tons. **21.** $40,000. **22.** $100. **23.** 9 tons. **24.** 24 hours. **25.** $1540. **26.** 81 thousand. **27.** $20. **28.** $146.25. **29.** $0.075. **30.** $4216.32. **31.** 15 ft. **32.** 6 T. 5 cwt. **33.** 1 lb. **34.** 8888 tons. **35.** 35,552 rev. **36.** $31.25. **37.** $600. **38.** 18 cords. **39.** 99 da. **40.** 1 lb 6 ʒ 6 ʒ. **41.** 45 bbl. **42.** 840 men. **43.** $36.45. **44.** $0.50. **45.** $6635.52. **46.** 15 bu. **47.** 1875 bbl. **48.** $0.45. **49.** $180. **50.** $148.50. **51.** $100.80. **52.** 135 mi. **53.** 58 bu. **54.** 24 sq. yd. **55.** 25 min. **56.** $270. **57.** 7 hr. **58.** $49.95. **59.** 640 rev. **60.** 1 lb. **61.** 2160 sills. **62.** 2 mi. 64 rd. **63.** 42 gal. **64.** $100.40. **65.** $0.025. **66.** 6 hr.

Art. 66. — **1.** 100 bu. 1 pk. 1 qt. 1 pt. **2.** 75 gal. **3.** 39 T. 8 cwt. 87 lb. 3 oz. **4.** 228 cu. yd. **5.** 99 A. 28 sq. yd. 4 sq. ft. **6.** 75 C. 125 cu. ft. 1500 cu. in. **7.** 52 wk. 1 da. 5 hr. 48 min. 50 sec. **8.** 99 mi. 319 rd. 5 yd. 1 ft. 6 in. **9.** 32 lb. 6 oz. 9 pwt. 8 gr. **10.** 50 ℔. **11.** 239 bu. 2 pk. 5 qt. **12.** 300 gal. 2 qt. 3 gi. **13.** 16 T. **14.** 360 cu. yd. 20 cu. ft. **15.** 150 cords. **16.** 575 A. 30 sq. yd. **17.** 75 da. 1 hr. 50 min. **18.** 37 lb. 6 oz. **19.** 94 ℔ 7 ℥ 4 ʒ 15 gr. **20.** 40 mi. 319 rd. 5 yd. 1 ft. 6 in.

Art. 67. — **1.** 33 bu. 3 pk. 3 qt. **2.** 22 gal. 0 qt. 1 pt. 3 gi. **3.** 17 T. 13 cwt. 74 lb. 14 oz. **4.** 48 cu. yd. 26 cu. ft. 1299 cu. in. **5.** 79 A. 60 sq. rd. 11 sq. yd. 5 sq. ft. 143 sq. in. **6.** 50 mi. 250 rd. 0 yd. 2 ft. 5 in. **7.** 28 cu. ft. 1629 cu. in. **8.** 23 wk. 5 da. 22 hr. 56 min. 45 sec. **9.** 10 lb. 11 oz. 19 pwt. 23 gr. **10.** 3 ℔ 1 ℥ 4 ʒ 0 ℈ 10 gr. **11.** 150 bu. **12.** 248 gal. 1 qt. 1 pt. **13.** 1 T. 14 cwt. 50 lb. 12 oz. **14.** 80 mi. 202 rd. 3 yd. 1 ft. **15.** 159 A. 30 sq. yd. 2 sq. ft. **16.** 27 cu. yd. 17 cu. ft. 1172 cu. in. **17.** 40 cords. **18.** 2 wk. 2 da. 16 hr. 40 min. 50 sec. **19.** 1 ℔ 10 ℥ 3 ʒ 1 ℈. **20.** 3 lb. 9 oz. 8 pwt.

Art. 68. — **1.** 56 yr. 2 mo. 3 da. **2.** 4 yr. 3 mo. 10 da. **3.** 50 yr. **4.** 194 yr. 9 mo. 2 da. **5.** 368 yr. 6 mo. **6.** 44 yr. 4 mo. 7 da. **7.** 107 yr. 10 mo. 4 da. **8.** Aug. 8, 1899. **9.** Sister, July 18, 1868; Brother, Sept. 21, 1863. **10.** 78 yr. 2 mo. 23 da.

Art. 69. — **1.** 111 da. **2.** 147 da. **3.** 179 da. **4.** 150 da. **5.** 182 da. **6.** 300 da. **7.** $675. **8.** Aug. 25. **9.** March 26. **10.** Nov. 30.

Art. 70. — **1.** 55 bu. 7 qt. 1 pt. **2.** 96 gal. 1 pt. 2 gi. **3.** 63 T. 12 cwt. 90 lb. **4.** 315 rd. 5 yd. 2 ft. 6 in. **5.** 640 A. 80 sq. rd. 29 sq. yd. 1 sq. ft. 12 sq. in. **6.** 400 C. 4 cu. ft. **7.** 52 wk. 1 da. 6 hr. 18 min. **8.** 212 lb. **9.** 101 ℔. **10.** 875 cu. yd. **11.** 774 bu. 1 pk. **12.** 1148 gal. 1 qt. 1 pt. 2 gi. **13.** 1000 yards. **14.** 46 T. 17 cwt. 50 lb. **15.** 15 A. 30 sq. yd. **16.** 313 days. **17.** 6 ft. **18.** 125 cords. **19.** $45,000. **20.** 99 ℔ 1 ℥.

Art. 71. — **1.** 14 bu. 1 pk. 2 qt. 1 pt. **2.** 5 T. 15 cwt. 25 lb. 5 oz. **3.** 16 gal. 2 qt. 1 pt. 2 gi. **4.** 12 lb. 8 oz. 10 pwt. 12 gr. **5.** 22 yd. 2 ft. 9 in. **6.** 5 ℔ 4 ℥ 3 ʒ 2 ℈ 1 gr. **7.** 2 wk. 14 hr. 37 min. 57 sec. **8.** 4 C. 21 cu. ft. 576 cu. in. **9.** 15 cu. yd. 20 cu. ft. 25 cu. in. **10.** 1 A. 45 sq. rd. 4 sq. ft. 108 sq. in. **11.** 24 bu. 2 pk. 5 qt. 1 pt. **12.** 1 T. 5 cwt. 20 lb. 8 oz. **13.** 480 times. **14.** 18 sq. yd. 6 sq. ft. 108 sq. in. **15.** 1 C. 5 cu. ft. 576 cu. in. **16.** 1875 medals. **17.** 25 yd. 2 ft. 3 in. **18.** 6 da. 12 hr. 30 min. **19.** 1 ℔ 5 ʒ 16 gr. **20.** 660 ft.

Art. 73. — **1.** 1 hr. 36 min. **2.** 1 hr. 15 min. **3.** 2 hr. 10 min. 6 sec. **4.** 35 min. 20 sec. **5.** 3 hr. 9 min. 7 sec. **6.** 1 hr. 4 min. 49 sec. **7.** 1 hr. 6 min. 17 sec. **8.** 8 hr. 59 min. 35 sec. P.M. **9.** 6 hr. 57 min. A.M. **10.** 23° 20′. **11.** 28° 20′. **12.** 52° 25′. **13.** 79° 52′ west. **14.** 90° 2′ 30″ west. **15.** 37° 33′ east. **16.** 9 hr. 9 min. 30 sec. A.M.

Art. 78. — **1.** $2 \cdot 2 \cdot 3 \cdot 3$. **2.** $3 \cdot 3 \cdot 5$. **3.** $2 \cdot 2 \cdot 2 \cdot 7$. **4.** $2 \cdot 2 \cdot 3 \cdot 5$. **5.** $2 \cdot 5 \cdot 7$. **6.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$. **7.** $2 \cdot 3 \cdot 13$. **8.** $2 \cdot 2 \cdot 3 \cdot 7$. **9.** $2 \cdot 7 \cdot 7$. **10.** $2 \cdot 2 \cdot 2 \cdot 13$. **11.** $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$. **12.** $3 \cdot 3 \cdot 13$. **13.** $7 \cdot 19$. **14.** $11 \cdot 13$. **15.** $2 \cdot 3 \cdot 5 \cdot 5$. **16.** $2 \cdot 2 \cdot 3 \cdot 13$. **17.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$. **18.** $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$. **19.** $2 \cdot 2 \cdot 7 \cdot 7$. **20.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$. **21.** $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ **22.** $17 \cdot 17$. **23.** $3 \cdot 3 \cdot 37$. **24.** $2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$. **25.** $2 \cdot 3 \cdot 5 \cdot 19$. **26.** $2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$. **27.** $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. **28.** $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$. **29.** $2 \cdot 5 \cdot 7 \cdot 13$. **30.** $3 \cdot 7 \cdot 47$. **31.** $2 \cdot 3 \cdot 5 \cdot 5 \cdot 7$. **32.** $2 \cdot 3 \cdot 11 \cdot 17$. **33.** $3 \cdot 11 \cdot 37$. **34.** $2 \cdot 3 \cdot 3 \cdot 7 \cdot 11$. **35.** $2 \cdot 3 \cdot 13 \cdot 19$. **36.** $2 \cdot 2 \cdot 5 \cdot 7 \cdot 11$. **37.** $2 \cdot 7 \cdot 7 \cdot 17$. **38.** $2 \cdot 2 \cdot 19 \cdot 23$. **39.** $2 \cdot 5 \cdot 11 \cdot 17$. **40.** $2 \cdot 2 \cdot 3 \cdot 7 \cdot 23$.

Art. 79. — **1.** $2 \cdot 3$. **2.** $2 \cdot 7$. **3.** $2 \cdot 3 \cdot 3$. **4.** $3 \cdot 5$. **5.** $2 \cdot 2 \cdot 3$. **6.** $2 \cdot 3 \cdot 7$. **7.** $2 \cdot 3 \cdot 5$. **8.** $7 \cdot 11$. **9.** $3 \cdot 13$. **10.** $5 \cdot 7$. **11.** $2 \cdot 2 \cdot 3$. **12.** $3 \cdot 7$. **13.** $2 \cdot 3 \cdot 7$. **14.** $3 \cdot 17$. **15.** $2 \cdot 19$. **16.** $2 \cdot 3 \cdot 7$. **17.** $2 \cdot 3 \cdot 17$. **18.** $2 \cdot 3 \cdot 11$. **19.** $2 \cdot 3 \cdot 23$. **20.** $2 \cdot 7 \cdot 11$.

Art. 80. — **1.** 35. **2.** 37. **3.** 43. **4.** 51. **5.** 59. **6.** 67. **7.** 49. **8.** 55. **9.** 75. **10.** 39. **11.** 63. **12.** 48. **13.** 99. **14.** 111. **15.** 1. **16.** 12. **17.** 57. **18.** 69. **19.** 44. **20.** 105. **21.** 44 gal. **22.** 12 dozen. **23.** 2686 rails. **24.** \$164,400. **25.** 50 bins, 99 bu.

Art. 81. — **1.** 90. **2.** 120. **3.** 144. **4.** 168. **5.** 96. **6.** 210. **7.** 216. **8.** 240. **9.** 280. **10.** 308. **11.** 336. **12.** 360. **13.** 396. **14.** 450. **15.** 468. **16.** 480. **17.** 540. **18.** 660. **19.** 720. **20.** 546. **21.** 3060. **22.** 2520. **23.** 3465. **24.** 3315. **25.** 4935. **26.** 3927. **27.** \$1440. **28.** 960 acres. **29.** \$21,600. **30.** 2160 bu. **31.** 1 hr. 28 min.

Art. 82. — **1.** 6. **2.** 5. **3.** 3 times. **4.** $3\frac{1}{4}$ times. **5.** 1. **6.** 18 bu. **7.** 1500 sods. **8.** \$150. **9.** 24 dozen. **10.** \$118.80. **11.** 50 days. **12.** \$20. **13.** 27 cents. **14.** \$16. **15.** 12 dozen. **16.** 1296 boxes. **17.** \$135. **18.** 1 day. **19.** \$4320.

Art. 93. — **1.** $\frac{180}{12}$. **2.** $\frac{380}{20}$. **3.** $\frac{289}{17}$. **4.** $\frac{600}{25}$. **5.** $\frac{741}{19}$. **6.** $\frac{1440}{32}$. **7.** $\frac{1512}{24}$. **8.** $\frac{2024}{40}$. **9.** $\frac{5888}{34}$. **10.** $\frac{3159}{39}$. **11.** $\frac{4256}{44}$. **12.** $\frac{2925}{75}$. **13.** $\frac{8400}{80}$. **14.** $\frac{1617}{33}$. **15.** $\frac{5992}{87}$. **16.** $\frac{2450}{49}$. **17.** $\frac{3136}{98}$. **18.** $\frac{2024}{36}$.

Art. 94. — **1.** $\frac{125}{8}$. **2.** $\frac{208}{11}$. **3.** $\frac{364}{15}$. **4.** $\frac{816}{25}$. **5.** $\frac{1752}{36}$. **6.** $\frac{2425}{43}$. **7.** $\frac{3250}{51}$. **8.** $\frac{4825}{75}$. **9.** $\frac{6292}{72}$. **10.** $\frac{8788}{88}$. **11.** $\frac{10000}{99}$. **12.** $\frac{3149}{144}$. **13.** $\frac{9425}{256}$. **14.** $\frac{14666}{325}$. **15.** $\frac{24042}{480}$. **16.** $\frac{8124}{125}$. **17.** $\frac{15555}{216}$. **18.** $\frac{11111}{132}$. **19.** $\frac{10818}{175}$. **20.** $\frac{10101}{100}$.

Art. 95. — **1.** $49\frac{8}{9}$. **2.** $37\frac{5}{12}$. **3.** $52\frac{7}{15}$. **4.** 63. **5.** $70\frac{24}{25}$. **6.** $39\frac{28}{39}$. **7.** $45\frac{37}{48}$. **8.** 57. **9.** $74\frac{12}{13}$. **10.** $65\frac{59}{77}$. **11.** $50\frac{21}{85}$. **12.** $99\frac{88}{89}$. **13.** 101. **14.** $64\frac{8}{203}$. **15.** 77. **16.** $44\frac{39}{444}$.

Art. 96. — **1.** $\frac{9}{24}$. **2.** $\frac{25}{45}$. **3.** $\frac{27}{48}$. **4.** $\frac{44}{56}$. **5.** $\frac{48}{75}$. **6.** $\frac{100}{108}$. **7.** $\frac{104}{120}$. **8.** $\frac{133}{168}$. **9.** $\frac{150}{210}$. **10.** $\frac{304}{380}$. **11.** $\frac{441}{486}$. **12.** $\frac{576}{648}$. **13.** $\frac{455}{676}$. **14.** $\frac{624}{756}$. **15.** $\frac{1085}{1085}$. **16.** $\frac{1152}{1350}$. **17.** $\frac{1617}{1880}$. **18.** $\frac{1028}{2013}$. **19.** $\frac{2376}{2688}$. **20.** $\frac{3250}{3744}$.

Art. 97. — **1.** $\frac{4}{7}$. **2.** $\frac{21}{28}$. **3.** $\frac{16}{21}$. **4.** $\frac{28}{33}$. **5.** $\frac{32}{44}$. **6.** $\frac{33}{50}$. **7.** $\frac{27}{35}$. **8.** $\frac{14}{33}$. **9.** $\frac{11}{28}$. **10.** $\frac{17}{27}$. **11.** $\frac{45}{56}$. **12.** $\frac{28}{27}$. **13.** $\frac{13}{24}$. **14.** $\frac{28}{25}$. **15.** $\frac{18}{25}$. **16.** $\frac{11}{23}$. **17.** $\frac{91}{108}$. **18.** $\frac{14}{17}$. **19.** $\frac{18}{28}$. **20.** $\frac{17}{28}$. **21.** $\frac{35}{42}$. **22.** $\frac{17}{27}$. **23.** $\frac{13}{15}$. **24.** $\frac{75}{169}$. **25.** $\frac{546}{1273}$. **26.** $\frac{135}{308}$. **27.** $\frac{25}{46}$. **28.** $\frac{3876}{5291}$.

Art. 98. — 1. $\frac{4}{12}$, $\frac{9}{12}$, $\frac{10}{12}$. 2. $\frac{8}{18}$, $\frac{4}{18}$, $\frac{5}{18}$. 3. $\frac{14}{24}$, $\frac{15}{24}$, $\frac{14}{24}$. 4. $\frac{20}{48}$, $\frac{15}{48}$, $\frac{10}{48}$. 5. $\frac{16}{40}$, $\frac{25}{40}$, $\frac{12}{40}$. 6. $\frac{27}{45}$, $\frac{35}{45}$, $\frac{24}{45}$. 7. $\frac{18}{48}$, $\frac{20}{48}$, $\frac{27}{48}$, $\frac{14}{48}$. 8. $\frac{54}{80}$, $\frac{55}{80}$, $\frac{52}{80}$, $\frac{38}{80}$. 9. $\frac{80}{75}$, $\frac{20}{75}$, $\frac{36}{75}$, $\frac{18}{75}$. 10. $\frac{42}{72}$, $\frac{44}{72}$, $\frac{39}{72}$, $\frac{18}{72}$. 11. $\frac{33}{84}$, $\frac{30}{84}$, $\frac{20}{84}$, $\frac{13}{84}$. 12. $\frac{88}{96}$, $\frac{54}{96}$, $\frac{68}{96}$, $\frac{57}{96}$. 13. $\frac{45}{105}$, $\frac{98}{105}$, $\frac{80}{105}$, $\frac{72}{105}$. 14. $\frac{64}{108}$, $\frac{63}{108}$, $\frac{42}{108}$, $\frac{28}{108}$. 15. $\frac{16}{120}$, $\frac{85}{120}$, $\frac{52}{120}$, $\frac{68}{120}$. 16. $\frac{35}{126}$, $\frac{102}{126}$, $\frac{75}{126}$, $\frac{108}{126}$. 17. $\frac{60}{135}$, $\frac{72}{135}$, $\frac{80}{135}$, $\frac{84}{135}$. 18. $\frac{135}{144}$, $\frac{138}{144}$, $\frac{140}{144}$, $\frac{141}{144}$. 19. $\frac{40}{150}$, $\frac{48}{150}$, $\frac{65}{150}$, $\frac{87}{150}$, $\frac{128}{150}$. 20. $\frac{128}{168}$, $\frac{112}{168}$, $\frac{76}{168}$, $\frac{75}{168}$, $\frac{74}{168}$. 21. $\frac{132}{180}$, $\frac{130}{180}$, $\frac{153}{180}$, $\frac{145}{180}$, $\frac{113}{180}$. 22. $\frac{147}{210}$, $\frac{185}{210}$, $\frac{112}{210}$, $\frac{50}{210}$, $\frac{96}{210}$. 23. $\frac{112}{240}$, $\frac{125}{240}$, $\frac{66}{240}$, $\frac{128}{240}$, $\frac{117}{240}$. 24. $\frac{275}{330}$, $\frac{297}{330}$, $\frac{240}{330}$, $\frac{308}{330}$, $\frac{168}{330}$. 25. $\frac{285}{360}$, $\frac{276}{360}$, $\frac{290}{360}$, $\frac{296}{360}$, $\frac{284}{360}$. 26. $\frac{288}{384}$, $\frac{294}{384}$, $\frac{325}{384}$, $\frac{273}{384}$, $\frac{256}{384}$. 27. $\frac{295}{390}$, $\frac{299}{390}$, $\frac{280}{390}$, $\frac{264}{390}$, $\frac{295}{390}$. 28. $\frac{425}{450}$, $\frac{216}{450}$, $\frac{193}{450}$, $\frac{290}{450}$, $\frac{408}{450}$. 29. $\frac{287}{483}$, $\frac{352}{483}$, $\frac{327}{483}$, $\frac{341}{483}$, $\frac{384}{483}$. 30. $\frac{312}{315}$, $\frac{295}{315}$, $\frac{314}{315}$, $\frac{296}{315}$, $\frac{220}{315}$. 31. $\frac{510}{540}$, $\frac{443}{540}$, $\frac{520}{540}$, $\frac{516}{540}$, $\frac{524}{540}$. 32. $\frac{418}{528}$, $\frac{357}{528}$, $\frac{350}{528}$, $\frac{481}{528}$, $\frac{408}{528}$. 33. $\frac{351}{520}$, $\frac{312}{520}$, $\frac{375}{520}$. 34. $\frac{285}{532}$, $\frac{315}{532}$, $\frac{500}{532}$. 35. $\frac{622}{816}$, $\frac{303}{816}$, $\frac{728}{816}$. 36. $\frac{688}{1008}$, $\frac{621}{1008}$, $\frac{875}{1008}$. 37. $\frac{27}{45}$, $\frac{30}{45}$, $\frac{25}{45}$, $\frac{24}{45}$. 38. $\frac{1513}{1547}$, $\frac{1257}{1547}$, $\frac{1544}{1547}$. 39. $\frac{1809}{3087}$, $\frac{2114}{3087}$. 40. $\frac{3341}{4368}$, $\frac{3833}{4368}$.

Art. 99. — 1. $\frac{4}{45}$. 2. $\frac{5}{42}$. 3. $\frac{7}{75}$. 4. $\frac{18}{576}$. 5. $\frac{82}{1001}$. 6. $1\frac{7}{20}$. 7. $\frac{64}{135}$. 8. $\frac{118}{120}$. 9. $\frac{77}{180}$. 10. $\frac{25}{144}$. 11. \$30,000. 12. 1928 rails.

Art. 100. — 1. $10\frac{1}{4}$. 2. $6\frac{9}{16}$. 3. $11\frac{1}{2}$. 4. $20\frac{1}{4}$. 5. $9\frac{15}{17}$. 6. $52\frac{1}{4}$. 7. $133\frac{7}{15}$. 8. $68\frac{4}{5}$. 9. $92\frac{3}{5}$. 10. $112\frac{7}{13}$. 11. $600\frac{3}{5}$ A.; \$30,000. 12. 15 days.

Art. 101. — 1. $1\frac{18}{20}$. 2. 2. 3. $1\frac{1}{4}$. 4. $2\frac{1}{3}$. 5. 3. 6. $1\frac{11}{24}$. 7. $2\frac{8}{14}$. 8. $1\frac{3}{10}$. 9. $4\frac{7}{9}$. 10. 4. 11. $18\frac{4}{7}$. 12. $8\frac{1}{30}$. 13. 24. 14. $32\frac{11}{15}$. 15. 100. 16. $2\frac{4}{15}$. 17. 1000. 18. $2\frac{1}{21}$. 19. 1. 20. 10. 21. $6\frac{1}{7}$ tons. 22. \$49.90. 23. \$100. 24. $99\frac{44}{45}$ A. 25. \$25. 26. $1001\frac{1}{2}$ bbl. 27. $66\frac{2}{3}$ da. 28. 30 tons. 29. $199\frac{1}{2}$ cu. ft. 30. 320 acres.

Art. 102. — 1. $\frac{7}{18}$. 2. $\frac{11}{24}$. 3. $\frac{5}{36}$. 4. $\frac{18}{45}$. 5. $\frac{13}{48}$. 6. $\frac{1}{12}$. 7. $\frac{7}{18}$. 8. $\frac{13}{35}$. 9. $\frac{9}{20}$. 10. $\frac{19}{63}$. 11. $4\frac{4}{15}$. 12. $6\frac{1}{3}$. 13. $10\frac{1}{15}$. 14. $19\frac{83}{91}$. 15. $25\frac{1}{20}$. 16. $501\frac{1}{3}$. 17. $99\frac{22}{35}$. 18. $110\frac{11}{30}$. 19. $234\frac{69}{91}$. 20. $125\frac{19}{63}$. 21. $\frac{4}{77}$. 22. $25\frac{27}{28}$. 23. $6\frac{3}{4}$. 24. $99\frac{15}{16}$. 25. $124\frac{1}{42}$. 26. $\frac{8}{35}$. 27. $45\frac{4}{45}$. 28. $10\frac{25}{36}$. 29. $27\frac{3}{5}$. 30. $99\frac{3}{8}$. 31. \$120$\frac{24}{25}$. 32. $201\frac{11}{15}$ bbl. 33. $10\frac{9}{80}$ tons. 34. B, $27\frac{8}{15}$ mi.; C, $24\frac{8}{9}$ mi. 35. \$$\frac{1}{2}$. 36. $68\frac{9}{20}$ days. 37. 160 rods. 38. 250 acres. 39. \$156$\frac{1}{4}$. 40. \$2880.

Art. 103. — 1. $4\frac{3}{8}$. 2. $3\frac{1}{3}$. 3. $4\frac{1}{2}$. 4. $6\frac{3}{4}$. 5. $5\frac{5}{8}$. 6. $12\frac{1}{2}$. 7. $7\frac{1}{7}$. 8. $10\frac{5}{9}$. 9. $12\frac{5}{8}$. 10. $9\frac{9}{10}$. 11. $13\frac{1}{2}$. 12. 36. 13. $9\frac{3}{5}$. 14. $14\frac{2}{7}$. 15. $33\frac{3}{10}$. 16. $22\frac{1}{2}$. 17. $33\frac{1}{3}$. 18. 38. 19. $52\frac{4}{11}$. 20. $59\frac{8}{9}$. 21. $66\frac{2}{3}$. 22. 75. 23. $87\frac{1}{2}$. 24. 160. 25. $18\frac{3}{4}$. 26. 36. 27. $32\frac{1}{2}$. 28. $24\frac{1}{6}$. 29. 72. 30. 220. 31. $\frac{5}{18}$. 32. $\frac{8}{9}$. 33. $\frac{1}{10}$. 34. 18. 35. 40. 36. 15. 37. $16\frac{2}{3}$. 38. 50. 39. 1. 40. $\frac{1}{15}$. 41. $\frac{6}{25}$. 42. $37\frac{1}{2}$. 43. $33\frac{1}{3}$. 44. 7. 45. 256. 46. $\frac{3}{4}$. 47. 25. 48. $2\frac{1}{4}$. 49. $2\frac{1}{2}$. 50. 2.

Art. 104. — 1. $\frac{24}{35}$. 2. $\frac{8}{10}$. 3. $\frac{24}{40}$. 4. $\frac{20}{63}$. 5. $\frac{2}{5}$. 6. $1\frac{7}{9}$. 7. $2\frac{8}{9}$. 8. $3\frac{5}{8}$. 9. $1\frac{19}{21}$. 10. $3\frac{18}{25}$. 11. $\frac{25}{64}$. 12. $\frac{8}{9}$. 13. $\frac{8}{15}$. 14. 1. 15. $2\frac{1}{2}$. 16. $3\frac{1}{3}$. 17. $\frac{9}{14}$. 18. $1\frac{17}{25}$. 19. 4. 20. 1.

Art. 105. — 1. \$2.25. 2. \$5.50. 3. \$31.25. 4. \$3.12$\frac{1}{2}$. 5. \$31.50. 6. \$15$\frac{3}{4}$. 7. \$1500. 8. \$6.60. 9. \$123$\frac{3}{4}$.

10. $137\frac{1}{2}$. **11.** 81. **12.** $21. **13.** 225 miles. **14.** $960. **15.** 100 barrels. **16.** $1008. **17.** $123\frac{3}{4}$. **18.** $300. **19.** $5\frac{5}{36}$. **20.** $2000. **21.** $350. **22.** $15,625. **23.** $60. **24.** 160 acres. **25.** 4 acres.

Art. 106. — **1.** $\frac{3}{25}$. **2.** $\frac{4}{39}$. **3.** 18. **4.** $37\frac{1}{2}$. **5.** $\frac{4}{7}$. **6.** $\frac{5}{8}$. **7.** $2\frac{1}{4}$. **8.** $7\frac{1}{2}$. **9.** $\frac{9}{11}$. **10.** $3\frac{1}{3}$. **11.** $2\frac{1}{10}$. **12.** $\frac{9}{20}$. **13.** $\frac{13}{35}$. **14.** 12. **15.** $\frac{8}{9}$. **16.** $\frac{5}{8}$. **17.** $\frac{4}{7}$. **18.** $\frac{15}{16}$. **19.** $\frac{1}{2}$. **20.** 4. **21.** $\frac{5}{16}$. **22.** $\frac{2}{3}$. **23.** $4\frac{4}{9}$. **24.** $\frac{15}{16}$. **25.** 5. **26.** $3\frac{3}{4}$. **27.** $12\frac{3}{4}$ lb. **28.** $3\frac{1}{8}$ da. **29.** $\frac{16}{25}$. **30.** $2\frac{5}{8}$ acres. **31.** $18\frac{3}{4}$ mi. **32.** $20\frac{5}{8}$ da. **33.** $13\frac{3}{8}$. **34.** $\frac{15}{28}$. **35.** 35 bbl. **36.** $\frac{18}{25}$. **37.** $56\frac{1}{4}$ bu. **38.** $\frac{133}{135}$. **39.** $20. **40.** 65 rods. **41.** $10\frac{1}{2}$ acres. **42.** $32\frac{2}{3}$. **43.** 10 hours. **44.** 28 men. **45.** 48 miles. **46.** 100 miles. **47.** 75. **48.** $18\frac{3}{4}$. **49.** $\frac{5}{8}$. **50.** 30 boxes.

Art. 107. — **1.** $\frac{15}{32}$. **2.** $\frac{9}{10}$. **3.** $\frac{16}{25}$. **4.** $\frac{8}{15}$. **5.** $\frac{5}{12}$. **6.** $\frac{3}{10}$. **7.** $\frac{21}{44}$. **8.** $\frac{25}{27}$. **9.** $\frac{18}{25}$. **10.** $\frac{20}{49}$. **11.** $\frac{28}{45}$. **12.** $\frac{4}{25}$. **13.** $\frac{33}{40}$. **14.** $\frac{8}{15}$. **15.** $\frac{4}{9}$. **16.** $\frac{7}{12}$. **17.** $\frac{18}{25}$. **18.** $\frac{2}{5}$. **19.** $\frac{3}{5}$. **20.** $\frac{25}{32}$. **21.** $\frac{3}{5}$. **22.** $\frac{6}{25}$. **23.** $\frac{5}{12}$. **24.** $\frac{28}{45}$. **25.** $\frac{9}{20}$.

Art. 108. — **1.** $\frac{5}{12}$. **2.** $\frac{8}{9}$. **3.** $\frac{9}{10}$. **4.** $\frac{2}{3}$. **5.** $\frac{16}{21}$. **6.** $\frac{2}{15}$. **7.** $\frac{5}{48}$. **8.** $26\frac{2}{3}$. **9.** $49\frac{1}{2}$. **10.** $\frac{9}{10}$. **11.** $\frac{18}{14}$. **12.** $\frac{14}{27}$. **13.** $\frac{49}{49}$. **14.** $\frac{12}{25}$. **15.** $\frac{33}{40}$. **16.** $1\frac{5}{9}$. **17.** 2. **18.** $\frac{1}{2}$. **19.** $\frac{4}{5}$. **20.** $\frac{8}{9}$. **21.** 1. **22.** $1\frac{1}{2}$.

Art. 109. — **1.** $3\frac{3}{7}$. **2.** $4\frac{1}{8}$. **3.** $3\frac{3}{20}$. **4.** $\frac{16}{45}$. **5.** $\frac{12}{25}$. **6.** $16\frac{2}{3}$ hr. **7.** $33\frac{3}{10}$. **8.** $5\frac{3}{25}$. **9.** $\frac{5}{12}$. **10.** 33 suits. **11.** 15 men. **12.** $2560. **13.** 9 hr. **14.** 50 sheep. **15.** $67\frac{1}{2}$. **16.** $66\frac{2}{3}$. **17.** 25 boxes. **18.** $31\frac{1}{4}$ bbl. **19.** $10\frac{1}{2}$. **20.** A, $18\frac{3}{4}$; B, $15. **21.** $13\frac{1}{2}$. **22.** $6\frac{3}{4}$. **23.** $220\frac{1}{2}$. **24.** $56\frac{1}{4}$. **25.** $\frac{2}{3}$.

Art. 110. — **1.** $\frac{5}{8}$ pt. **2.** $\frac{3}{8}$ gi. **3.** $\frac{5}{6}$ oz. **4.** $\frac{2}{3}$ ft. **5.** $\frac{2}{3}$ sq. yd. **6.** $1\frac{5}{12}$ cu. ft. **7.** $\frac{3}{4}$ min. **8.** $\frac{24}{25}$ gr. **9.** $\frac{9}{20}$ ʒ. **10.** $\frac{5}{9}$ ft. **11.** $\frac{1}{2}$ gi. **12.** $\frac{2}{3}$ sq. ft.

Art. 111. — **1.** 2 pk. 7 qt. 1 pt. **2.** 2 qt. 1 pt. 1 gi. **3.** 4 cwt. 12 lb. 8 oz. **4.** 169 rd. 3 yd. 2 ft. 6 in. **5.** 88 cu. ft. 1536 cu. in. **6.** 5 da. 11 hr. 2 min. 24 sec. **7.** 7 oz. 8 pwt. 12 gr. **8.** 6 ℥ 7 ʒ 1 ℈ $13\frac{1}{3}$ gr. **9.** 7 cu. ft. 864 cu. in. **10.** 5 cwt. 85 lb. 15 oz. **11.** 220 A. 61 sq. rd. 21 sq. yd. 3 sq. ft. 108 sq. in. **12.** 25 gal. 1 pt. $2\frac{2}{3}$ gi.

Art. 112. — **1.** $\frac{5}{312}$ bu. **2.** $\frac{8}{128}$ gal. **3.** $\frac{1}{31125}$ ton. **4.** $\frac{1}{300}$ rd. **5.** $\frac{1}{2178}$ sq. rd. **6.** $\frac{1}{2170}$ wk. **7.** $\frac{1}{60000}$ lb. **8.** $\frac{1}{400}$ ℔. **9.** $\frac{1}{5460}$ hhd. **10.** $\frac{1}{176368}$ mi. **11.** $\frac{1}{48850}$ A. **12.** $\frac{1}{10880}$ bbl.

Art. 113. — **1.** $\frac{25}{64}$ bu. **2.** $\frac{839}{1600}$ ton. **3.** $\frac{15}{32}$ gal. **4.** $\frac{25}{33}$ rd. **5.** $\frac{8}{11}$ A. **6.** $\frac{18}{25}$ da. **7.** $\frac{5}{9}$. **8.** $\frac{7}{11}$. **9.** $\frac{5}{12}$. **10.** $\frac{4}{5}$. **11.** $\frac{9}{10}$. **12.** $\frac{15}{16}$.

Art. 114. — **1.** 1 gal. 1 pt. 3 gi. **2.** 4 cwt. 62 lb. 9 oz. **3.** 78 sq. rd. 29 sq. yd. 6 sq. ft. **4.** 5 da. 19 hr. 10 min. 50 sec. **5.** 1 ℔. **6.** 17 gal. **7.** 10 oz. 1 pwt. 1 gr. **8.** 15 rd. 1 ft. 10 in.

Art. 115. — **1.** $3750. **2.** $100. **3.** $12\frac{5}{18}$. **4.** $393\frac{3}{4}$. **5.** $3\frac{11}{16}$ tons. **6.** $\frac{4}{5}$. **7.** $125\frac{3}{4}$. **8.** 9 lots. **9.** $\frac{5}{8}$. **10.** A, $60; B, 37\frac{1}{2}$; C, 26\frac{1}{4}$. **11.** $14\frac{2}{3}$. **12.** $46\frac{2}{25}$. **13.** $8\frac{1}{3}$ hr. **14.** $\frac{5}{8}$. **15.** $\frac{3}{5}$. **16.** $341\frac{1}{4}$. **17.** 9. **18.** 100 rods. **19.** $36\frac{3}{4}$. **20.** 150 A. **21.** $\frac{24}{25}$. **22.** $76\frac{4}{5}$. **23.** 5 da. **24.** 24 men. **25.** $\frac{8}{9}$. **26.** $\frac{1}{16}$. **27.** $\frac{11}{25}$. **28.** 25 bu. **29.** 360 rev.

30. 2250 bu. **31.** $1\frac{1}{2}$. **32.** $100. **33.** 88 chickens, 75 ducks. **34.** $1500. **35.** $\frac{1}{8}$. **36.** 6 men. **37.** 15. **38.** $5062\frac{1}{2}$. **39.** $\frac{1}{2}$. **40.** $7\frac{1}{5}$ hr.; A, $40\frac{1}{2}$ mi.; B, 32 mi. **41.** 60 da. **42.** $3840. **43.** $12\frac{1}{2}$. **44.** $2\frac{1}{2}$. **45.** A, $36; B, $20\frac{1}{2}$. **46.** 1000 tiles. **47.** $\frac{3}{4}$. **48.** 1125 bu. **49.** $26\frac{2}{3}$ A. **50.** $\$\frac{33}{35}$. **51.** $442\frac{1}{2}$. **52.** 6 da. **53.** $562\frac{1}{2}$. **54.** 30 rugs. **55.** $4\frac{1}{2}$. **56.** $52\frac{1}{2}$. **57.** 15 horses. **58.** $187\frac{1}{2}$. **59.** $41\frac{2}{3}$ acres. **60.** $720. **61.** 150 acres. **62.** $6\frac{5}{35}$. **63.** $10\frac{9}{10}$ min. **64.** $15\frac{1}{5}$. **65.** 70 peach trees. **66.** $\frac{3}{4}$. **67.** 160 acres. **68.** $87\frac{1}{2}$ mi. **69.** $273\frac{3}{8}$ ft. **70.** $18\frac{7}{10}$, $14\frac{1}{8}$. **71.** $12\frac{1}{2}$. **72.** $\frac{5}{8}$. **73.** $\frac{1}{2}$ acre. **74.** $11,137\frac{1}{2}$. **75.** $410\frac{2}{3}$. **76.** $\frac{5}{9}$. **77.** $446\frac{1}{4}$. **78.** $59\frac{2}{25}$. **79.** A, $31\frac{1}{3}$ mi.; B, $25\frac{5}{9}$ mi. **80.** $7\frac{1}{2}$ days. **81.** $\frac{3}{5}$. **82.** $\frac{4}{15}$. **83.** 88 bu. **84.** $1687\frac{1}{2}$. **85.** $1362\frac{1}{2}$ mi. **86.** 50 bbl. **87.** $62\frac{1}{2}$. **88.** $187\frac{1}{2}$. **89.** 150 bbl. **90.** 250 mi. **91.** $\frac{4}{5}$. **92.** $25. **93.** $330. **94.** $43,875. **95.** $123\frac{3}{4}$. **96.** $712\frac{1}{2}$. **97.** $25. **98.** $150.

Art. 125. — **1.** $\frac{9}{25}$. **2.** $\frac{47}{50}$. **3.** $\frac{3}{4}$. **4.** $\frac{13}{20}$. **5.** $\frac{6}{125}$. **6.** $\frac{7}{8}$. **7.** $\frac{9}{40}$. **8.** $\frac{49}{200}$. **9.** $\frac{1}{16}$. **10.** $\frac{32}{625}$. **11.** $\frac{83}{80}$. **12.** $\frac{25}{32}$. **13.** $\frac{101}{160}$. **14.** $\frac{243}{400}$. **15.** $\frac{729}{800}$. **16.** $\frac{45}{64}$. **17.** $\frac{1728}{3125}$. **18.** $\frac{343}{320}$. **19.** $\frac{125}{128}$. **20.** $6\frac{2}{5}$. **21.** $8\frac{3}{4}$. **22.** $9\frac{13}{25}$. **23.** $15\frac{1}{40}$. **24.** $40\frac{7}{16}$. **25.** $\frac{5}{8}$. **26.** $\frac{5}{12}$. **27.** $\frac{81}{250}$. **28.** $\frac{9}{16}$. **29.** $2\frac{13}{24}$. **30.** $4\frac{24}{35}$. **31.** $6\frac{3}{64}$. **32.** $8\frac{1}{3}$.

Art. 126. — **1.** .85. **2.** .72. **3.** .98. **4.** .325. **5.** .625. **6.** .512. **7.** .195. **8.** .484. **9.** .694. **10.** .2048. **11.** .7425. **12.** .7992. **13.** .5875. **14.** .6875. **15.** .09375. **16.** .05625. **17.** .390625. **18.** .059375. **19.** .3515625. **20.** .03168. **21.** 2.225. **22.** 4.096. **23.** 9.1125. **24.** 4.6875. **25.** 3.78125. **26.** .694444+. **27.** .653061+. **28.** .0470588+. **29.** .991666$\frac{2}{3}$. **30.** .868055$\frac{5}{9}$.

Art. 127. — **1.** 32.625. **2.** 21.03125. **3.** 300. **4.** 1072.75. **5.** 1. **6.** 123.456789. **7.** 100. **8.** 987.5. **9.** 50.05050505. **10.** 17. **11.** 2500 mi. **12.** 7500 tons. **13.** 640 acres. **14.** 306.8046875 bu. **15.** 64.085 perches. **16.** 375.125 cu. ft. **17.** 2000.6015625 gal. **18.** $42,958.75. **19.** $406.25. **20.** $1987.50.

Art. 128. — **1.** 190.42. **2.** 267.035. **3.** 158.9312. **4.** .41508. **5.** 90.9091. **6.** 48.943752. **7.** 223.4925. **8.** 24.6913569. **9.** 9.009. **10.** 12.4875. **11.** 628.4375. **12.** 109.74705. **13.** 243.5625. **14.** 1250.0125. **15.** 1. **16.** 104.584. **17.** 29.048. **18.** 40.5 yr. **19.** 75 miles. **20.** 9.75 acres. **21.** 150 tons. **22.** $216.75. **23.** 1500 bu. **24.** 62.5 gal. **25.** 3.125 acres.

Art. 129. — **1.** 1825.74. **2.** 1487.12. **3.** 133.285. **4.** 3.725. **5.** 1.92. **6.** 3.51. **7.** 13.5. **8.** 1. **9.** .096. **10.** 100. **11.** 400. **12.** 3774.13344. **13.** 3.2. **14.** .024. **15.** .008. **16.** 945. **17.** 12.5. **18.** 10. **19.** 48. **20.** 100. **21.** $160. **22.** 610.5 mi. **23.** $687.5. **24.** $93.75. **25.** 50. **26.** 337.5 bu. **27.** $368.64. **28.** $560. **29.** 75 head. **30.** $25. **31.** 160 tons. **32.** $540. **33.** $210. **34.** 337.5 bu. **35.** $2320.

Art. 130. — **1.** 31.4. **2.** 548. **3.** 237.3. **4.** 92.4. **5.** 4.56. **6.** 73.18. **7.** .864. **8.** 1.728. **9.** .4375. **10.** .06875. **11.** .00312.

12. .572649. **13.** 1250. **14.** 102.4. **15.** .46875. **16.** 5.12. **17.** .078125. **18.** .00486. **19.** .64. **20.** 160. **21.** 93.75. bu. **22.** $4.6. **23.** $.35. **24.** 3.125 lb. **25.** 37.5 mi. **26.** $.225. **27.** 12.5 hr. **28.** 56.25 yd. **29.** 51.2 rd. **30.** .76. **31.** 187.5 cords. **32.** 48 gal. **33.** .48. **34.** 76.8 bu. **35.** $3.75. **36.** 4.5 da. **37.** $4250. **38.** $27.5. **39.** 12.25. **40.** 25.

Art. 131. — **1.** .16 pt. **2.** .72 gi. **3.** .88 oz. **4.** .66 yd. **5.** .968 sq. yd. **6.** .16 cu. ft. **7.** .756 min. **8.** .99 pwt. **9.** .864 gr. **10.** .756 gi. **11.** .33 ft. **12.** .648 gr. **13.** .256 sq. rd. **14.** .49 pt. **15.** .5 oz.

Art. 132. — **1.** 1 pk. 4 qt. 1 pt. **2.** 2 qt. 1 pt. 1 gi. **3.** 4 cwt. 68 lb. 12 oz. **4.** 187 rd. 2 yd. 2 ft. 3 in. **5.** 112 sq. rd. 15 sq. yd. 1 sq. ft. 18 sq. in. **6.** 4 cu. ft. 864 cu. in. **7.** 4 hr. 41 min. 15 sec. **8.** 6 oz. 11 pwt. 6 gr. **9.** 10 ℥ 2 ʒ 1 ℈ 10 gr. **10.** 9 gal. 3 qt. 3 gi. **11.** 25 cu. ft. 1269 cu. in. **12.** 57 A. 128 sq. rd. **13.** 3 yd. 1 ft. 3.75 in. **14.** 1 ℥ 6 ʒ 2 ℈ 11 gr. **15.** 123 da. 6 hr. 31 min. 30 sec. **16.** 1 ft. 7 in.

Art. 133. — **1.** .0135 bu. **2.** .01125 gal. **3.** .00048 cwt. **4.** .0005625 mi. **5.** .00015 acre. **6.** .006875 cord. **7.** .004375 wk. **8.** .003375 lb. **9.** .00015625 lb. **10.** .00025 bbl. **11.** .390625 bu. **12.** .46875 gal. **13.** .1875 wk. **14.** .765625 lb. **15.** .703125 lb.

Art. 134. — **1.** $532.80. **2.** 9.6 da. **3.** $226.80. **4.** 50. **5.** 16 da. 10 hr. 28 min. 12 sec. **6.** $247.50. **7.** 25 sq. yd. **8.** 121.5. **9.** $31.25. **10.** 1.0125. **11.** $360. **12.** $3750. **13.** $361.90. **14.** 1 lb. **15.** $4500. **16.** 24 hr. **17.** $300. **18.** .48. **19.** 1. **20.** 31.25 yd. **21.** $29.40. **22.** A, 150; B, 144; C, 90. **23.** $1.20. **24.** $12,500. **25.** 1070 mi. **26.** $112.50. **27.** 1 bu. 5 qt. 1 pt. **28.** 150 acres. **29.** $480. **30.** $250. **31.** .9. **32.** $.875. **33.** $1417.50. **34.** $2750. **35.** 25 pigs. **36.** 160 mi. **37.** $.875. **38.** $.75. **39.** 640 rev. **40.** 32 bbl. **41.** .5 acre. **42.** 32 men. **43.** 39.6. **44.** 32.4. **45.** $78.75. **46.** 144 acres. **47.** A, 64 rd.; B, 50 rd.; C, 46 rd. **48.** 1.6. **49.** $1350. **50.** $124. **51.** $4620. **52.** $.50. **53.** 300 coins. **54.** 4 hours. **55.** $36. **56.** $135. **57.** $2812.50. **58.** $2.44. **59.** $123.75. **60.** .875. **61.** 78 A., 160 A. in farm. **62.** 1500 mi. **63.** 101.25 bu. **64.** 1. **65.** 120 acres. **66.** $1620. **67.** $3456. **68.** $450. **69.** $1260.

Art. 136. — **1.** 1245.6 cm. **2.** 1680 m. **3.** 432 mm. **4.** 57.6 dm. **5.** 31.25 cm. **6.** 3.9724 dm. **7.** 418.32 Hm. **8.** .028296 m. **9.** .004375 Km. **10.** 5.28 Dm.

Art. 137. — **1.** 1832.4 ca. **2.** 1010 a. **3.** 25.6 m^2. **4.** 687.5 ca. **5.** 1.728 Ha. **6.** .0864 a. **7.** .216 Ha. **8.** .004375 Ha.

Art. 138. — **1.** 1080 cl. **2.** 1512 dl. **3.** 62.5 cl. **4.** 2.4 l. **5.** 78.54 cl. **6.** .8136 l. **7.** 1.76 Hl. **8.** .0128 Kl. **9.** 7.8125 Dl. **10.** 10.172 m^3.

Art. 139. — **1.** 3840 cg. **2.** 51.2 g. **3.** 87.5 dg. **4.** 18.64 Dg.

5. 45.6 cg. **6.** .0256 Hg. **7.** 12.5 Dg. **8.** .0048 Kg. **9.** .00864 Mg. **10.** 3.90625 M.T.

Art. 140. — **1.** 70 bu. 3 pk. 6 qt. **2.** 149 sq. yd. 4 sq. ft. 72 sq. in. **3.** 298 rd. 1 yd. 1 ft. 3 in. **4.** 3 lb. 4 oz. 3 pwt. 18 gr. **5.** 1 ℔ 8 ℥ 2 ℈ 5 gr. **6.** 77 A. 35 sq. rd. **7.** 2 T. 15 cwt. 11 lb. 8 oz. **8.** 375 m². **9.** 45 Hl. **10.** 1125 m³. **11.** 15 Kg. **12.** 50 Km.

Art. 141. — **1.** $6000. **2.** $140. **3.** $187.50. **4.** $14.40. **5.** 1.75 m. **6.** $50. **7.** $13,778.75. **8.** $24.60. **9.** $57.87. **10.** 480 blocks. **11.** 120 bu. **12.** $83.19. **13.** 1280 loads. **14.** $187.40. **15.** $7925.25. **16.** 13,750 tonneaux.

Art. 143. — **1.** $\frac{1}{25}$, .04. **2.** $\frac{1}{20}$, .05. **3.** $\frac{1}{50}$, .06. **4.** $\frac{1}{10}$, .1. **5.** $\frac{1}{5}$, .2. **6.** $\frac{1}{4}$, .25. **7.** $\frac{1}{2}$, .5. **8.** $\frac{3}{4}$, .75. **9.** $\frac{1}{40}$, .025. **10.** $\frac{1}{16}$, .0625. **11.** $\frac{1}{8}$, .125. **12.** $\frac{3}{16}$, .1875. **13.** $\frac{3}{8}$, .375. **14.** $\frac{9}{16}$, .5625. **15.** $\frac{5}{8}$, .625. **16.** $\frac{7}{8}$, .875. **17.** $\frac{15}{16}$, .9375. **18.** $\frac{1}{400}$, .0025. **19.** $\frac{1}{250}$, .004. **20.** $\frac{1}{160}$, .00625. **21.** $\frac{3}{1000}$, .003. **22.** $\frac{9}{500}$, .006. **23.** $\frac{1}{125}$, .008. **24.** $\frac{3}{400}$, .0075. **25.** $\frac{5}{4}$, 1.25. **26.** $\frac{1}{15}$, .06$\frac{2}{3}$. **27.** $\frac{1}{12}$, .08$\frac{1}{3}$. **28.** $\frac{1}{6}$, .16$\frac{2}{3}$.

Art. 144. — **1.** $50. **2.** 45 bu. **3.** 24 gal. **4.** 325 lb. **5.** 48 mi. **6.** 108 A. **7.** 27 cords. **8.** 119 da. **9.** 342 bbl. **10.** 660 men. **11.** $12. **12.** 286 sheep. **13.** 145 horses. **14.** 160 tons. **15.** 3360 bu. **16.** 999 rods. **17.** $3840. **18.** 55 yr. **19.** 32 mi. **20.** 7.5. **21.** .045. **22.** 1.25. **23.** 5.12. **24.** 11.25. **25.** .35. **26.** 1. **27.** .0081. **28.** .1. **29.** 4. **30.** 1.28. **31.** 4.8. **32.** 25.6. **33.** 36. **34.** 2.8. **35.** .3. **36.** 58.8. **37.** 80. **38.** 15.36. **39.** 3.85. **40.** 3200. **41.** $\frac{5}{6}$. **42.** $\frac{5}{8}$. **43.** 1$\frac{5}{8}$. **44.** 5$\frac{1}{4}$. **45.** 4. **46.** 11$\frac{1}{9}$. **47.** 16$\frac{2}{3}$. **48.** 46$\frac{7}{8}$. **49.** 40$\frac{1}{2}$. **50.** 99$\frac{9}{10}$. **51.** 7\frac{1}{2}$. **52.** 2 bu. **53.** 16 gal. **54.** 21 lb. **55.** 8 min. **56.** 6 cu. ft. **57.** 11$\frac{1}{4}$ mi. **58.** 1 A. **59.** 55 cu. yd. **60.** 20 men. **61.** $5.55. **62.** 3 gal. 2 qt. **63.** 1 T. 11 cwt. 50 lb. **64.** 7 mi. 160 rd. **65.** 28 A. 20 sq. rd. **66.** 7 cords, 64 cu. ft. **67.** 15 da. 13 hr. 20 min. **68.** 3 lb. 4 oz. 10 pwt. **69.** 1 bu. 2 pk. 3 qt. 1 pt. **70.** 152 da. 4 hr. 30 min. **71.** $415.80. **72.** 1250 lb. **73.** 33$\frac{1}{3}$ mi. **74.** $3. **75.** $3.75. **76.** 25 bbl. **77.** 11,125 men. **78.** 11 lots. **79.** $1440. **80.** 15 mules. **81.** 3 acres. **82.** $8.75. **83.** $1000. **84.** 68.2 acres. **85.** $787.50. **86.** $2250. **87.** 6 hr. 15 min. **88.** $11,250. **89.** 121 yd. **90.** 220 A. **91.** $99. **92.** $75. **93.** $160. **94.** 50 bbl. **95.** $157.50. **96.** 12.25. **97.** $1856.25. **98.** $33.75. **99.** $750. **100.** 1$\frac{3}{8}$.

Art. 145. — **1.** 15%. **2.** 28%. **3.** 8%. **4.** 12$\frac{1}{2}$%. **5.** 16%. **6.** 62$\frac{1}{2}$%. **7.** 22$\frac{1}{2}$%. **8.** 39%. **9.** 64%. **10.** 37$\frac{1}{2}$%. **11.** 55$\frac{5}{9}$%. **12.** 31$\frac{1}{4}$%. **13.** 16$\frac{2}{3}$%. **14.** 18$\frac{3}{4}$%. **15.** 44%. **16.** 36%. **17.** 6$\frac{1}{4}$%. **18.** 41$\frac{2}{3}$%. **19.** 17$\frac{1}{2}$%. **20.** 93$\frac{1}{3}$%. **21.** 64%. **22.** 3$\frac{1}{3}$%. **23.** 48%. **24.** 7$\frac{1}{2}$%. **25.** 22$\frac{2}{9}$%. **26.** 333$\frac{1}{3}$%. **27.** 32%. **28.** 12$\frac{1}{2}$%. **29.** 6$\frac{2}{3}$%. **30.** 62$\frac{1}{2}$%. **31.** 75%. **32.** 12%. **33.** 87$\frac{1}{2}$%. **34.** 18$\frac{3}{4}$%. **35.** 16$\frac{2}{3}$%. **36.** 28%. **37.** 68$\frac{3}{4}$%. **38.** 8$\frac{1}{3}$%. **39.** 80%. **40.** 33$\frac{1}{3}$%. **41.** 64%. **42.** 60%. **43.** 0. **44.** 31$\frac{1}{4}$%. **45.** 37$\frac{1}{2}$%. **46.** 35%.

Art. 146. — **1.** 350. **2.** 225. **3.** 250. **4.** 180. **5.** 320. **6.** 75. **7.** 625. **8.** 324. **9.** 875. **10.** 1250. **11.** 468. **12.** 37.5. **13.** 1024. **14.** 93.75. **15.** 187.5. **16.** 2750. **17.** 175. **18.** 512. **19.** 132. **20.** 150. **21.** 93¾. **22.** 160. **23.** 337½. **24.** 64. **25.** $324. **26.** 195 bu. **27.** 450 mi. **28.** 240 men. **29.** 28 A. **30.** 1375 gal. **31.** $125. **32.** $375. **33.** 75 A. **34.** 1850 bu. **35.** $1080. **36.** $356.40. **37.** 999 men. **38.** $1541.25. **39.** $318.75. **40.** $0.84. **41.** $510. **42.** 6.25 A. **43.** $285.60. **44.** $80. **45.** 339 pupils. **46.** $1000. **47.** 42 acres. **48.** $12,500. **49.** $111. **50.** $46.50. **51.** $5775. **52.** A, 168; B, 174. **53.** $3.75. **54.** 120 A.; 75 A. **55.** $3675. **56.** 937.5. **57.** 64 bu.

Art. 147. — **1.** 75. **2.** 280. **3.** 325. **4.** 460. **5.** 250. **6.** 625. **7.** 775. **8.** 860. **9.** 1025. **10.** 1350. **11.** 1640. **12.** 2160. **13.** 450. **14.** 2048. **15.** 1845. **16.** 4444. **17.** 1250. **18.** 1875. **19.** 3125. **20.** 166⅔. **21.** $280. **22.** 50 yr. **23.** 75 A. **24.** $450. **25.** 1280 bbl. **26.** 1160. **27.** $3750. **28.** $2160. **29.** $1215. **30.** $507.50. **31.** $2.40. **32.** $3.85. **33.** $5000. **34.** $6875. **35.** B, $2640; C, $3450. **36.** $3250. **37.** $87,500. **38.** $15,625. **39.** 80 head. **40.** $62.50. **41.** $800. **42.** $150. **43.** $180. **44.** $157.50. **45.** 1250.

Art. 149. — **1.** 25 lots. **2.** 37½%. **3.** $94.50. **4.** $3.60. **5.** $1.50. **6.** 20%. **7.** $576.10. **8.** 12½ acres. **9.** 32 mules. **10.** $1875. **11.** $59.40. **12.** 62½%. **13.** $276.75. **14.** 75%. **15.** $1.10. **16.** 44%. **17.** $285. **18.** $703.80. **19.** $945. **20.** 12½%. **21.** $525. **22.** $4.80. **23.** $0.45. **24.** 16⅔%. **25.** 150 A.; sold 84 A. **26.** 96 cows. **27.** 145 hogs. **28.** 13⅓%. **29.** $12,500; $3000 for bonds. **30.** $5.75. **31.** $1050. **32.** 28%. **33.** $640. **34.** $585.90. **35.** $180. **36.** 25%. **37.** 102.4 A.; 160 A. **38.** $135. **39.** 64 sheep. **40.** 1280.

Art. 154. — **1.** $45. **2.** $60. **3.** $2240. **4.** $2090. **5.** $2530. **6.** $2520. **7.** 4½%. **8.** 8⅓%. **9.** 6¼%. **10.** 6⅔%. **11.** $1653.75. **12.** $1890. **13.** $5320. **14.** $4375. **15.** $893.75. **16.** $4750. **17.** $2250. **18.** $4120. **19.** $1740. **20.** $150. **21.** $735. **22.** 25%. **23.** 16%. **24.** $80. **25.** $30. **26.** $50. **27.** $37.50. **28.** $312.50. **29.** 0. **30.** $67.50. **31.** $525. **32.** 25%. **33.** $320. **34.** $375.

Art. 152. — **1.** $320. **2.** $665. **3.** $700. **4.** $380. **5.** $450. **6.** $2160. **7.** $157.50. **8.** $877.50. **9.** $1377. **10.** $1350. **11.** $682.50. **12.** $275.40. **13.** $651. **14.** $1086.75. **15.** $1080. **16.** $598.50. **17.** $566.50. **18.** $365.05. **19.** $708.75. **20.** $437.50. **21.** 60%. **22.** $270. **23.** All are alike.

Art. 154. — **1.** $97.50. **2.** $50. **3.** $130.50. **4.** $250.80. **5.** $106,110. **6.** 2½%. **7.** 3⅛%. **8.** $5760. **9.** 3%. **10.** $89.10. **11.** $6599.50. **12.** $3000; $105. **13.** $147.75. **14.** $10,500. **15.** $80. **16.** $97.50. **17.** $75. **18.** $50. **19.** 1280 bu.

Art. 156. — 1. $42.50. 2. ⅜%. 3. 50 shares. 4. $45. 5. ⅛%. 6. 125 shares. 7. $94.50. 8. ¾%. 9. 150 shares; $37.50. 10. $63.

Art. 157. — 1. $18,750. 2. 52 shares. 3. 45 shares. 4. $187,500. 5. 8½%. 6. 12½%. 7. 12½%. 8. 1875 shares. 9. $150,000. 10. 13½%; $337.50. 11. $312.50. 12. 3750 shares. 13. 8½%. 14. $562.50. 15. $13,750.

Art. 158. — 1. $2512.50. 2. $3340. 3. $4350. 4. $5262.50. 5. $6412.40. 6. 72 shares. 7. 99 shares. 8. 108 shares. 9. 160 shares. 10. 84 shares. 11. 88. 12. 106. 13. 168 shares. 14. 1375 shares. 15. $10,437.50. 16. 62½ cents. 17. $1.25. 18. 111 shares. 19. $14,775. 20. 105. 21. 103¼.

Art. 159. — 1. $315. 2. $360. 3. $15. 4. $400. 5. $1260. 6. 125. 7. 80. 8. 6¼%. 9. $9450. 10. $11,550. 11. $4687.50. 12. $806.40. 13. $450.50. 14. $1080. 15. $75. 16. 0. 17. The latter by 1%. 18. The former by $30. 19. $23,850. 20. $19,250.

Art. 160. — 1. 144 shares. 2. 150 shares. 3. 108 shares. 4. $7500. 5. $4925. 6. $6688. 7. $2750. 8. $2750. 9. 166⅔. 10. 33⅓%. 11. 132 shares. 12. $45. 13. $234. 14. $666. 15. 150 shares. 16. ¼%. 17. The former by $37.50. 18. 5% premium. 19. 9% discount. 20. 103. 21. $13,078.125. 22. $3075. 23. The former by $\frac{5}{14}$%. 24. $6664. 25. 4% premium.

Art. 162. — 1. $24.30. 2. $157.50. 3. $180. 4. $325.65. 5. 1½%. 6. 1⅘%. 7. 2¼%. 8. 3⅓%. 9. $3150. 10. $5280. 11. $13,260. 12. $10,000. 13. $1000. 14. ⅓. 15. ⅞. 16. A, $4687.50; B, $6250; C, $7812.50. 17. $121,875. 18. $84,525. 19. $4800. 20. $3200.

Art. 163. — 1. $793.50. 2. $27.50. 3. $6145.

Art. 165. — 1. $5142.40. 2. $197,250. 3. 18 mills. 4. $59.40. 5. $109.82. 6. $1,375,800. 7. $15,895. 8. $325,600. 9. $7475.75. 10. $646,560. 11. Rate, .024; tax, $128.50. 12. $137.58. 13. $101.25. 14. $167.40. 15. $287,480. 16. $15,253.75. 17. $875,750. 18. $12,329.10. 19. $819,200. 20. $10,000.

Art. 166. — 1. $33.90. 2. $57. 3. $19.44. 4. $64.23. 5. $17.22. 6. $61.11. 7. $149.04. 8. 24 mills. 9. $22.22. 10. $469.92. 11. $6175.68. 12. $25.63. 13. $1545. 14. $260.64. 15. $7603.06.

Art. 168. — 1. $0.50. 2. $4000. 3. $875. 4. $38.40.

Art. 169. — 1. $10. 2. $960. 3. $366. 4. $521.10. 5. $999. 6. $4755.52. 7. $3929.10. 8. $11.20.

Art. 172. — 1. $5.40. 2. $16. 3. $37.80. 4. $75.60. 5. $139. 6. $115.74. 7. $880.60. 8. $1004.30. 9. $1298.70. 10. $1435. 11. $1477.50. 12. $1805.25. 13. $2.10. 14. $4.40. 15. $7.80. 16. $12.96. 17. $19.25. 18. $30. 19. $866.25. 20. $990.60. 21. $1154.25. 22. $1200.80. 23. $1321.25. 24. $1451.25. 25. $1.05. 26. $1.65. 27. $1.75. 28. $1.44. 29. $2.70. 30. $1.80.

31. $676.50. **32.** $482.24. **33.** $1027.20. **34.** $1172.60. **35.** $1357.50. **36.** $1448.40. **37.** $6.30. **38.** $13.75. **39.** $20.80. **40.** $33.75. **41.** $51.75. **42.** $82.50. **43.** $108. **44.** $141.68. **45.** $211.60. **46.** $218.70. **47.** $145.20. **48.** $293.70. **49.** $443.63. **50.** $653.35. **51.** $624.45. **52.** $821.70. **53.** $952.25. **54.** $1043.20. **55.** $1162.50. **56.** $555.55.

Art. 173. — **1.** $0.042. **2.** $0.0575. **3.** $0.029. **4.** $0.0535. **5.** $0.0345. **6.** $0.038. **7.** $0.0215. **8.** $0.046. **9.** 0.081\frac{3}{4}$. **10.** $0.0905. **11.** $0.147. **12.** 0.162\frac{5}{8}$. **13.** 0.219\frac{1}{8}$. **14.** $0.2255. **15.** 0.246\frac{3}{8}$. **16.** 0.298\frac{1}{2}$. **17.** $5. **18.** $9.45. **19.** $7.20. **20.** $13.50. **21.** $21.34. **22.** $31.90. **23.** $17.55. **24.** $38.40. **25.** $39.60. **26.** $43.74. **27.** $1.25. **28.** $2.25. **29.** $3.60. **30.** $4.20. **31.** $6.72. **32.** $8.25. **33.** $10.50. **34.** $11. **35.** $12.42. **36.** $14.80. **37.** $13.95. **38.** $15.84. **39.** $18.20. **40.** $22.50. **41.** $9. **42.** $12.90. **43.** $17.15. **44.** $24.30. **45.** $57.40. **46.** $68.40. **47.** $73.60. **48.** $97.50. **49.** $144. **50.** $166.50. **51.** $210.72. **52.** $228.50. **53.** $345.15. **54.** $512. **55.** $552.50. **56.** $728. **57.** $826.92. **58.** $1026.52. **59.** $1158.05. **60.** $1334.34. **61.** $1405.30. **62.** $1570. **63.** $1706.60. **64.** $1825.20. **65.** $1991.34.

Art. 174. — **1.** $4.50. **2.** $12.50. **3.** $13.36. **4.** $52.35. **5.** $11.11. **6.** $577.60. **7.** $528.75. **8.** $748. **9.** $1190.70. **10.** $1902.50.

Art. 175. — **1.** $9.50. **2.** $32.50. **3.** $43.56. **4.** $21. **5.** $80.25. **6.** $66.55. **7.** $177.55. **8.** $47.56. **9.** $230.40. **10.** $169.05. **11.** $408.50. **12.** $167.75. **13.** $424.65. **14.** $333.33. **15.** $661.85. **16.** $28.35. **17.** $39. **18.** $43.20. **19.** $56. **20.** $58.32. **21.** $60.75. **22.** $227.50. **23.** $248.40. **24.** $300. **25.** $86.10. **26.** $179.40. **27.** $291.20. **28.** $107.25. **29.** $634.90. **30.** $914.40. **31.** $190.35. **32.** $258.75. **33.** $370.50. **34.** $494.95. **35.** $629.75. **36.** $719.68. **37.** $891.25. **38.** $993.28. **39.** $1166.64. **40.** $1402.50. **41.** $1605.50. **42.** $1833.30. **43.** $1423.95. **44.** $2272.80. **45.** $1712.50. **46.** $1957.50. **47.** $1931.18. **48.** $2315.70. **49.** $2255.22. **50.** $2713.28.

Art. 176. — **1.** 1 yr. 9 mo. 0 da. **2.** 2 yr. 6 mo. 0 da. **3.** 2 yr. 7 mo. 15 da. **4.** 2 yr. 4 mo. 24 da. **5.** 1 yr. 9 mo. 18 da. **6.** 3 yr. 7 mo. 6 da. **7.** 2 yr. 10 mo. 24 da. **8.** 3 yr. 2 mo. 12 da. **9.** 0 yr. 10 mo. 24 da. **10.** 2 yr. 10 mo. 15 da. **11.** 4 yr. 9 mo. 18 da. **12.** 1 yr. 4 mo. 15 da. **13.** 1 yr. 7 mo. 6 da. **14.** 2 yr. 0 mo. 24 da. **15.** 2 yr. 8 mo. 12 da. **16.** 4 yr. 4 mo. 24 da. **17.** 3 yr. 3 mo. 18 da. **18.** 3 yr. 1 mo. 15 da. **19.** 2 yr. 1 mo. 6 da. **20.** 4 yr. 10 mo. 24 da. **21.** 1 yr. 2 mo. 12 da. **22.** 3 yr. 4 mo. 15 da. **23.** 3 yr. 7 mo. 15 da. **24.** 2 yr. 9 mo. 18 da. **25.** 1 yr. 3 mo. 18 da. **26.** 4 yr. 3 mo. 6 da. **27.** 2 yr. 4 mo. 24 da. **28.** 2 yr. 1 mo. 6 da. **29.** 13 yr. 4 mo. 0 da. **30.** 22 yr. 2 mo. 20 da.

Art. 177. — **1.** 3%. **2.** 4%. **3.** 5%. **4.** 6%. **5.** 7%. **6.** 8%. **7.** 9%. **8.** 3%. **9.** 3½%. **10.** 4%. **11.** 4½%. **12.** 5%. **13.** 6%. **14.** 6¼%. **15.** 6¾%. **16.** 7%. **17.** 7½%. **18.** 8%. **19.** 8½%. **20.** 9%. **21.** 4%. **22.** 4½%. **23.** 3⅓%. **24.** 5%. **25.** 7%. **26.** 7½%. **27.** 6¼%. **28.** 8%. **29.** 3¾%. **30.** 4½%.

Art. 178. — **1.** $185. **2.** $225. **3.** $395. **4.** $445. **5.** $525. **6.** $640. **7.** $750. **8.** $888. **9.** $936. **10.** $1062.50. **11.** $1160. **12.** $1275. **13.** $1350. **14.** $1425. **15.** $1555. **16.** $1666. **17.** $1728. **18.** $1875. **19.** $1920. **20.** $2160. **21.** $2225. **22.** $2480. **23.** $2562.50. **24.** $2687.50. **25.** $2812.50. **26.** $2937.50. **27.** $1024. **28.** $1280. **29.** $1620. **30.** $1562.50.

Art. 179. — **1.** $75. **2.** $180. **3.** $250. **4.** $325. **5.** $444. **6.** $625. **7.** $750. **8.** $875. **9.** $998. **10.** $1025. **11.** $1168. **12.** $1250. **13.** $1345. **14.** $1475. **15.** $1567.50. **16.** $1687.50. **17.** $1780. **18.** $1864. **19.** $1996. **20.** $2062.50. **21.** $15.75. **22.** $22.05. **23.** $51. **24.** $60. **25.** $134.40. **26.** $168. **27.** $210. **28.** $420. **29.** $361.11. **30.** $418.44.

Art. 180. — **1.** $798.20. **2.** 3 yr. 1 mo. 15 da. **3.** 7%. **4.** $555.50. **5.** $45. **6.** $225. **7.** Mar. 2, 1902. **8.** 4½%. **9.** $324. **10.** $78. **11.** $405.50. **12.** Jan. 3, 1900. **13.** 3%. **14.** $1687.80. **15.** 3 yr. 3 mo. 6 da. **16.** 5%. **17.** $1687.50. **18.** $337.50. **19.** $3009.

Art. 181. — **1.** $154.50. **2.** $133.75. **3.** $375. **4.** $497.90. **5.** $449.50. **6.** $725. **7.** $269. **8.** $400. **9.** $368.90. **10.** $562.90. **11.** $204.20. **12.** $555. **13.** $364.50. **14.** $401.45. **15.** $884.85. **16.** $100. **17.** $453.25. **18.** $916.50. **19.** $1033.25. **20.** $517.20. **21.** $1000. **22.** $720.

Art. 182. — **1.** $253. **2.** $107.54. **3.** $294.50. **4.** $100. **5.** $276.75.

Art. 185. — **1.** May 25, $2.20, $397.80. **2.** Sept. 9, $3.15, $446.85. **3.** June 20, $8.37, $531.63. **4.** May 28, $13.86, $646.14. **5.** Sept. 30, $7.25, $717.25. **6.** June 18, $7.20, $352.80. **7.** April 5, $9.25, $545.75. **8.** Dec. 7, $7.50, $987.50. **9.** July 26, $10, $740. **10.** April 14, $20.48, $1617.92. **11.** $1100.40. **12.** $360. **13.** $365. **14.** $198.75. **15.** $312. **16.** $11.55, $538.45. **17.** $26.25, $1023.75. **18.** $3.91, $308.89. **19.** $5.85, $481.65. **20.** $10.35, $564.65. **21.** $12.60, $707.40. **22.** $656.01. **23.** $798.72. **24.** $617.78. **25.** $936.32.

Art. 186. — **1.** $15.50, $759.50. **2.** $13, $767. **3.** $13.25, $649.25. **4.** $15.60, $1024.40. **5.** $40, $1560. **6.** $25.76, $1446.24. **7.** $24.75, $800.25. **8.** $29.70, $1320.30. **9.** $22.50, $1177.50. **10.** $50.25, $1959.75. **11.** $1253.75. **12.** $1033.50. **13.** $938.40. **14.** $1004.70. **15.** $992.25.

Art. 187. — **1.** $400. **2.** $640. **3.** $480. **4.** $819.20. **5.** $562.50. **6.** $937.50. **7.** $720. **8.** $649. **9.** $5.12. **10.** $32. **11.** $1320. **12.** 1250 bu. **13.** 256 bbl., $563.20.

Art. 189. — 1. \$450, \$67.50. 2. \$375, \$45. 3. \$564, \$42.30. 4. \$512.80, \$128.20. 5. \$625. 6. \$60.50. 7. \$130. 8. \$687.50. 9. \$819.20. 10. \$243.75. 11. \$7.50. 12. \$16. 13. \$40. 14. \$25. 15. \$5. 16. The former by \$35. 17. \$1366.25. 18. \$692.25. 19. \$870.60. 20. \$1893.75. 21. \$743.06. 22. \$1200, \$60. 23. \$2531.25, \$202.50. 24. \$77.50. 25. \$75.

Art. 192. — 1. \$730.80. 2. \$833.70. 3. \$1006.25. 4. \$620. 5. \$640. 6. \$850. 7. \$960. 8. \$775. 9. \$740. 10. \$709.20. 11. \$875. 12. \$925. 13. \$1063.80. 14. \$1050. 15. \$3.90.

Art. 194. — 1. \$2733.75. 2. \$1986.56. 3. £312 10*s*. 4. £468 15*s*. 5. \$480. 6. 9675 fr. 7. 5670 marks. 8. \$1267.20.

Art. 196. — 1. 9. 2. 7. 3. $\frac{4}{5}$. 4. $\frac{11}{16}$. 5. $5\frac{5}{8}$. 6. 16. 7. $1\frac{1}{4}$. 8. $\frac{12}{48}$. 9. $\frac{14}{35}$. 10. $2\frac{2}{15}$. 11. $\frac{2}{25}$. 12. 1.85. 13. .875. 14. 6.4. 15. 9.25. 16. 52.8. 17. 51.2. 18. .27. 19. $\frac{25}{64}$. 20. $5\frac{5}{8}$. 21. .03125. 22. 102.4. 23. $\frac{5}{8}$. 24. 37. 25. 99. 26. 25. 27. $\frac{1}{12}$. 28. $\frac{9}{20}$. 29. $\frac{15}{16}$. 30. $1\frac{3}{5}$.

Art. 197. — 1. 196. 2. 585. 3. 2000. 4. 560. 5. 294. 6. 125. 7. $\frac{9}{26}$. 8. 26. 9. .999. 10. 18.75. 11. 18 rd. 1 yd. 12. 1000 bu.

Art. 198. — 1. 7. 2. 24. 3. 2.7. 4. $\frac{64}{81}$. 5. 1215. 6. $\frac{2}{15}$. 7. $\frac{8}{40}$. 8. 625. 9. .0016. 10. 8.75. 11. 9.6. 12. .4.

Art. 199. — 1. $2\frac{1}{2}$. 2. $\frac{1}{5}$. 3. 3. 4. 4. 5. 1. 6. $3\frac{1}{3}$. 7. .6. 8. $\frac{7}{12}$. 9. $1\frac{1}{2}$. 10. 100.

Art 202. — 1. 52. 2. 49. 3. 84. 4. 95. 5. 68. 6. 30. 7. $6\frac{3}{4}$. 8. $\frac{1}{4}$. 9. 1. 10. $166\frac{2}{3}$. 11. $4\frac{1}{2}$. 12. 54. 13. 27. 14. 4. 15. 10. 16. .01.

Art. 204. — 1. $37\frac{1}{2}$ A. 2. \$68.25. 3. $13\frac{1}{3}$ da. 4. \$27.50. 5. \$200. 6. 87.5 ft. 7. \$350. 8. 49.95 bu. 9. 4.5 hr. 10. 68.75 ft. 11. $\$37\frac{1}{2}$. 12. 10 oz. 13. 45. 14. $44\frac{4}{5}$. 15. $31\frac{1}{2}$ bbl. 16. 125 rotations. 17. \$0.15. 18. $6\frac{3}{4}$ days. 19. $6\frac{3}{4}$ bbl. 20. 1625 bu. 21. \$2750. 22. \$3375. 23. $33\frac{1}{3}$ A. 24. \$45. 25. \$4400. 26. $112\frac{1}{2}$ bu. 27. $\frac{3}{10}$. 28. 540 mi. 29. \$1520. 30. \$120. 31. 100 hogs. 32. \$0.50. 33. 1 mi. 34. \$100. 35. 75 yd.

Art. 206. — 1. A, \$200; B, \$250. 2. A, \$240; B, \$400. 3. A, \$780; B, \$1040; C, \$1300. 4. A, \$125; B, \$150; C, \$175. 5. 224, 288. 6. 210, 330. 7. 360, 375. 8. 306, 337.5, 369. 9. 125, 131.25, 137.5. 10. $373\frac{1}{3}$, $341\frac{1}{3}$, 352. 11. C, 81 acres; D, 91.5; E, 112.5. 12. E, \$206.25; F, \$168.75. 13. Eldest, \$1890; Second, \$1680; Youngest, \$1470. 14. 1st, \$2625; 2d, \$2250; 3d, \$1875; 4th, \$1500. 15. A, \$24; B, \$28.80; C, \$36; D, \$43.20; E, \$48. 16. \$180. 17. 1st, 2880 bu.; 2d, 3072 bu.; 3d, 2400 bu.; 4th, 2688 bu. 18. A, \$112.50; B, \$147.50; C, \$100. 19. 1st, \$27.50; 2d, \$23.10; 3d, \$26.40. 20. A, \$866.25; B, \$490.05; C, \$623.70. 21. 150 acres.

Art. 207. — 1. \$0.75. 2. 88%. 3. \$1088. 4. \$240. 5. \$351. 6. \$625. 7. A, \$1760; B, \$2145; C, \$2530; D, \$2970. 8. \$12,500. 9. \$320,000.

Art. 209. — **1.** 9801. **2.** 59,319. **3.** 4,100,625. **4.** 335.54432. **5.** 11.390625. **6.** 3906.25. **7.** $1953\frac{1}{4}$. **8.** $1975\frac{25}{81}$. **9.** $\frac{125}{3375}$. **10.** $5\frac{5}{7}\frac{1}{2}$. **11.** 622,521. **12.** 976.5625. **13.** $4726\frac{9}{16}$. **14.** $\frac{2401}{2500}$. **15.** $93\frac{1}{1}\frac{2}{2}\frac{7}{5}\frac{3}{6}$. **16.** $623\frac{1}{8}\frac{1}{1}\frac{5}{5}$. **17.** $7346\frac{4}{7}\frac{8}{3}$. **18.** .499849. **19.** 99.8001. **20.** 50,653. **21.** 2,097,152. **22.** 8,242,408. **23.** $6591\frac{5}{6}\frac{1}{4}$. **24.** $\frac{1}{1}\frac{8}{5}\frac{8}{6}\frac{2}{2}\frac{4}{5}$. **25.** 1860.867. **26.** 1.030301. **27.** $37,037\frac{1}{2}\frac{1}{7}$. **28.** $364\frac{4}{8}\frac{8}{3}\frac{8}{3}$. **29.** 194,481. **30.** 5,764,801. **31.** 915.0625. **32.** .01679616. **33.** $\frac{4096}{3561}$. **34.** $1525\frac{2}{1}\frac{2}{5}\frac{5}{6}\frac{5}{8}$. **35.** 254,803,968. **36.** 23,730.46875. **37.** $1,286,008\frac{56}{243}$.

Art. 212. — **1.** 31. **2.** 49. **3.** 87. **4.** 101. **5.** 123. **6.** 246. **7.** 298. **8.** 333. **9.** 372. **10.** 408. **11.** 480. **12.** 555. **13.** 624. **14.** 729. **15.** 876. **16.** 999. **17.** 1250. **18.** 1875. **19.** 7.6. **20.** 3.24. **21.** .268. **22.** .037. **23.** .0306. **24.** 1.001. **25.** 562.5. **26.** 123.45. **27.** .0196. **28.** $\frac{28}{99}$. **29.** $\frac{125}{333}$. **30.** $\frac{324}{999}$. **31.** $88\frac{1}{8}$. **32.** $55\frac{5}{9}$. **33.** $102\frac{3}{8}$. **34.** $2\frac{1}{32}$. **35.** 3.79473 +. **36.** .7905694 +.

Art. 213. — **1.** 27. **2.** 39. **3.** 44. **4.** 63. **5.** 75. **6.** 66. **7.** 98. **8.** 102. **9.** 7.8. **10.** 154. **11.** 168. **12.** 330.

Art. 214. — **1.** 35 ft. **2.** 56 ft. **3.** 60 ft. **4.** 305 yd. **5.** 169 rd. **6.** 28 ft. **7.** 125 ft. **8.** 530 yd. **9.** 48 rd. **10.** 57 ft. **11.** 90 ft. 9 in. **12.** 40 in.

Art. 215. — **1.** 16 ft. **2.** $240. **3.** 4 rd. **4.** 100 rd. **5.** 137 rd. **6.** A's, 284 rd.; B's, 388 rd.; C's, 340 rd. **7.** $150.

Art. 216. — **1.** 36 pipes. **2.** 25 acres. **3.** 1 hour. **4.** 120 rd. long, 100 rd. wide. **5.** $375. **6.** 38 ft. **7.** 125 rd. long, 80 rd. wide. **8.** Base, 105 ft.; perpen., 140 ft.; hypot., 175 ft. **9.** 125 ft. **10.** $1575.

Art. 217. — **1.** 2.75. **2.** 6.25. **3.** 325 yd. **4.** $5. **5.** 126 ft. **6.** 1624 mi. **7.** 6 rd. **8.** 30 ft. **9.** 40 ft. 2 in. **10.** 1 rod. **11.** 30 rd. **12.** 50 ft. **13.** $120. **14.** 77 ft. **15.** $83\frac{1}{3}$ ct. **16.** 48 rd. wide, 75 rd. long. **17.** $8\frac{3}{4}$. **18.** 85 ft. **19.** 99 ft. **20.** 32 ft. 6 in. **21.** $135.

Art. 219. — **1.** 24. **2.** 32. **3.** 46. **4.** 51. **5.** 67. **6.** 75. **7.** 88. **8.** 93. **9.** 101. **10.** 249. **11.** 456. **12.** 532. **13.** 777. **14.** 808. **15.** 97.5. **16.** .39. **17.** .055. **18.** .999. **19.** 1.001. **20.** 1.93899 +. **21.** $\frac{2}{4}\frac{8}{5}$. **22.** $\frac{4}{5}\frac{7}{3}$. **23.** $\frac{1}{1}\frac{1}{1}\frac{9}{1}$. **24.** $\frac{4}{5}\frac{8}{5}$. **25.** $20\frac{1}{6}$. **26.** $15\frac{5}{8}$. **27.** $43\frac{3}{4}$. **28.** 105. **29.** .09. **30.** 13. **31.** 21. **32.** 5. **33.** 720. **34.** 2. **35.** 5.925. **36.** 33.25.

Art. 220. — **1.** 18 in. **2.** 33 in. **3.** 15 ft. **4.** 72 in. **5.** 50 in. **6.** 3960 ft.

Art. 221. — **1.** 6000 gal. **2.** 6 × 4 × 3 ft. **3.** 1500 cu. ft. **4.** 10,000 bbl. **5.** 100 × 100 × 150 ft. **6.** 5 × 4 × $3\frac{1}{2}$ ft. **7.** 8 × $3\frac{1}{2}$ × 2 ft.

Art. 222. — **1.** 38 in. **2.** 48 acres. **3.** 36 ft. **4.** 27 in. **5.** 15,625. **6.** 9 ft. **7.** $30. **8.** 45 in. **9.** $36.30. **10.** 15.625 cu. ft. **11.** $25. **12.** 80 sq. ft. **13.** 5 ft. 7.2 in. **14.** 36 × 24 × 15 in. **15.** contents, 40.5 cu. ft.; dimen., 18 × 1.5 × 1.5 ft. **16.** 225 × 75 × 15 ft. **17.** 120 × 9 × 6 ft.

Art. 224. — **1.** 1 sq. yd. **2.** 24 A. $66\frac{1}{4}$ sq. rd. **3.** 34 A. 60 sq. rd.

4. $231. **5.** $150. **6.** 207 boards. **7.** $750. **8.** $4.80. **9.** 16 ft. 8 in. **10.** 12 boxes. **11.** $180. **12.** 36 sq. yd. **13.** 300 rd. **14.** 9 A. 96 sq. rd. **15.** $183.

Art. 225. — **1.** 50 sq. yd. **2.** 30 acres. **3.** $4500. **4.** $5000 **5.** $2500. **6.** 66 ft. **7.** 65 rd. **8.** 75 ft.; 60 ft. **9.** 125 ft. **10.** 56 rd.; 64 rd. **11.** 15 rd. **12.** 48 rd. **13.** 80%.

Art. 226. — **1.** 200 sq. yd. **2.** 2 acres. **3.** $2250. **4.** $625. **5.** 48 rd. and 73 rd. **6.** $1260. **7.** $240. **8.** $45. **9.** $189. **10.** $6.50. **11.** $8.80. **12.** 87 ft. **13.** 997.66 sq. ft.

Art. 227. — **1.** 800 sq. yd. **2.** 80 sq. rd. **3.** $26.25. **4.** 9 acres. **5.** $1000. **6.** $3840. **7.** 1 A. 118 sq. rd. 14.5 sq. yd. **8.** $4680.

Art. 228. — **1.** 785.4 ft. **2.** 235.62 ft. **3.** 238 rods. **4.** 58 ft. 4 in. **5.** 150 ft. **6.** 5 ft. **7.** 11 mi. 288 rd. **8.** 50 ft. **9.** 1 mile. **10.** 33 ft. 4 in. **11.** 28.56 rd. **12.** 20 ft.

Art. 229. — **1.** 176.715 sq. yd. **2.** 78.54 sq. rd. **3.** $392.70. **4.** $1570.80. **5.** 16 sq. rd. $6\frac{7}{9}$ sq. yd. **6.** 54 sq. yd. 4.875 sq. ft. **7.** 50 rd. **8.** 120 rd. **9.** 71.4 rd. **10.** 4 A. 81 sq. rd. $6\frac{5}{11}$ sq. yd. **11.** 62 ft. 6 in. **12.** 6 A. 78 sq. rd. 16.5 sq. yd. **13.** 12 A. 27 sq. rd. 8.25 sq. yd.

Art. 231. — **1.** 15 sq. ft. 90 sq. in. **2.** 7 sq. ft. **3.** 6 sq. ft. 56 sq. in. **4.** 12 sq. ft. 72 sq. in. **5.** 65 sq. ft. 64.8 sq. in. **6.** 34 sq. ft. 12.75 sq. in. **7.** 30 sq. ft. 78.24 sq. in.

Art. 232. — **1.** 324 cu. in. **2.** 2 cu. ft. **3.** 16 ft. 8 in. **4.** 20 cu. ft. 783 cu. in. **5.** 113 cu. ft. 1086 cu. in. **6.** 37 ft. 6 in.

Art. 233. — **1.** 15 sq. ft. **2.** 75 sq. ft. 36 sq. in. **3.** 100 sq. in. **4.** 13 sq. ft. 12.96 sq. in. **5.** 240 sq. ft. **6.** 19 sq. ft. 80 sq. in. **7.** 150 sq. ft. **8.** 490 sq. ft. 126 sq. in.

Art. 234. — **1.** 125 cu. yd. **2.** 3 cu. ft. **3.** 99 ft. **4.** 353.43 cu. ft. **5.** 1963.5 cu. ft. **6.** 8 ft. **7.** 96 cu. yd. **8.** 70 cu. yd. **9.** 641.41 cu. yd. **10.** 236 cu. yd. 12 cu. ft.

Art. 235. — **1.** 8 sq. ft. 104.64 sq. in. **2.** 44 sq. ft. 25.74 sq. in. **3.** $17.67. **4.** 181 sq. yd. 7 sq. ft. 36 sq. in. **5.** 144 globes.

Art. 236. — **1.** 8 cu. ft. 313.2 cu. in. **2.** 163 lb. 10 oz. **3.** 3375 marbles. **4.** 15 in. **5.** 4 cu. ft. 1269.25 cu. in. **6.** $13.09.

Art. 237. — **1.** $101.25. **2.** $105.60. **3.** 48 boxes. **4.** $22.50. **5.** $353.43. **6.** $217.50. **7.** $17.80. **8.** $13.09. **9.** $31.25. **10.** $99.77.

Art. 238. — **1.** 36 ft. **2.** $29.16. **3.** $92.40. **4.** 48 planks. **5.** $56\frac{1}{4}$ ft. **6.** $12.50.

Art. 239. — **1.** 120 perches. **2.** $66\frac{2}{3}$ perches. **3.** $180. **4.** $400. **5.** $90. **6.** $3780. **7.** $65. **8.** 220 ft.

Art. 240. — **1.** 56 bu. 1 pk. **2.** 675 bu. **3.** 450 bu. **4.** $90. **5.** 56.1 bu.; 522.24 gal. **6.** 48 bbl. **7.** 154 cu. ft; 1152 gal.; 123 bu. 3 pk.

Art. 241. — **1.** 4 dozen. **2.** G. C. D. $\frac{5}{6}$; L. C. M. 30. **3.** $16\frac{2}{3}$ sq. ft.

4. $32,031.25. **5.** Length 232 ft. 6 in.; value, $1743.75. **6.** 100, and 156.25. **7.** House, $3000; lot, $1125; stable, $675. **8.** 50 ft. 9 in. **9.** 12½%. **10.** 5. **11.** 16 inches. **12.** A, 16 birds; B, 24 birds. **13.** $5. **14.** $4380. **15.** .010077696. **16.** 52.36 lb. **17.** 2 hr. 24 min. P.M. **18.** 1 hr.; 4 hr.; 20 min. **19.** 15%. **20.** Length, 18 ft.; area, 1104 sq. ft. 67.5 sq. in. **21.** 7 hr. 20 min.; 62½ miles. **22.** 500 coins. **23.** 2. **24.** Area, $90\frac{5}{24}$ sq. yd.; altitude, 37½ ft. **25.** 90 ft. 9 in. **26.** $187.50. **27.** $3125. **28.** 1st, 160 rods; 2d, 128 rods; 3d, 160 rods. **29.** 12½%. **30.** Larger, 13,125 gal.; smaller, 9375 gal. **31.** $1230. **32.** Rate of rowing, 7½ mi.; rate of current, 1½ mi. **33.** 3071 sq. ft. **34.** 9.6. **35.** 105 sq. rd. 23¾ sq. yd. **36.** 3 hr. 30 min. 20 sec. P.M. **37.** 10 dozen. **38.** 1½. **39.** $10.80. **40.** Selling price, $1800; gain, 25%. **41.** 8 mo. **42.** 22.5. **43.** 225 ft. long, 100 ft. wide, 30 ft. deep. **44.** 99. **45.** Second, 200 rd.; fourth, 248 rd. **46.** Real, $1,875,000; personal, $375,000. **47.** 19%. **48.** $937.50. **49.** 84 ft. 6 in. **50.** 64. **51.** A, $2400; B, $2250; C, $2000. **52.** 13.5 sq. ft. **53.** 375 mi. **54.** $600 gain. **55.** 10 ft. **56.** 1,000,405. **57.** $4.50. **58.** 1st farm, $2400; 2d farm, $3600. **59.** Base, 30 rods; perp., 72 rods; hypot., 78 rods. **60.** 10 × 7 × 3 ft. **61.** .3. **62.** 27 ft.; 12 ft.; 40 ft. **63.** $298.60. **64.** 16,777,216. **65.** Son, $13,125; daughter, $7875; widow, $10,500. **66.** Dimensions, 16 × 48 × 72 in.; diagonal, 88 in. **67.** 5's, $3500; 4's, $4500. **68.** 40 times. **69.** $102.50. **70.** 39.27 cu. ft. **71.** $93.60. **72.** 49. **73.** 16×60×160 ft. **74.** Flour, $3.75; pork, $11. **75.** 63,525 tons. **76.** In 5's, $23,850; in 4's, $22,475; brokerage, $81.25. **77.** Surface, 37½ sq. ft.; vol., 8 cu. ft. 313.2 cu. in. **78.** 50 bbl. **79.** Side, 19.134 ft.; perp., 23.097 ft. **80.** 7. **81.** 81, 192, 375, and 648 lb. **82.** ⅓, 192, and 455⅓ lb. **83.** 5 pineapples. **84.** 8.1 acres. **85.** Prin., $937.50; rates, 7½%, 6⅔%. **86.** 160 tons. **87.** Face, $562.50; bank dis., $16.50. **88.** 18.75. **89.** 7½ acres; 45 horses. **90.** $2.50. **91.** 50 rods. **92.** 36 rods; $112.50. **93.** $3750. **94.** 66⅔ and 112.5. **95.** $362.50. **96.** 11 × 13.75 × 55 in.; 16.5 × 24.75 × 49.5 in.; 19.25 × 38.5 × 38.5 in.; 10.5 × 31.5 × 47.25 in.; 20.79×27.72 × 46.2 in.

Important New Books

Crockett's Plane and Spherical Trigonometry

By C. W. CROCKETT, C.E., Professor of Mathematics and Astronomy in Rensselaer Polytechnic Institute, Troy, New York. With Tables. Cloth, 8vo. 310 pages . . $1.25

The Same. Without Tables 1.00

Logarithmic and Trigonometric Tables (separate) . . 1.00

A clear analytic treatment of the elements of Plane and Spherical Trigonometry and their practical applications to Surveying, Geodesy, and Astronomy, with convenient and accurate "five place" tables for the use of the student, engineer, and surveyor. Designed for High Schools, Colleges, and Technical Institutions.

Raymond's Plane Surveying

By W. G. RAYMOND, C.E., Member American Society of Civil Engineers, Professor of Geodesy, Road Engineering, and Topographical Drawing in Rensselaer Polytechnic Institute.

Cloth, 8vo. 485 pages. With Tables and Illustrations . $3.00

A modern text-book for the study and practice of Land, Topographical, Hydrographical, and Mine Surveying. Special attention is given to such practical subjects as system in office work, to labor-saving devices, to coördinate methods, and to the explanation of difficulties encountered by young surveyors. The appendix contains a large number of original problems, and a full set of tables for class and field work.

Todd's New Astronomy

By DAVID P. TODD, M.A., Ph.D., Professor of Astronomy and Director of the Observatory, Amherst College.

Cloth, 12mo. 500 pages. Illustrated $1.30

A new Astronomy designed for classes pursuing the study in High Schools, Academies, and other Preparatory Schools. The treatment throughout is simple, clear, scientific, and deeply interesting. The illustrations include sketches from the author's laboratory and expeditions, and numerous reproductions from astronomical photographs.

Copies of the above books will be sent, prepaid, to any address on receipt of the price by the Publishers:

American Book Company

New York • Cincinnati • Chicago

Eclectic School Readings

A Collection of Choice, Interesting, and Instructive Books, Handsomely Bound in Cloth.

Folk Stories

Lane's Stories for Children	$0.25
Baldwin's Fairy Stories and Fables	.35
Baldwin's Fifty Famous Stories Retold	.35
Baldwin's Old Stories of the East	.45
Baldwin's Old Greek Stories	.45
Bradish's Old Norse Stories	.45
Pyle's Prose and Verse for Children	.40

Famous Stories

Clarke's Stories from the Arabian Nights	.60
Defoe's Robinson Crusoe (Stephens)	.50
Dickens's Tale of Two Cities (Kirk)	.50
Scott's Kenilworth (Norris)	.50
Scott's Quentin Durward (Norris)	.50
Scott's Talisman (Dewey)	.50

Geographical Stories

Krout's Alice's Visit to the Hawaiian Islands	.45
Shaw's Big People and Little People of Other Lands	.30
Shaw's Discoverers and Explorers	.35

Historical Stories

Eggleston's Stories of Great Americans	.40
Eggleston's Stories of American Life and Adventure	.50
Guerber's Story of the Thirteen Colonies	.65
Guerber's Story of the Great Republic	.65
Guerber's Story of the English	.65
Guerber's Story of the Chosen People	.60
Guerber's Story of the Greeks	.60
Guerber's Story of the Romans	.60

Classical Stories

Clarke's Story of Troy	.60
Clarke's Story of Ulysses	.60
Clarke's Story of Aeneas	.45
Clarke's Story of Caesar	.45

Natural History Stories

Holder's Stories of Animal Life	.60
Kelly's Short Stories of Our Shy Neighbors	.50
Dana's Plants and Their Children	.65
Needham's Outdoor Studies	.40

Patriotic and Moral Readings

Markwick and Smith's True Citizen	.60
Persons's Our Country in Poem and Prose	.50

Copies sent, prepaid, on receipt of the price.

American Book Company

New York • Cincinnati • Chicago

Elementary English

FOR BEGINNERS IN THE STUDY OF LANGUAGE AND COMPOSITION.

LONG'S NEW LANGUAGE EXERCISES. Part I. . . 20 cents
LONG'S NEW LANGUAGE EXERCISES. Part II. . . 25 cents
LONG'S LESSONS IN ENGLISH (Grammar and Composition) 35 cents

A popular and carefully graded series, based on actual schoolroom work. Part I. for first and second years. Part II. for third and fourth year grades. The "Lessons in English" occupies the place of a primary Grammar and Composition.

LYTE'S ELEMENTARY ENGLISH . . . . . 35 cents

For primary and lower grammar grades. Divided into three parts, each representing a year's work.

MAXWELL'S FIRST BOOK IN ENGLISH . . . . 40 cents

Provides instruction for a three years' course in Language and Composition. Includes lessons, practice, and instruction in the elementary principles of the English language.

METCALF AND BRIGHT'S LANGUAGE LESSONS. Part I. 35 cents
METCALF AND BRIGHT'S LANGUAGE LESSONS. Part II. 55 cents

A graded series of lessons intended to cover the course in language in primary and intermediate grades. A study of literary examples is a marked feature of the second book.

SWINTON'S LANGUAGE PRIMER . . . . . 28 cents
SWINTON'S LANGUAGE LESSONS . . . . . 38 cents

The Primer, or Beginner's Lessons in Speaking and Writing, is designed for use in primary grades. The Language Lessons furnishes material for elementary instruction in Grammar and Composition.

Language Tablets and Blanks

NATIONAL LANGUAGE TABLETS . . . Per dozen, 90 cents

PATTERSON'S COMPOSITION BOOKS

No. 1.	Flexible.	36 pages . . .	Per dozen, 96 cents
No. 2.	Boards.	60 pages . . .	Per dozen, $1.80
No. 3.	Cloth.	84 pages . . .	Per dozen, 2 70
No. 4.	Extra.	108 pages . . .	Per dozen, 3 60

WARD'S GRAMMAR BLANKS. 2 Nos. . . Per dozen, 90 cents

Copies sent, prepaid, to any address on receipt of price.

American Book Company

New York • Cincinnati • Chicago

Overton's Applied Physiology

By FRANK OVERTON, A.M., M.D.
Late House Surgeon in the City Hospital, New York City.

OVERTON'S APPLIED PHYSIOLOGY—Primary . . . 30 cents
OVERTON'S APPLIED PHYSIOLOGY—Intermediate . . 50 cents
OVERTON'S APPLIED PHYSIOLOGY—Advanced . . 80 cents

These books form a complete series for the study of physiology in all grades. The Primary Book follows a natural order of treatment and presents the elementary facts and principles of physiology in so simple a way as to bring them within the comprehension of children. The Intermediate Book is designed to be a complete elementary text-book in itself and an introduction to the advanced study of anatomy and physiology. The Advanced Physiology is designed to meet the requirements of teachers and schools for a text-book of Physiology and Hygiene for Higher Grades, which should combine the latest results of study and research in biological, medical, and chemical science and the best pedagogical methods of science teaching. The book represents a new and radical departure from the old-time methods pursued in the teaching of physiology.

The fundamental principle of the series is that the study of anatomy and physiology should be the study of the *cell*, from the most elementary structure in organic life, to its highest and most complex forms in the human body. The effects of alcohol and other stimulants and narcotics are treated very fully in each book of the series.

Illustrations and outline diagrams are inserted wherever needed to explain the text.

Copies of Overton's Physiologies will be sent, prepaid, to any address on receipt of the price.

American Book Company

New York • Cincinnati • Chicago

(150)

Supplementary Reading

FOR ELEMENTARY GRADES

FAMOUS STORIES

FIFTY FAMOUS STORIES RETOLD. By JAMES BALDWIN 35 cents

This is a collection of the most famous tales of ancient and modern times; some historical and some legendary. Among them are the stories of King Alfred and the Cakes, Robin Hood, Bruce and the Spider, George Washington and his Hatchet, William Tell, How Napoleon Crossed the Alps, and other favorite stories for the young. The stories are told in a form that is easily intelligible to children, yet in no way distorted from the established versions.

OLD STORIES OF THE EAST. By JAMES BALDWIN . . 45 cents

The stories contained in this book are taken from the Hebrew Scriptures, and are among the most famous and most intensely interesting that the world has ever known. While it is neither possible nor desirable to omit allusions to their Scriptural origin, great care has been taken not to trespass on the domain of the religious teacher.

OLD GREEK STORIES. By JAMES BALDWIN . . . 45 cents

No other stories have been told more often or listened to with more pleasure than these classic tales of ancient Greece. They have become so incorporated into our own language and thought and so interwoven with our literature that they form almost as important a part of our intellectual life as they did that of the Greeks.

DEFOE'S ROBINSON CRUSOE

Adapted to School Purposes by KATE STEPHENS . . 50 cents

From the date of its first appearance this has been one of the most widely read books in English fiction. It has here been made suitable for school and supplementary reading, but the editor has taken great care that the essential features of the work should in no way be changed.

STORIES FROM THE ARABIAN NIGHTS

Edited by M. CLARKE . . . . . 60 cents

Of all the famous stories handed down to us from the past none has afforded more entertainment and delight to young and old of every land than these. Although the oldest stories in existence they are as fresh and interesting to-day as when they were first told. The best of these are here related with such skill as to preserve all their original charm and attractiveness, while adapting them in form and expression for school and home reading.

Copies of any of these books will be sent, prepaid, to any address on receipt of the price.

American Book Company

New York • Cincinnati • Chicago

The Natural Course in Music

BY

FREDERIC H. RIPLEY AND THOMAS TAPPER

NATURAL MUSIC READERS

Natural Music Primer	$0.30
Natural Music Reader, No. 1	.30
Natural Music Reader, No. 2	.35
Natural Music Reader, No. 3	.35
Natural Music Reader, No. 4	.35
Natural Music Reader, No. 5	.50
Natural Advanced Music Reader	1.00

NATURAL MUSIC CHARTS

In seven series, A, B, C, D, E, F, and G, comprising 110 charts, substantially bound and mounted on iron tripod supporter, each series, per set $4.00

The Natural Course in Music consists of a carefully prepared series of books and charts, reasonable in price and designed to cover the whole graded course in Primary and Grammar Schools. The plan contemplates the constant use of charts, at first independent of the books, and afterwards as a systematic preparation for the lessons to be learned from day to day in the music readers.

NATURAL MUSIC SERIES—SHORT COURSE

BOOK I.	For Elementary Grades	35 cents
BOOK II.	For Advanced Grades	40 cents

The Short Course in Music is embraced in two books and is designed for use in graded or ungraded schools in which a more complete course is deemed unnecessary or impracticable. In both books familiar and standard songs are made the basis of elementary music instruction. The Short Course embraces over 200 such songs—sacred, patriotic, occasional, and miscellaneous. Throughout, the teaching is constantly applied to the compositions of the best song writers. Exercises in two and three parts in simple form are included in the course. Few definitions are given, but a brief summary of the Elements of Music is included in an appendix. The books are illustrated by numerous portraits of authors and composers.

The books and charts of the Natural Music Course will be sent, prepaid, to any address on receipt of the price by the Publishers:

American Book Company

New York • Cincinnati • Chicago

Books for Teachers

FOR THE STUDY OF PEDAGOGY

Aiken's Methods of Mind-Training	$1.00
Aiken's Exercises in Mind-Training	1.00
Alling-Aber's Experiment in Education	1.25
Hailmann's History of Pedagogy	.60
Hewett's Pedagogy for Young Teachers	.85
Hinsdale's The Art of Study	1.00
King's School Interests and Duties	1.00
Mann's School Recreations and Amusements	1.00
Page's Theory and Practice of Teaching	1.00
Palmer's Science of Education	1.00
Payne's School Supervision	1.00
Roark's Method in Education	1.00
Roark's Psychology in Education	1.00
Seeley's History of Education	1.25
Shoup's History and Science of Education	1.00
Swett's American Public Schools	1.00
White's Elements of Pedagogy	1.00
White's School Management	1.00
Putnam's Text-Book of Psychology	1.00

Copies sent, prepaid, to any address on receipt of the price.

American Book Company

New York • Cincinnati • Chicago

For Teachers and School Officers

KING'S SCHOOL INTERESTS AND DUTIES

Developed from "Page's Mutual Duties of Parents and Teachers," from various Public Records and Documents, and from the Bulletins of the National Bureau of Education. By ROBERT M. KING.

Cloth, 12mo, 336 pages $1.00

This new work, original in its scope and plan, presents in one volume interesting and valuable expositions of the modern demands, best methods, and most important interests of our Public School Systems. Its central idea is to show the importance and value of co-operation in school work and the mutual duties of teachers, school officers, and parents. It also embodies synopses of the discussions on leading educational topics from the various fugitive reports and manuals issued, from time to time, by school officials and State Departments of Education. It will be found an invaluable manual and guide for school superintendents, officers, and patrons, and, indeed, for every one interested in educational work.

MANN'S SCHOOL RECREATIONS AND AMUSEMENTS

By CHARLES W. MANN, A.M., Dean of the Chicago Academy.

Cloth, 12mo, 352 pages $1.00

This volume not only opens up a new field of much needed information and direction in the matter of physical training of pupils, but also furnishes suggestions for intellectual recreations which will greatly add to the interest and value of school work and lend a charm to school life in all its phases. Some of the subjects treated in this work are: Morning Exercises, Care and Equipment of Schoolrooms, Singing Games and Songs, Indoor Exercises and Outdoor Games, Experiments in Physics and Chemistry, Recreations in Latin, Outline for Reading Circles, etc.

Copies of the above books will be sent, prepaid, to any address on receipt of the price by the Publishers:

American Book Company

New York • Cincinnati • Chicago


Acme Library Card Pocket
Under Pat. "Ref. Index File"
Made by Library Bureau
530 Atlantic Ave., Boston
Keep Your Card in this Pocket

